JEE MAIN

Class XI

1 Kinematics

- **Scalars and Vectors, Vector addition and subtraction, zero vectors, scalar and vector products, unit vector, resolution of a vector.**
- **Frame of reference, Motion in straight line : Position-time graph, speed and velocity.**
- **Uniform and non-uniform motion, average speed and instantaneous velocity. Uniformly accelerated motion, velocity-time, position-time graphs, relations for uniformly accelerated motion.**
- **Relative Velocity, Motion in a plane, Projectile motion, Uniform circular motion.**

SCALARS, VECTORS AND TENSORS

- Scalars are those quantities which have only magnitudes but no direction. For example mass, length, time, speed, work, temperature, current etc.
- Vectors are those quantities which have magnitude as well as direction. For example displacement, velocity, acceleration, force, momentum, etc.
- A physical quantity which has different values in different directions at the same point is called a **tensor.**
- Pressure, stress, moduli of elasticity, moment of inertia, radius of gyration, refractive index, wave velocity, dielectric constant, conductivity, resistivity and density are a few examples of tensor. Magnitude of tensor is not unique.

> **Note** **Scalars vectors, matrices can be expressed in the form of tensors like scalars are zero order tensor vectors are first order tensors and matrices are second order tensors.**

Types of Vectors

- Graphically a vector A is represented by a directed segment of a straight line, whose direction is that of the vector it represents and whose length corresponds to the magnitude $|\vec{A}|$ of $\vec{A}$.
- **A unit vector** of a given vector $\vec{A}$ is a vector of unit magnitude and has the same direction as that of the given vector. A unit vector of $\vec{A}$ is written as $\hat{A}$, where $\hat{A} = \vec{A}/|\vec{A}|$. A unit vector is unitless and dimensionless vector and represents direction only.

  For example,
  If the velocity of an object is 15 m s^{-1} due east
  Its unit vector

  $$\vec{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{15\,\text{m s}^{-1}\text{due east}}{15}$$

  $$\hat{v} = 1\text{ m s}^{-1}\text{due east}$$

  So, unit vector has same direction but unit magnitude.
- The symbol $\hat{i}, \hat{j}, \hat{k}$ represent unit vectors of x, y and z directions of coordinate axes respectively.
- **Null vector** is a vector which has zero magnitude and an arbitrary direction. It is represented by $\vec{0}$ and is also known as zero vector. Velocity of a stationary object, acceleration of an object moving with uniform velocity and resultant of two equal and opposite vectors are the examples of null vector.

- **Equal vectors :** Two vectors are said to be equal if they have equal magnitude and same direction.
- **A negative vector** of a given vector is a vector of same magnitude but acting in a direction opposite to that of the given vector. The negative vector of $\vec{A}$ is represented by $-\vec{A}$.
- A vector whose initial point is fixed is called a ***localised vector*** or fixed vector and whose initial point is not fixed is called ***non-localised vector*** or free vector.
- **Collinear vectors :** Vectors having equal or unequal magnitude but drawn in the same direction are called collinear vectors. Two colinear vectors having the same direction (θ = 0°) are called like or parallel vectors. Two collinear vectors having the opposite directions are called unlike or antiparallel vectors.

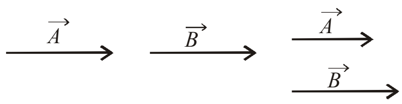


Fig. Like vectors

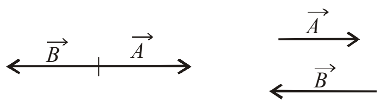


Fig. Unlike vectors

- **Co-initial vectors :** The vectors which have the same initial point are called co-initial vectors. In figure, $\vec{A}, \vec{B}$ and $\vec{C}$ are co-initial vectors.

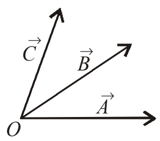
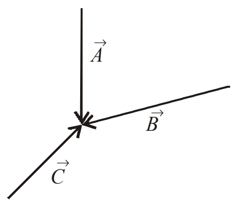
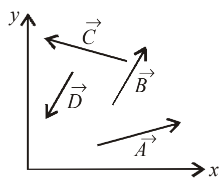


Fig. Co-initial vectors.

- **Co-terminus vectors :** The vectors which have the same terminal point are called co-terminus vector. In given figure $\vec{A}, \vec{B}$ and $\vec{C}$ are co-terminus vectors.



- **Co-planar vectors :** All the vectors in the same plane are called co-planar vectors.



As all the vectors shown in figure are in *xy* plane.

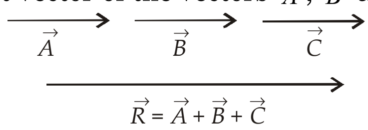
ADDITION AND SUBTRACTION OF VECTORS

Linear Addition

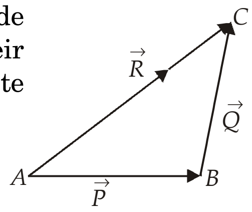
- The resultant of a number of vectors is a single vector which would produce the same effect as all the original vectors put together.

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

where $\vec{R}$ is the sum or resultant vector of the vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$.



Triangle Law of Vector Addition



Triangle law

- If two vectors acting simultaneously at a point are represented in magnitude and direction by two sides of a triangle taken in the same order, then their resultant is represented by the third side of the triangle taken in the opposite order.

  According to triangle law of vector addition.

  $\vec{P}+\vec{Q} = \vec{R}$ or $\overrightarrow{AB}+\overrightarrow{BC} = \overrightarrow{AC}$

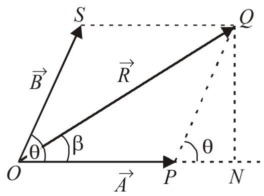
  or $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA} = \overrightarrow{AC}+\overrightarrow{CA} = \overrightarrow{AC}-\overrightarrow{AC} = 0$

  Therefore, if three vectors acting simultaneously at a point be represented both in magnitude and direction by three sides of a triangle, taken in order, the resultant will be zero.

Parallelogram Law of Vectors

- It states that if two vectors acting on a particle at the same time be represented in magnitude and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant vector is represented in magnitude and direction by the diagonal of the parallelogram drawn from the same point, (See figure).

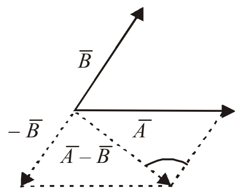
> **Note** **Resultant, R is maximum if two vectors A and B having same direction (*i.e* $\theta = 0°$). If two vectors A and B in opposite direction(*i.e* $\theta = 180°$) then R will be minimum.**



  If $\vec{R}$ is the resultant of $\vec{A}$ and $\vec{B}$ and β is angle between resultant vector $\vec{R}$ and $\vec{A}$ vector then,

  $R=\sqrt{A^2+B^2+2AB\cos\theta}$ and $\tan\beta=\dfrac{B\sin\theta}{A+B\cos\theta}$

Subtraction of Vectors



- Subtraction of a vector $\vec{B}$ from a vector $\vec{A}$ is defined as the addition of vector $-\vec{B}$ (negative of vector $\vec{B}$) to vector $\vec{A}$.

  Thus, $\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

  $\vec{A}-\vec{B}\neq\vec{B}-\vec{A}$

Properties of Vector Addition

If $\vec{A}+\vec{B}=\vec{B}+\vec{A}$ (commutative)

$(\vec{A}+\vec{B})+\vec{C}=\vec{A}+(\vec{B}+\vec{C})$ (Associativity)

$\lambda(\vec{A}+\vec{B})=(\lambda\vec{A}+\lambda\vec{B})$ (Distributive)

REPRESENTATION OF VECTOR BY CO-ORDINATES

- In a three dimensional orthogonal rectangular coordinate system, the unit vectors along x, y and z are denoted by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively. If the point P has a co-ordinate (x, y, z), then the figure A the vector $\overrightarrow{OP}$ (known as position vector of P) is denoted by $\overrightarrow{OP} = x\hat{i}+y\hat{j}+z\hat{k}$.

