Guru Aanklan

Combined Paper	Std. X - Algebra Chapter 1 and 3	Solutions

Q.1 Attempt any four of the following :

(1) $t_n = n^2 + n$

For $n = 2,\ t_2 = 2^2 + 2$ **...[½ M]**

$t_2 = 4 + 2 = 6$

For $n = 3,\ t_3 = 3^2 + 3$ **...[½ M]**

$t_3 = 9 + 2 = 12$

$\therefore t_2 = 6$ and $t_3 = 12$

(2) $x - 4x^2 + 5 = 0$

$\therefore 4x^2 - x - 5 = 0$ **...[½ M]**

Comparing with $ax^2 + bx + c = 0$

$a = 4,\ b = -1, c = -5$ **...[½ M]**

(3) Find the next two terms of the sequence $192, -96, 48, -24 ....$

$t_5 = \frac{t_4}{-2}$

$t_5 = \frac{-24}{-2} = 12$ **...[½ M]**

$t_6 = \frac{t_5}{-2}$

$t_6 = \frac{12}{-2} = -6$ **...[½ M]**

$\therefore$ Next two terms are $t_5 = 12$ and $t_6 = -6$

(4) $(P - 4)P = 0$

$\therefore P^2 - 4P = 0$ **...[½ M]**

$\therefore$ Degree of the equation is 2 and $a \neq 0$

Hence it is a quadratic equation **...[½ M]**

(5) If $a = 2.5$ and $d = 1.5$

$\therefore$ First term $= a = 2.5$

Second term $= t_2 = a + d = 2.5 + 1.5$

$t_2 = 4$ ...[½ M]

Third term $= t_3 = t_2 + d = 4 + 2.5$

$t_3 = 6.5$ ...[½ M]

Q.2 Attempt any three of the following :

1) The given A.P. $1, 7, 13, 19 \ldots\ldots..$

First term = a = 1

Common difference = 7 - 1 = 6 ...[½ M]

$t_n = a + (n-1)d$ ...[½ M]

$t_{18} = 1 + (18-1) \times 6$

$t_{18} = 1 + 17 \times 6$ ...[½ M]

$t_{18} = 1 + 102 = 103$ ...[½ M]

Eighteen term of the given A.P. is 103

(2) Let the three consecutive terms in A.P. be

a - d, a, a + d ...[½ M]

According to the condition

$\therefore\ a - d + a + a + d = 27$

$\therefore\ 3a = 27$

$\therefore\ a = 9$ ...[½ M]

$a(a-d)(a+d) = 504$

$(9-d)(9+d) = \frac{504}{9}$ ...[½ M]

$9^2 - d^2 = \frac{504}{9}$

$81 - d^2 = 56$

$d^2 = 81 - 56$

$d^2 = 25$

$d = \pm 5$ ...[½ M]

The three terms are 14, 9, 4 OR 4, 9, 14 ...[½ M]

(3) One root of the quadratic equation is 4

Hence it satisfies the equation ...[½ M]

$\therefore$ Substitute $x = 4$ in $x^2 - 7x + k = 0$

$\therefore (4)^2 - 7 \times 4 + k = 0$ ...[½ M]

$\therefore\ 16 - 28 + k = 0$ ...[½ M]

$\therefore\ -12 + k = 0$

$\therefore k = 12$ ...[½ M]

Hence value of k is 12

(4) The given equation is $2x^2 + 5\sqrt{3}x + 16 = 0$

Compare it with $ax^2 + bx + c = 0$ ...[½ M]

$a = 2,\ b = 5\sqrt{3}, c = 16$

$\Delta = b^2 - 4ac$

$\Delta = \left(5\sqrt{3}\right)^2 - 4 \times 2 \times 16$ ...[½ M]

$\Delta 25 \times 3 - 128$

$\Delta = 75 - 128$ ...[½ M]

$\Delta = -53$

$\therefore\ \Delta < 0$ ...[½ M]

$\therefore$ No real roots for the equation.

Q.3 Attempt any Two of the following :

(1) The given equation is $3y^2 + 7y + 1 = 0$

Divide throughout by 3

$3y^2 + 7y + 1 = 0$

$y^2 + \frac{7}{3}y = -\frac{1}{3}$ ...(1) ...[½ M]

$\therefore$ Third term $= \left(\frac{1}{2} \times \text{co} - \text{efficient of } y\right)^2$

$= \left(\frac{1}{2} \times \frac{7}{3}\right)^2$

$= \left(\frac{7}{6}\right)^2$

$= \frac{49}{36}$ ...[½ M]

Adding this on both sides of equation (1)

$$y^2 + \frac{7}{3}y + \frac{49}{36} = -\frac{1}{3} + \frac{49}{36}$$

$$\left(y + \frac{7}{6}\right)^2 = \frac{-12 + 49}{36}$$

$$\left(y + \frac{7}{6}\right)^2 = \frac{37}{36}$$ ...[½ M]

$$\therefore\ y + \frac{7}{6} = \pm\sqrt{\frac{37}{36}}$$

$$\therefore\ y = -\frac{7}{6} + \sqrt{\frac{37}{36}} \text{ OR } y = -\frac{7}{6} - \sqrt{\frac{37}{36}}$$

$$\therefore\ y = \frac{-7 + \sqrt{37}}{6} \text{ OR } y = \frac{-7 - \sqrt{37}}{6}$$ ...[½ M]

$$\therefore \text{Solution set } \left\{\left(\frac{-7 + \sqrt{37}}{6}, \frac{-7 - \sqrt{37}}{6}\right)\right\}$$ ...[½ M]