  In figure B, $\overrightarrow{P_1P_2}$ is known as displacement vector.

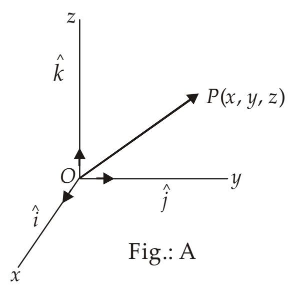


Fig.: A

  Now, $\overrightarrow{OP_1}+\overrightarrow{P_1P_2} = \overrightarrow{OP_2}$

  $\therefore \quad \overrightarrow{P_1P_2} = \overrightarrow{OP_2}-\overrightarrow{OP_1}$

  $\overrightarrow{OP_1}$ (position vector of P_1) = $x_1\hat{i}+y_1\hat{j}+z_1\hat{k}$

  $\overrightarrow{OP_2}$ (position vector of P_2) = $x_2\hat{i}+y_2\hat{j}+z_2\hat{k}$

  $\therefore \quad \overrightarrow{P_1P_2} = x_2\hat{i}+y_2\hat{j}+z_2\hat{k}-(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})$

  $= (x_2-x_1)\hat{i}+(y_2-y_1)\hat{j}+(z_2-z_1)\hat{k} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k}.$

So any vector $\vec{A}$ can be represented as

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Now, magnitude of $\vec{A}$

$= |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\therefore$ Unit vector along $\vec{A} = \hat{a} = \dfrac{\vec{A}}{|\vec{A}|}$

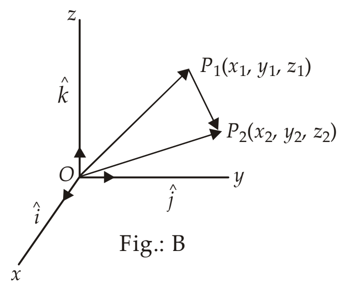


Fig.: B

or $\quad \hat{a} = \dfrac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{|\vec{A}|} = \dfrac{A_x}{|\vec{A}|}\hat{i} + \dfrac{A_y}{|\vec{A}|}\hat{j} + \dfrac{A_z}{|\vec{A}|}\hat{k}$

or $\quad \hat{a} = \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$

where, $\cos\alpha = \dfrac{A_x}{|\vec{A}|} = \dfrac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$, $\cos\beta = \dfrac{A_y}{|\vec{A}|} = \dfrac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$, $\cos\gamma = \dfrac{A_z}{|\vec{A}|} = \dfrac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$

Where α, β and γ are the angles made by the vector $\vec{A}$ with the x, y and z axes respectively. cos α, cos β and cos γ are known as direction cosines of the vector $\vec{A}$..

MULTIPLICATION OF A VECTOR BY A SCALAR

- When a vector $\vec{A}$ is multiplied by a scalar s, it becomes a vector $s\vec{A}$, whose magnitude is s times the magnitude of $\vec{A}$ and it acts along the direction of $\vec{A}$. The unit of $s\vec{A}$ may be different from the unit of vector $\vec{A}$.

  If $\quad \vec{A} = A_x\hat{i} + A_y\hat{i} + A_z\hat{k}$

  Then $s\vec{A} = sA_x\hat{i} + sA_y\hat{j} + sA_z\hat{k}$

Dot or scalar product of two vectors

- The dot product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\cdot\vec{B}$ and is given by $\vec{A}\cdot\vec{B} = AB\cos\theta$, where θ is angle between $\vec{A}$ and $\vec{B}$. The dot product of two vectors is a scalar.

Geometrical interpretation of dot product of two vectors

- It is the product of the magnitude of one vector with the magnitude of the component of other vector in the direction of first vector.
- $\vec{A}\cdot\vec{A} = A^2$ or $A = (\vec{A}\cdot\vec{A})^{1/2}$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$ and $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

  $\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = A_xB_x + A_yB_y + A_zB_z.$

  $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

ILLUSTRATION

1 *If* $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{b} = -\hat{i} + 2\hat{j} - \hat{k}$

then $\vec{a}\cdot\vec{b}$ equals what?

Soln.: $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{b} = -1\hat{i} + 2\hat{j} - 1\hat{k}$

$\vec{a}\cdot\vec{b} = (2)(-1) + (-3)(2) + (4)(-1)$

$= -2 - 6 - 4 = -12.$

Cross Product or Vector Product of two Vectors

- The cross product of two vectors $\vec{A}$ and $\vec{B}$ is denoted by $\vec{A}\times\vec{B}$. It is a vector whose magnitude is equal to the product of the magnitudes of the two vectors and sine of the smaller angle between them. If θ is smaller angle between $\vec{A}$ and $\vec{B}$, then $\vec{A}\times\vec{B} = \vec{C} = AB\sin\theta\,\hat{n}$, where $\hat{n}$ is a unit vector in the direction of $\vec{C}$.

- **Geometrical interpretation of vector product of two vectors**
  The magnitude of vector product of two vectors is equal.

(i) to the area of the parallelogram whose two sides are represented by two vectors.
(ii) to twice the area of a triangle whose two sides are represented by the two vectors.

Physical Examples of Vector Product

(i) **Torque $\vec{\tau}$:** The torque acting on a particle is equal to the vector product of its position vector $(\vec{r})$ and force vector $(\vec{F})$. Thus $\vec{\tau} = \vec{r} \times \vec{F}$

(ii) **Angular Momentum $\vec{L}$:** The angular momentum of a particle is equal to the cross product of its position vector $(\vec{r})$ and linear momentum $(\vec{p})$. Thus $\vec{L} = \vec{r} \times \vec{p}$.

(iii) **Instantaneous Velocity $\vec{v}$:** The instantaneous velocity of a particle is equal to the cross product of its angular velocity $(\vec{\omega})$ and the position vector $(\vec{r})$. Thus $\vec{v} = \vec{\omega} \times \vec{r}$.

Properties of cross product

– Cross product of two parallel vectors is zero $(i.e.\ \vec{A} \times \vec{A}) = 0$.

(i) $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

(ii) $-\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j}$

Cross product in cartesian co-ordinates

$$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Direction of Vector Cross Product

- When $\vec{C} = \vec{A} \times \vec{B}$, the direction of $\vec{C}$ is at right angles to the plane containing the vectors $\vec{A}$ and $\vec{B}$. The **direction** is determined by the right hand screw rule and the right hand thumb rule.
- **Right hand screw rule :** Rotate a right handed screw from first vector $(\vec{A})$ towards second vector $(\vec{B})$. The direction in which the right handed screw moves gives the direction of vector $\vec{C}$ (Fig. 1).
- **Right hand thumb rule :** Curl the fingers of your right hand from $\vec{A}$ to $\vec{B}$. Then the direction of the erect right thumb will point in the direction of $\vec{A} \times \vec{B}$ (Fig. 2).

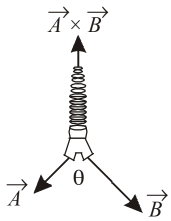


Fig. 1

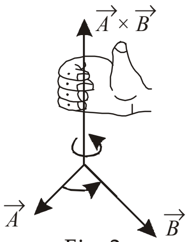


Fig. 2

ILLUSTRATION

(2) If $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + \hat{j} - 2\hat{k}$, then find $\vec{p} \times \vec{q}$ *and* $|\vec{p} \times \vec{q}|$.

Soln.: $\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 1 & -2 \end{vmatrix}$

$= \hat{i}\,[(3)(-2) - (1)(1)] - \hat{j}\,[(2)(-2) - (1)(1)] + \hat{k}[(2)(1) - (3)(1)]$

$= i(-7) - \hat{j}\,(-5) + \hat{k}\,(-1)$

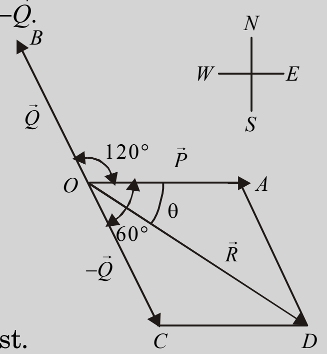
$\Rightarrow \vec{p} \times \vec{q} = -7\hat{i} + 5\hat{j} - \hat{k}$

and $|\vec{p} \times \vec{q}| = \sqrt{7^2 + 5^2 + 1^2} = \sqrt{75} = 5\sqrt{3}$

(3) $\vec{P}$ is a vector, having magnitudes 6 cm/s and directed eastward. Another vector $\vec{Q}$ is acting in north-west direction making an angle 120° with the vector $\vec{P}$. If the magnitude of $\vec{Q}$ be 8 cm/s, find the difference of $\vec{P}$ and $\vec{Q}$.

Soln.: $\overrightarrow{OA}$ represents vector $\vec{P}$ whose magnitude is 6 cm/s. $\overrightarrow{OB}$ represents vector $\vec{Q}$ whose magnitude is 8 cm/s.

BO is produced to C such that $BO = OC$ then $\overrightarrow{OC}$ will represent vector $-\vec{Q}$.



We have to find out $\vec{P}-\vec{Q}$

Let $\vec{R} = P - \vec{Q} = \vec{P} + (-\vec{Q})$

$\therefore$ Magnitude of $\vec{R} = \sqrt{P^2 + Q^2 + 2PQ\cos 60°}$

$$= \sqrt{6^2 + 8^2 + 2\times 6\times 8\times \frac{1}{2}} = 12.17 \text{ cm/s}$$

and $\tan\theta = \dfrac{8\sin 60°}{6 + 8\cos 60°}$ $\quad\therefore\ \theta = 34°42'$

Hence, vector difference = 12.17 cm/s and direction is 34°42′ south of east.

4 **Find a vector whose magnitude is 5 and which is perpendicular to each of the vectors $\vec{A} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ and $\vec{B} = \hat{i} + \hat{j} + \hat{k}$.**

Soln.: Let us first find a unit vector perpendicular to $\vec{A}$ and $\vec{B}$

$$\vec{C} = \vec{A}\times\vec{B},\ \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 5 \\ 1 & 1 & 1 \end{vmatrix}$$

$\vec{C} = \hat{i}\ [-3-5] - \hat{j}[2-5] + \hat{k}\ [2 + 3], \vec{C} = -8\hat{i} + 3\hat{j} + 7\hat{k}$

Unit vector, $\hat{C} = \dfrac{\vec{C}}{|\vec{C}|} = \dfrac{-8\hat{i} + 3\hat{j} + 7\hat{k}}{\sqrt{122}} = \dfrac{-8\hat{i} + 3\hat{j} + 7\hat{k}}{11.05}$

Now a vector with magnitude 5 and perpendicular to $\vec{A}$ and $\vec{B}$ is say $\vec{D}$

$\vec{D} = 5\hat{C},\ \vec{D} = 5\left[\dfrac{-8\hat{i} + 3\hat{j} + 7\hat{k}}{11.05}\right],\ \vec{D} = \dfrac{-40\hat{i} + 15\hat{j} + 35\hat{k}}{11.05}.$

5 **Determine angle between the vectors $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$.**

Soln.: We know the relation $|\vec{A}\times\vec{B}| = AB\sin\theta$

$\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{AB}$ $\quad \vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ $\quad |\vec{A}| = \sqrt{9+1+4} = \sqrt{14}$

$\vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Similarly, $|\vec{B}| = \sqrt{4+4+16} = \sqrt{24}$

$$\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$$

$\vec{C} = \vec{A}\times\vec{B} = \hat{i}\lfloor 4+4 \rfloor - \hat{j}\lfloor 12-4 \rfloor + \hat{k}\lfloor -6-2 \rfloor$

$\vec{C} = \vec{A}\times\vec{B} = 8\hat{i} - 8\hat{j} - 8\hat{k}$ $\quad |\vec{C}| = |\vec{A}\times\vec{B}| = 8\sqrt{3}$

Now, $\sin\theta = \dfrac{|\vec{A}\times\vec{B}|}{|\vec{A}||\vec{B}|} = \dfrac{8\sqrt{3}}{\sqrt{14}\sqrt{24}}$ $\quad \sin\theta = \dfrac{2}{\sqrt{7}}$

or $\quad \theta = \sin^{-1}\left(\dfrac{2}{\sqrt{7}}\right).$

ILLUSTRATION

6 **If $\vec{a},\vec{b},\vec{c}$ are three vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ where $\vec{a}\neq 0$ then prove that $\vec{b}=\vec{c}$.**

Soln.: Given that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$, $\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}=0$

$\vec{a}\cdot(\vec{b}-\vec{c})=0$ ...(i)

In equation (i) we can observe than either $\vec{b}=\vec{c}$ or $\vec{a}$ is perpendicular to $(\vec{b}-\vec{c})$ as it is given $\vec{a}\neq 0$

Similarly $\vec{a}\times\vec{b}=\vec{a}\times\vec{c}$ $\vec{a}\times\vec{b}-\vec{a}\times\vec{c}=0$

$\vec{a}\times(\vec{b}-\vec{c})=0$ ...(ii)

Again, as $\vec{a}\neq 0$ so either $\vec{b}=\vec{c}$ or $\vec{a}$ is parallel to $\vec{b}-\vec{c}$

Now, we can conclude finally that $\vec{b}=\vec{c}$ and $\vec{a}$ cannot be perpendicular and parallel simultaneously to $\vec{b}-\vec{c}$.`

FRAME OF REFERENCE

- The frame of reference is a system of coordinates axes attached to an observer having a clock with him, with respect to which, the observer can describe position, displacement, acceleration etc of a moving body.

Types of Frame of References

(i) Inertial frame of reference
(ii) Non inertial frame of reference

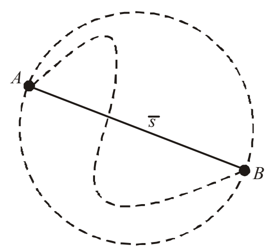
- **Inertial frame of reference :** In which Newton's first law of motion holds good. For example, a frame of reference attached to a boy in a train at rest or moving with a uniform velocity along straight path.
- **Non inertial frame of reference :** In which Newton's first law of motion does not hold good. For example a frame of reference attached to a boy in a train moving with variable velocity or moving with acceleration along a straight path.

MOTION

- Description of motion of a body regardless of the cause that effects (change) the motion is called Kinematics. The study of motion deals with two questions: where and when the body is.
- In order to define "where", it is necessary to have a reference frame. A reference system is an arbitrary point as origin and a co-ordinate system attached to it, with respect to which a given motion can be considered. Usually a fixed reference system is assumed to be a system of co-ordinate axes attached to the earth. The curve described by a particle as it moves in space with respect to a chosen reference system is called its *path*. These paths may be rectilinear or curved.