2) Number of rows in the meeting hall are

20, 24, 28…… ...[½ M]

which is in A.P. with

First term = a = 20

Common difference = d = 24-20 = 4 ...[½ M]

Hall has 30 rows $\therefore\ n = 30$

To find the total number of seats in the hall

i.e. S_{30}

$$\therefore\ S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$ ...[½ M]

$$\therefore S_{30} = \frac{30}{2}\left[2 \times 20 + (30-1) \times 4\right]$$

$$\therefore\ S_{30} = 15\left[40 + (29) \times 4\right]$$ ...[½ M]

$$\therefore S_{30} = 15\left[40 + 116\right]$$

$$\therefore\ S_{30} = 15\left[156\right]$$ ...[½ M]

$$\therefore S_{30} = 2340$$ ...[½ M]

Hence total number of seats in the hall are 2340

(3) Let the breadth of the rectangle be 'x' cm

Hence length of the rectangle is 'x+2' cm ...[½ M]

According to the condition

$\ell \times b = \text{area of rectangle}$ ...[½ M]

$x \times (x+2) = 24$

$x^2 = 2x - 24 = 0$

$x^2 + 6x - 4x - 24 = 0$ ...[½ M]

$x \times (x+6) - 4 \times (x+6) = 0$

$(x+6)(x-4) = 0$ ...[½ M]

$x + 6 = 0$ or $x - 4 = 0$

$\therefore x = -6$ or $x = 4$ ...[½ M]

But the length cannot be negative $\therefore x = -6$ is not acceptable

Hence breadth of the rectangle is 4cms ...[½ M]

Hence length of the rectangle is $x + 2 = 4 + 2 = 6$cms

Q.4 Attempt any one of the following :

(1) $2\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) + 14 = 0 \qquad ...(1)$

Let $\left(x + \frac{1}{x}\right) = m$

$\therefore \left(x^2 + \frac{1}{x^2}\right) = m^2 - 2$ ...[½ M]

Substitute in equation (1)

$2(m^2 - 2) - 9(m) + 14 = 0$

$2m^2 - 4 - 9m + 14 = 0$ ...[½ M]

$2m^2 - 9m + 10 = 0$

$2m^2 - 4m - 5m + 10 = 0$

$2m \times (m-2) - 5 \times (m-2) = 0$

$(2m-5)(m-2) = 0$ ...[½ M]

$\therefore 2m - 5 = 0$ OR $m - 2 = 0$

$\therefore 2m = 5$

$\therefore m = \frac{5}{2}$ or $m = 2$ ...[½ M]

Re-substitute $\left(x+\frac{1}{x}\right)=m$

$x+\frac{1}{x}=\frac{5}{2}$ OR $x+\frac{1}{x}=2$ **...[½ M]**

Considering $x+\frac{1}{x}=\frac{5}{2}$

Multiplying throughout by 2x

$2x^2+2=5x$

$\therefore 2x^2-5x+2=0$

$\therefore 2x^2-4x-x+2=0$

$\therefore 2x\times(x-2)-1\times(x-2)=0$

$\therefore (x-2)(2x-1)=0$

$\therefore x-2=0$ OR $2x-1=0$ **...[½ M]**

$\therefore x=2$ OR $x=\frac{1}{2}$

Considering $x+\frac{1}{x}=2$

Multiplying x on both sides

$x^2+1=2x$

$\therefore x^2-2x+1=0$

$\therefore (x-1)^2=0$ **...[½ M]**

$\therefore x-1=0$

$\therefore x=1$

$\therefore$ Solution set $=\left\{1,2\frac{1}{2}\right\}$ **...[½ M]**

(2) $t_{11}=16$ and $t_{21}=29$

Let the first term = a, common difference = d

$t_n=\left[a+(n-1)d\right]$ **...[½ M]**

$t_{11}=\left[a+(11-1)d\right]$

$16=\left[a+10d\right]$ **...[½ M]**

$a + 10d = 16$ .... (1)

$t_{21} = [a + (21 - 1)d]$

$29 = [a + 20d]$

$a + 20d = 29$ ....(2) **...[½ M]**

Substracting (1) from (2)

$10d = 13$

$\therefore\ d = \frac{13}{10} = 1.3$ **...[½ M]**

Substitute in equation (1)

$a + 10 \times \frac{13}{10} = 16$

$a + 13 = 16$

$a = 16 - 13 = 3$

$\therefore\ a = 3$ **...[½ M]**

$t_n = [a + (n - 1)d]$

$\therefore\ t_{34} = [3 + (34 - 1) \times 1.3]$

$\therefore\ t_{34} = [3 + (33) \times 1.3]$

$\therefore\ t_{34} = [3 + 42.9]$

$\therefore\ t_{34} = 45.9$ **...[½ M]**

n such that $t_n = 55$

$t_n = [a + (n - 1)d]$

$\therefore\ 55 = [3 + (n - 1) \times 1.3]$

$\therefore\ 55 - 3 = [(n - 1) \times 1.3]$

$\therefore\ 52 = [(n - 1) \times 1.3]$

$\therefore\ \frac{52}{1.3} = (n - 1)$

$\therefore\ 40 = n - 1$

$\therefore\ n = 40 + 1$

$\therefore\ n = 41$ **...[½ M]**

First term $= a = 3$

Common difference $= d = 1.3$

$t_{34} = 45.9$

n such that $t_n = 55 = 41$ **...[½ M]**