DISTANCE AND DISPLACEMENT

- Distance between two points is the length of the path between the two points. It is a scalar and its value cannot be negative. It measured in metres in S.I. unit.
- The shortest distance between the initial position to final position is called displacement of the object. It is a vector quantity. SI unit of displacement is meter (m). CGS unit of displacement is centimeter (cm). Its direction is from initial position to final position.



- The lengths of the dotted lines are the various distances, whereas $\overrightarrow{AB}$ is the displacement. A displacement is often designated by $\vec{s}$. The magnitude of the displacement represents the shortest distance.

SPEED

- Speed of an object is the distance travelled in unit time.

$$\text{Speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- Speed has only magnitude and no direction, it is a scalar quantity. Also the distance travelled by object is either positive or zero, so the speed may be positive or zero but never negative.
  The SI unit of speed is ms^{-1}.
  The CGS unit of speed is cms^{-1}.
  The dimensional formula of speed is $[M^0 L T^{-1}]$.

Types of speed

- ***Average speed*** of an object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time. Average speed of a given motion is the ratio of total distance travelled to total time taken.

  $$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

  The two types of special cases are :

  Case 1 : If a particle travels distances s_1, s_2, s_3 etc. with speeds v_1, v_2, v_3 etc. respectively in some direction then total distance travelled = $s_1 + s_2 + s_3 + \ldots$, and

  $$\text{total time taken} = \left(\frac{s_1}{v_1}\right) + \left(\frac{s_2}{v_2}\right) + \left(\frac{s_3}{v_3}\right) + \ldots$$

  $$\Rightarrow \text{Average speed} = \frac{s_1 + s_2 + s_3 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \ldots\right)}$$

  When $s_1 = s_2 = s$, then

  $$v_{av} = \frac{s+s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}.$$

  Case 2 : If a particle travels with speeds v_1, v_2, v_3 etc. during time intervals t_1, t_2, t_3 etc. respectively, then

  total distance travelled = $v_1t_1 + v_2t_2 + \ldots$

  total time taken = $t_1 + t_2 + \ldots$

  $$v_{av} = \frac{v_1t_1 + v_2t_2 + \ldots}{t_1 + t_2 + \ldots}$$

  If $t_1 = t_2 = \ldots = t$, then $v_{av} = \left(\frac{v_1 + v_2 + \ldots v_n}{n}\right)$

Instantaneous Speed and Instantaneous Velocity

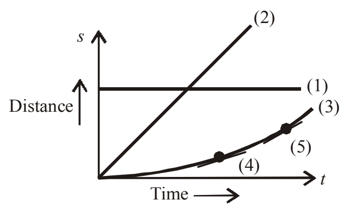
- When an object is travelling with variable speed, it possesses different speeds at different instants. It's speed at a given instant of time or at a given point of path. We call as instantaneous speed $V = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$
- Corresponding, to the concepts of uniform speed, variable speed, average speed, instantaneous speed, we have the concepts of uniform velocity, variable velocity, average velocity and instantaneous velocity. In the earlier definitions the word distance has to be replaced by the word displacement.
  Thus,

  $$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$

  Instantaneous velocity

  $$\bar{v} = \frac{d\bar{s}}{dt}$$

  where $\bar{s}$ represents the displacement.



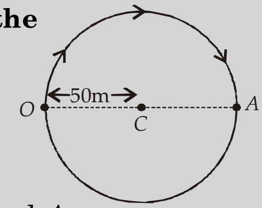
For slop (1) $\frac{ds}{dt} = 0$,

For slope(2) $\frac{ds}{dt}$ = constant,

For slope (3) $\frac{ds}{dt}$ = varies,

ILLUSTRATION

7 **A person walks along a circular path of radius 50 m at a constant pace. He completes one circle in 10 minutes. After 5 minutes, what is the**



(a) distance covered
(b) magnitude of displacement
(c) average speed
(d) magnitude of average velocity
(e) magnitude of instantaneous velocity

Soln.: The person after 5 minutes will reach the diametrically opposite end A,

(a) Distance covered by the person is length of the semicircle = πR = 50π m

(b) Displacement is the length of the diameter

$|\overline{OA}|$ = 50 × 2 m = 100 m

(c) Average speed = $\dfrac{\text{Total distance travelled}}{\text{Time taken}} = \dfrac{50\pi \text{ m}}{5 \text{ minutes}} = \dfrac{50\pi \text{ m}}{300 \text{ s}} = \dfrac{\pi}{6}\dfrac{\text{m}}{\text{s}}$

(d) $|\text{Average velocity}| = \dfrac{|\text{Total displacement}|}{\text{Total time taken}}$

$|\text{Average velocity}| = \dfrac{100 \text{ m}}{5 \text{ minutes}} = \dfrac{100 \text{ m}}{300 \text{ s}} = \dfrac{1}{3}\dfrac{\text{m}}{\text{s}}$.

(e) The magnitude of instantaneous velocity, is the same as instantaneous speed.

As the person is walking with uniform speed it is the same as average speed. Further, its value will match with instantaneous speed at all instants. Here,

$|\text{Instantaneous velocity}|$ = Instantaneous speed

= Uniform speed = Average speed = $\dfrac{\pi}{6}\dfrac{\text{m}}{\text{s}}$.

ACCELERATION

- If the velocity of a body changes with respect to time, then the body is said to accelerate. Rate of change of velocity is called acceleration.

  Average acceleration = $\dfrac{\text{Net change in velocity}}{\text{Total time elapsed}}$

  If $\Delta\vec{v}$ represents the change in velocity which takes place in time interval Δt, then the acceleration during this interval is given by,

  $$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

  The direction of acceleration is the same as the direction of change in velocity $\Delta\vec{v}$.

- For one dimensional motion, average acceleration,

  $$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

- Acceleration of the particle at any given instant of time is given by,

  $\vec{a} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{v}}{\Delta t} = \dfrac{d\vec{v}}{dt}$, is called the instantaneous acceleration.

- When the instantaneous acceleration is same over entire time interval of the motion or when the average acceleration calculated over different time intervals chosen randomly turns out to be the same the motion of the particle is said to be uniformly accelerated.

EQUATIONS OF MOTION AND THEIR GRAPHS

- Consider a velocity-time graph. The velocity-time graph gives three pieces of information.

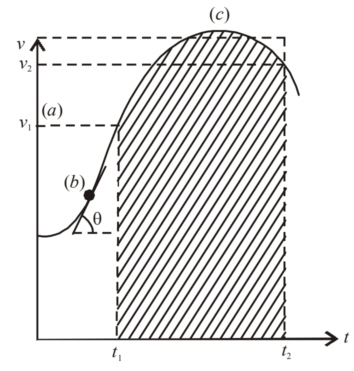
(*a*) The instantaneous velocity, e.g. at t_1 the instantaneous velocity is v_1 and at t_2 it is v_2.

(*b*) The slope of the tangent to the curve at any point gives the instantaneous acceleration.

$$a = \frac{dv}{dt} = \tan\theta$$

(c) The shaded area under the curve gives total displacement of the object

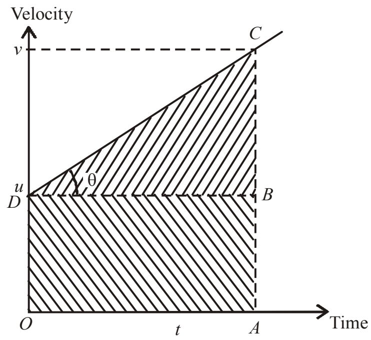
$s = \int_{t_1}^{t_2} v dt$.



> **Note**
>
> **While dealing with vectors in 1-D, we take one of the directions as positive and the opposite as negative. We dispense of with the vector symbol $\vec{v}$ and instead use positive or negative.**

Uniformly Accelerated Motion

- Let us consider the velocity-time graph of uniform acceleration. The acceleration (a) is the slope of graph here.



$a = \tan\theta = \frac{BC}{BD}$

$\Rightarrow a = \left(\frac{v-u}{t}\right)$

$\Rightarrow v = u + at$ ...(i)

The total displacement of the object is the area $OABCD$

s = Area $OABCD$ = Area $OABD$ + Area BCD

$= ut + \frac{1}{2}\cdot(v-u) \times t$

$= ut + \frac{1}{2}(at) \times t$

$\Rightarrow s = ut + \frac{1}{2}at^2$ ...(ii)

Again, s = Area $OABCD$ (a trapezium)

$= \frac{1}{2}\cdot$ (Sum of the parallel sides) × (Perpendicular distance between them)

$= \frac{1}{2}(AC + OD) \times OA = \frac{1}{2}(v+u) \times t = \frac{1}{2}(v+u) \times \frac{(v-u)}{a}$

$s = \frac{(v^2 - u^2)}{2a}$

or $v^2 = u^2 + 2as$ ...(iii)

Average velocity ($<v>$)

$<v> = \frac{s}{t} = \frac{ut + \frac{1}{2}at^2}{t}$

$= u + \frac{at}{2} = \frac{2u + at}{2} = \frac{u + (u + at)}{2}$

$<v> = \frac{u+v}{2}$ ...(iv)

For displacement in n^{th} second, we subtract the displacement in $(n - 1)$ seconds from the displacement in n seconds.

$s_n = un + \frac{1}{2}an^2$

$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$

$s_{n^{\text{th}}} = s_n - s_{n-1}$

$= \left[un + \frac{1}{2}an^2\right] - \left[u(n-1) + \frac{1}{2}a(n-1)^2\right]$

$= [un - u(n - 1)] + \frac{1}{2}a[n^2 - (n - 1)^2]$

$= [u] + \frac{1}{2}a[(n + n - 1)]$

$\Rightarrow 8_{n^{th}} = u + \frac{a}{2}(2n - 1)$ ...(v)

- **Equations of motion**
  Let u = initial velocity of particle/body, v = final velocity of particle/body, a = uniform acceleration. $-a$ = retardation or –ve acceleration, t = time of journey, s = distance travelled in the journey D_n= distance covered in n^{th} second.

(a) The equations of motion are

(i) $v = u \pm at$ (ii) $s = ut \pm \frac{1}{2}at^2$ (iii) $v^2 = u^2 \pm 2as$ (iv) $D_n = u + \frac{a}{2}(2n - 1)$

(b) The equations of motion under gravity when body falls vertically

(i) $v = u + gt$ (ii) $s = ut + \frac{1}{2}gt^2$ (iii) $v^2 = u^2 + 2gs$ (iv) $D_n = u + \frac{g}{2}(2n - 1)$

(c) The equations of motion under gravity when body rises vertically

(i) $v = u - gt$ (ii) $s = ut - \frac{1}{2}gt^2$ (iii) $v^2 = u^2 - 2gs$ (iv) $D_n = u - \frac{g}{2}(2n - 1)$

MOTION OF FREELY FALLING BODIES

Motion of a body vertically downward

- When a body is released from rest at a certain height h, then equations of motion are reduced to

  (a) $v = gt$ (b) $h = \frac{1}{2}gt^2$ (c) $v^2 = 2gh$.

  Where g is the acceleration due to gravity, whose value varies from 9.781 ms^{-2} at the equator to the value 9.831 ms^{-2} at the poles.
- If a body is falling from a certain height h, then distance fallen in n^{th} sec.

  $h_{nth} = h_n - h_{n-1} = \frac{1}{2}gn^2 - \frac{1}{2}g(n-1)^2 = \frac{1}{2}g(2n-1)$

Projection of a body vertically upward

- Suppose a body is projected vertically upward with velocity u. In this case acceleration due to gravity will be negative ($-g$). When the body reaches the maximum height, its velocity v = 0. This occurs when $t = u/g$. The maximum height attained by the body $h = \frac{u^2}{2g}$. Afterward, the body start falling freely downward.
- At the point of projection displacement $s = 0$, hence according to the equation,

  $s = ut - \frac{1}{2}gt^2$ or $0 = ut - \frac{1}{2}gt^2 \Rightarrow t = \frac{2u}{g}$

  i.e. the total time of flight = $\frac{2u}{g}$

ILLUSTRATION

8 A **body is projected upwards with a velocity of 98 m/s. If g = 9.8 m/s^2, find**
(a) the maximum height reached.
(b) the time taken to reach maximum height
(c) its velocity at a height of 196 m from the point of projection.

Soln.: We take up as positive. Hence u = + 98 m/s, $a = -g = -9.8$ m/s^2
(a)At the maximum height, v = 0,
Using $v^2 = u^2 + 2as$

or $0^2 = 98^2 + 2(-9.8) \cdot H_{max} \Rightarrow H_{max} = \dfrac{98^2}{2 \times 9 \cdot 8} = 490$ m.

(b) Now using $v = u + at$, At maximum height,
$v = 0$, $0 = 98 + (-9.8)t$

$\Rightarrow t = \dfrac{98}{9.8} = 10$ s

(c) For velocity at 196 m, $s = 196$ m, $u = +98\ \dfrac{\text{m}}{\text{s}}$, $a = -9.8$ m/s^2

Using, $v^2 = u^2 + 2as$

$\Rightarrow v^2 = (98)^2 + 2(-9.8)(196) = (9604) - (3841.6) = 5762.4$

$\Rightarrow v = \pm 75.91 = \pm 76$ m/s.

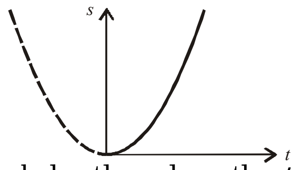
+ 76 m/s is the velocity when the object is on its flight upwards and –76 m/s is its velocity at the same point while coming down.

GRAPHS FOR UNIFORMLY ACCELERATED MOTION

- ***s-t* graph for uniform acceleration**

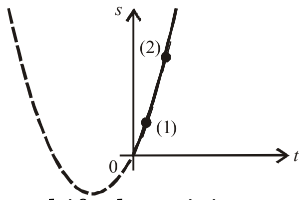
$s = ut + \dfrac{1}{2}at^2$

The quadratic equation represents that it is a parabola. Different cases and their corresponding graphs are as follows:



Case 1 : At $t = 0$, $u = 0$, $a > 0$. It means the body starts accelerating from rest.

Since $a > 0$, it is an upward looking parabola. It passes through origin since at $t = 0$,
$s = 0$. The dotted line represents $t < 0$, we make the complete parabola, then drop the $t < 0$ section, that is represented as the dotted part.

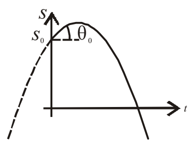


Case 2 : $u > 0$, $a > 0$, $s = 0$ at $t = 0$

As $a > 0$, the parabola looks upward. At $t = 0$, $s = 0$, parabola passes origin. For $t > 0$, slope goes on increasing as $a > 0$ means velocity is increasing. Hence slope at (2) > slope at (1).

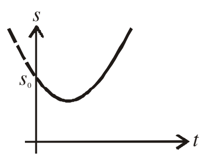
Case 3 : $u > 0$, $a < 0$, $s = s_0$, at $t = 0$

As $a < 0$, the parabola faces down. At $t = 0$, $s = s_0$, so we have to shift the origin to s_0 in the equation, $s = ut + \dfrac{1}{2}at^2$. We replace s by $(s - s_0)$, hence the relevant equation is



$(s - s_0) = ut + \dfrac{1}{2}at^2$

or$s = s_0 + ut + \dfrac{1}{2}at^2$



This is the equation for which the graph is drawn.

Note that at $t = 0$, $\theta_0 > 0$, $\tan\theta_0 > 0$.

$\Rightarrow \left(\dfrac{ds}{dt}\right)_{t=0} > 0$ or $u > 0$

Case 4 : $u < 0$, $a > 0$, $s = s_0$ at $t = 0$

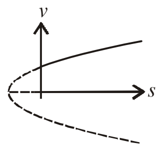
The analysis is similar to the previous case.

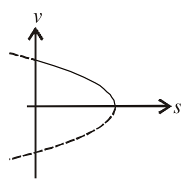
- **The *v-s* graph for uniform acceleration.**

$v^2 = u^2 + 2as$

Case 1: $a > 0$, $u > 0$

The graph is a parabola facing to the right as $a > 0$.





At $t = 0$, $u > 0$. As $a > 0$, $v > u$ thereafter.
Hence we have chosen the
portion of the graph where $v > u$.

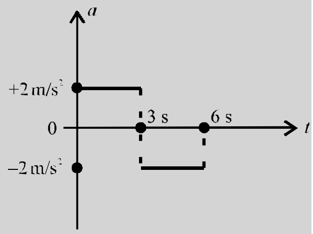
Case 2: $a < 0$, $u > 0$

ILLUSTRATION

9 At $t = 0$, a particle is at rest at origin. Its acceleration is $2\ ms^{-2}$ for the first 3 s and $-2\ ms^{-2}$ for the next 3 s. Plot the acceleration vs time and velocity vs time graphs.

Soln.: Here, $a = +2\ m/s^2$ for $0 \le t \le 3$ s

$a = -2\ m/s^2$ for $3\ s \le t \le 6$ s.



For the first phase of motion,

$u = 0$, $a = +2\ m/s^2$

$\Rightarrow v = 2t$ for $0 \le t \le 3$ s

The velocity at the end 3 s is the initial velocity for the second phase of motion.

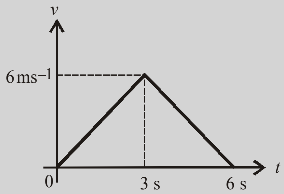
$u = 2(3\ s) = 6\ m/s$

Here $a = -2\ m/s^2$

$\Rightarrow v = u + a(t - 3) = 6 - 2(t - 3) = 12 - 2t$

Thus, $v = 2t$ for $0 \le t \le 3$ s;

$v = 12 - 2t$ for $3\ s \le t \le 6$ s



EQUATIONS OF MOTION USING CALCULUS

- The essential equations of kinematics are $v = \dfrac{ds}{dt}$ and $a = \dfrac{dv}{dt}$.

If acceleration is constant, then $dv = a \cdot dt$

$\Rightarrow \int dv = a\int dt$, At $t = 0$, $v = u$,

Hence $\int_u^v dv = a\int_0^t dt$

$\Rightarrow v - u = a(t - 0)$ or $v = u + at$

Now, $ds = vdt$

$ds = (u + at)dt$, $\int ds = \int (u + at)dt$

At $t = 0$, $s = s_0$,

hence $\int_{s_0}^{s} ds = \int_0^t (u + at)dt$

$s - s_0 = ut + \frac{1}{2}at^2$

$s = s_0 + ut + \frac{1}{2}at^2$

Then, $a = \dfrac{dv}{dt} = \dfrac{dv}{dS} \cdot \left(\dfrac{dS}{dt}\right)$

$\Rightarrow ads = vdv$

$\int ads = \int vdv$

At, $s = s_0$, $v = u$

$a\int_{s_0}^{s} ds = \int_u^v vdv \Rightarrow a(s - s_0) = \dfrac{v^2}{2} - \dfrac{u^2}{2}$

or $v^2 = u^2 + 2a(s - s_0)$

ILLUSTRATION

(10) **A particle moving in a straight line has an acceleration of (3t – 4) m/s^2 at time *t* seconds. The particle is initially 1 m from O, a fixed point on the line, travelling with a velocity of 2 ms^{-1}. Find the times when the velocity is zero.**

Soln.: Here $a = 3t - 4$

As $a = \frac{dv}{dt}$, $dv = adt$

$\int dv = \int adt$

At $t = 0$, $v = 2$ m/s

$\int_2^v dv = \int_0^t (3t - 4)dt$

$\Rightarrow (v - 2) = \left[\frac{3t^2}{2} - 4t\right]_0^t$

$v = \frac{3t^2}{2} - 4t + 2$

When $v = 0$, $\frac{3t^2}{2} - 4t + 2 = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0$, or $3t^2 - 6t - 2t + 4 = 0$

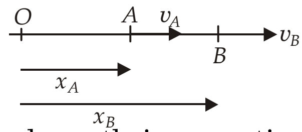
$3t(t - 2) - 2(t - 2) = 0$, or $(3t - 2) \cdot (t - 2) = 0$

$\Rightarrow t = \frac{2}{3}$ s or $t = 2$ s

These are the times when velocity is 0.

RELATIVE VELOCITY

- Let two particles A and B move along the same straight line and at time t, their displacements measured from some fixed origin O on the line be X_A and X_B respectively. The velocities of A and B are,

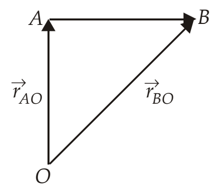


$v_A = \frac{dX_A}{dt}$ and $v_B = \frac{dX_B}{dt}$

- The relative velocities of A and B with respect to each other depends on their respective displacements.
- The displacement of B relative to A (i.e. displacement of B as measured from A) = $(X_B - X_A)$.
- The rate of change of this displacement is called the velocity of B relative to $A = \frac{d}{dt}(X_B - X_A)$.

  $\therefore$ The velocity of B relative to $A = \frac{dX_B}{dt} - \frac{dX_A}{dt} = v_B - v_A$

  This is the velocity of B appears to have, when seen from A.
- The above idea of one dimensional relative motion can be easily extended to motion in two dimensions.



Let $\vec{r}_{AO}$ and $\vec{r}_{BO}$ be the position vectors at time t, of two moving particles with respect to fixed origin O. The velocities $\vec{v}_{AO}$ and $\vec{v}_{BO}$ are then given by,

$\vec{v}_{AO} = \frac{d\vec{r}_{AO}}{dt}$ and $\vec{v}_{BO} = \frac{d\vec{r}_{BO}}{dt}$

By the triangle law of vectors, $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$

$\therefore \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_{BO} - \vec{r}_{AO} = \vec{r}_{BA}$

$\overrightarrow{AB}$ is the displacement of B relative to A and $\vec{r}_{BA}$ is the position vector of B relative to A.

The velocity of B relative to A is $\vec{v}_{BA} = \frac{d\vec{r}_{BA}}{dt} = \frac{d}{dt}(\vec{r}_{BO} - \vec{r}_{AO})$

$$= \frac{d\vec{r}_{BO}}{dt} - \frac{d\vec{r}_{AO}}{dt}$$

$\therefore \quad \vec{v}_{BA} = \vec{v}_{BO} - \vec{v}_{AO}$

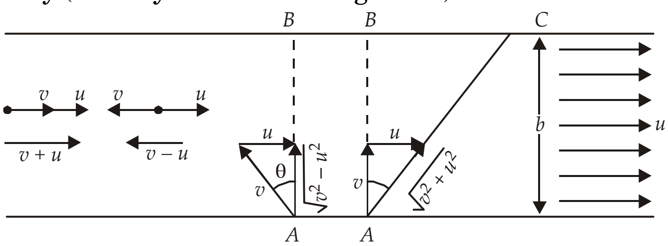
This shows that the relative velocity of two moving particles is the vector difference of their velocities with respect to origin.

> **Note** **Observers on different frames of reference that move at constant velocity relative to each other will measure the same acceleration for a moving particle.**

- **An Example:** Analysis of different situations of the motion of a boat (or a swimmer) in running water, say a river.

v = Velocity of the boat w.r.t. water

u = River flow velocity (velocity of water w.r.t. ground).



If the boat is sailed down stream, velocity of boat with respect to ground = $v + u$.

If the boat is sailed up stream, velocity of the boat with respect to ground = $v - u$.

If the boat is to be sailed right across the river, it must be 'headed' some what upstream so that the upstream component of the velocity of the boat w.r.t water is nullified by the river flow velocity. From the figure,

$$\sin\theta = \frac{u}{v} \Rightarrow \theta = \sin^{-1}\left(\frac{u}{v}\right)$$

In this case the boat will cross the river with velocity $v\cos\theta = \sqrt{v^2 - u^2}$ and will take time $t = \frac{b}{\sqrt{v^2 - u^2}}$ to cross the river, if b is the width of the river.

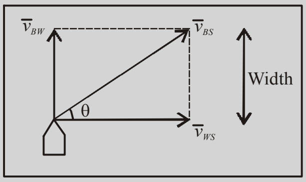
ILLUSTRATION

(11) The engine of a boat drives it across a river that is 1800 m wide. The velocity $\bar{v}_{BW}$ of boat relative to the water is 4 m/s directed perpendicular to the current, as in the figure. Given $\bar{v}_{WS}$ = 2 m/s.

(a) What is the velocity $\bar{v}_{BS}$ of the boat relative to the shore?

(b) How long does it take for the boat to cross the river?

Soln.: $\bar{v}_{BS} = \bar{v}_{BW} + \bar{v}_{WS}$



(a) As $\bar{v}_{BW}$ is normal to $\bar{v}_{WS}$, using Pythagoras theorem,

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2}$$

$$= \sqrt{(4.0\,\text{m/s})^2 + (2.0\,\text{m/s})^2}$$

$$\approx 4.5 \text{ m/s.}$$

The direction of the boat relative to the shore,

$$\tan\theta = \frac{v_{BW}}{v_{WS}} = \left(\frac{4.0 \text{ m/s}}{2.0 \text{ m/s}}\right) = 2 \Rightarrow \theta = \tan^{-1}2 \approx 63°.$$

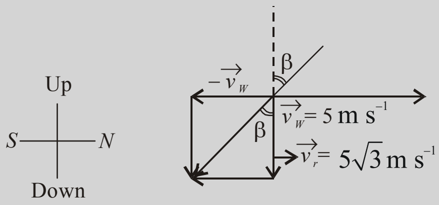
(b) The time t for the boat to cross the river is

$$t = \frac{\text{Width}}{(v_{BS}\cdot\sin\theta)} = \frac{\text{Width}}{v_{BW}} = \frac{1800 \text{ m}}{4.0 \text{ m/s}} = 450 \text{ s.}$$

ILLUSTRATION

(12) **A man can swim at a speed v_1 in still water. What time will be take to travel 'S' distance upstream and then to come back the starting point. The river is flowing at a speed v_2 ($v_1 > v_2$).**

Soln.: Velocity of rain with respect to woman, $\vec{v}rw = \vec{v}r - \vec{v}w$ $\tan\beta = \frac{5}{5\sqrt{3}} = \frac{1}{\sqrt{3}}$, $\beta = 30°$.



(13) **A man can swim at a velocity v_1, relative to water in a river flowing with a speed v_2 ($v_1 > v_2$). Show that it will take him $\frac{v_1}{\sqrt{v_1^2 - v_2^2}}$ times as long to swim a certain distance upstream and back as to swim the same distance and back perpendicular to the direction of stream.**

Soln.: Time to move S upstream, $t_1 = \frac{S}{v_1 - v_2}$

Time to move S downstream, $t_2 = \frac{S}{v_1 + v_2}$

Total time upstream and downstream

$$T_1 = t_1 + t_2 = \frac{2v_1 S}{v_1^2 - v_2^2} \quad \text{...(i)}$$

Time taken to cross the river of width S along shortest path $t_1' = \frac{S}{\sqrt{v_1^2 - v_2^2}}$

Time to return on shortest path, $t_2' = \frac{S}{\sqrt{v_1^2 - v_2^2}}$

Total time to move and come back on shortest path

$$T_2 = t_1' + t_2' = \frac{2S}{\sqrt{v_1^2 - v_2^2}} \quad \text{...(ii)}$$

Thus, $\frac{T_1}{T_2} = \frac{v_1}{\sqrt{v_1^2 - v_2^2}}$.

MOTION IN TWO DIMENSIONS OR MOTION IN PLANE

- If we apply the equations of one dimensional kinematics to x-direction and y-direction separately and then superimpose, we get equations of 2-D kinematics.

  For instance,

  $v_x = u_x + a_x t$

  $s_x = u_x t + \frac{1}{2} a_x t^2$

  $v_x^2 = u_x^2 + 2a_x s_x$

  $v_y = u_y + a_y t$

  $s_y = u_y t + \frac{1}{2} a_y t^2$

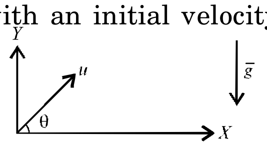
$v_y^2 = u_y^2 + 2a_y s_y$

$\vec{s}, \vec{u}, \vec{v}, \vec{a}$ have their x and y components, whereas time being a scalar is the same in both the sets.

Note: **Component of velocity in x-direction is uniform because components of acceleration in x- direction is zero.**

PROJECTILE MOTION

- "Projectile" means a body, projected with a velocity into the uniform gravitational field of the earth. The path described by a projectile is called the trajectory. The trajectory of a projectile is a parabola.
- Consider, a missile thrown with an initial velocity u at an angle θ to the horizontal



We choose x-axis along the horizontal and y-axis along the vertical. When u is resolved, it is

$u_x = u \cos\theta$, $u_y = u \sin\theta$

Further, $a_x = 0$; $a_y = -g$

Note: **In Projectile motion the horizontal motion and the vertical motion are independent of each other, that is neither motion affects the other**

- **Velocity after time t**

$v_x = u_x + a_x t \Rightarrow v_x = u \cos\theta$

$v_y = u_y + a_y t \Rightarrow v_y = u \sin\theta - gt$

$|v| = v = \sqrt{v_x^2 + v_y^2} = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2}$

$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta - 2u\sin\theta gt + g^2t^2}$

$\Rightarrow v = \sqrt{u^2 - 2u\sin\theta . gt + g^2t^2}$

As $\bar{v} = v_x\hat{i} + v_y\hat{j}$, the direction of $\bar{v}$ w.r.t. x-axis is

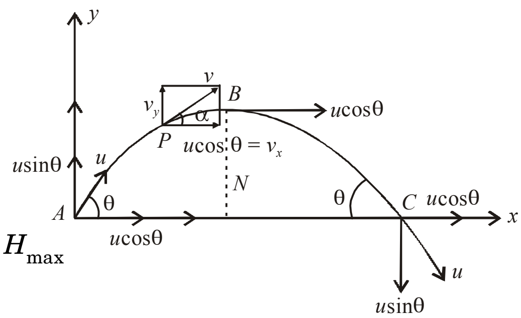
$\tan\alpha = \dfrac{v_y}{v_x}$ or $\alpha = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

- **Time taken to reach maximum height**

At maximum height $v_y = 0$, and $t = t'$, so

$v_y = u \sin\theta - gt' \Rightarrow 0 = u \sin\theta - gt'$

or $t' = \left(\dfrac{u\sin\theta}{g}\right)$



- **Maximum Height**

At maximum height $v_y = 0$

Hence, $v_y^2 = u_y^2 + 2a_y \cdot s \Rightarrow 0 = (u \sin\theta)^2 + 2(-g) H_{max}$

$\Rightarrow H_{max} = \dfrac{u^2\sin^2\theta}{2g}$

Alternatively, at maximum height

$t' = \dfrac{u\sin\theta}{g}$,

using $s_y = u_y t + \frac{1}{2}a_y t^2$

$H_{max} = (u\sin\theta)\cdot\left(\dfrac{u\sin\theta}{g}\right) + \dfrac{1}{2}(-g)\left(\dfrac{u\sin\theta}{g}\right)^2$

$= \dfrac{u^2\sin^2\theta}{g} - \dfrac{u^2\sin^2\theta}{2g} \Rightarrow H_{max} = \dfrac{u^2\sin^2\theta}{2g}$

- **Time of Flight of the Projectile**

  When the projectile comes down again, its $s_y = 0$

  Using $s_y = u_y t + \frac{1}{2}a_y t^2$

  $\Rightarrow 0 = (u \sin\theta)\cdot t + \frac{1}{2}(-g)t^2$

  $\Rightarrow 0 = t\left[u\sin\theta - \frac{gt}{2}\right] \Rightarrow t = 0$ or $t = \frac{2u\sin\theta}{g}$

  The first solution indicates the starting position and the second solution indicates the time taken to complete the flight.

  We note that time of flight is double the time of ascent. Hence time of ascent equals time of descent.

- **Range of the Projectile**

  When the projectile comes down, the time elapsed $t = \frac{2u\sin\theta}{g}$.

  Hence $d_x = u_x t + \frac{1}{2}a_x t^2$

  $R = (u \cos\theta)\left[\frac{2u\sin\theta}{g}\right] + \frac{1}{2}(0)\left[\frac{2u\sin\theta}{g}\right]^2$

  $= \frac{2u^2\cos\theta\sin\theta}{g}$; $R = \frac{u^2\cdot\sin 2\theta}{g}$

  When an object is projected with velocity u at an angle $(90° - \theta)$ with the horizontal, then the range will be

  $R' = \frac{u^2\sin 2(90° - \theta)}{g} = \frac{u^2\sin(180° - 2\theta)}{g} = \frac{u^2\cdot\sin 2\theta}{g} = R$

  Thus the horizontal range is the same, whether the angle of projection of an object is θ or $(90° - \theta)$ with the horizontal direction.

- **Trajectory of the Projectile**

  At any point let $s_x = x$ and $s_y = y$.

  Using $s_x = u_x t + \frac{1}{2}a_x t^2$,

  $x = (u \cos\theta)\cdot t \Rightarrow t = \frac{x}{u\cos\theta}$

  Now, using

  $s_y = u_y t + \frac{1}{2}a_y t^2$

  $y = (u \sin\theta)\, t + \frac{1}{2}(-g)t^2$

  Substituting $t = \frac{x}{u\cos\theta}$, we get

  $y = (u \sin\theta)\left[\frac{x}{u\cos\theta}\right] - \frac{g}{2}\left(\frac{x^2}{u^2\cos^2\theta}\right)$

  $y = (\tan\theta)\cdot x - \left(\frac{g}{2u^2\cos^2\theta}\right)x^2$

  The trajectory as we can see from its quadratic nature is a parabola. It is facing downwards as the coefficient of x^2 is negative.

ILLUSTRATION

(14) **A fire hose ejects a stream of water at an angle of 45° above the horizontal. The water leaves the nozzle with a speed of 25 m/s. Assuming that the water behaves like a projectile, how far from a building should the hose be located to hit the highest possible fire?**

Soln.: The range of a projectile = $\frac{u^2 \sin 2\theta}{g}$. The projectile's trajectory is parabolic and the maximum height happens at half the range. Hence for dousing the highest possible fire, the hose must be located at a distance of

$$\frac{\text{Range}}{2} = \frac{u^2 \sin 2\theta}{2g} = \frac{(25\,\text{m/s})^2 \cdot \sin 2(45°)}{2 \times 9.8\,\text{m/s}^2} = 31.9 \text{ m.}$$

(15) **A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting the angle of projection can one hope to hit a target 5 km away? Assume the muzzle speed to be fixed and neglect air resistance.**

Soln.: In the first case, R = 3 km = 3000 m, θ = 30°

Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore \quad 3000 = \frac{u^2 \sin 60°}{g}$

or $\quad \frac{u^2}{g} = \frac{3000}{\sin 60°} = \frac{3000 \times 2}{\sqrt{3}} = 2000\sqrt{3}$

Maximum horizontal range,

$R_{max} = \frac{u^2}{g} = 2000\sqrt{3}$ m = 3464 m = 3.46 km.

But distance of the target (5 km) is greater than the maximum horizontal range of 3.46 km, so the target cannot be hit by adjusting the angle of projection.

(16) **A fighter plane flying horizontally at an altitude of 15 km with a speed 720 km h^{-1} passes directly overhead an anti aircraft gun. At what angle from the vertical should the gun be fired for the shell muzzle speed 600 m s^{-1} to hit the plane? At what maximum altitude should the plane to avoid being hit? Take g = 10 m s^{-2}.**

Soln.: Speed of plane = 720 km h^{-1} = 200 m s^{-1}

The shell moves along curve OL. The plane moves along PL. Let them hit after a time t.

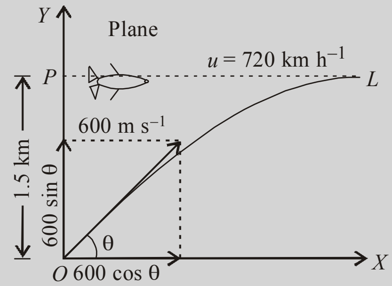
For hitting, horizontal distance travelled by the plane

= Horizontal distance travelled by the shell

or Horizontal velocity of plane × t

= Horizontal velocity of shell × t

200 × t = 600 cos θ × t



$\cos\theta = \frac{200}{600} = \frac{1}{3}$

or $\quad \theta = 70°30'$

The shell should be fired at an angle of 70°30′ with the horizontal or 19°30′ with the vertical.

The maximum height of flight of the shell is

$$h = \frac{u^2 \sin^2\theta}{2g} = \frac{u^2(1-\cos^2\theta)}{2g}$$

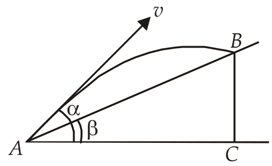
$$= \frac{(600)^2 \times (1-\frac{1}{9})}{2 \times 10} = 16000 \text{ m} = 16 \text{ km}$$

Thus, the pilot should fly the plane at a minimum altitude of 16 km to avoid being hit by the shell.

PROJECTILE ON AN INCLINED PLANE THROUGH THE POINT OF PROJECTION

A particle is projected from a point A on an inclined plane, inclined at an angle β, to the horizontal, with a velocity u at an angle α, to the horizontal. Let the particle strike the plane at B, so that AB is the range on the inclined plane.

The initial velocity u can be resolved into a component $u\cos(\alpha-\beta)$ along the plane and a component $u\sin(\alpha-\beta)$ perpendicular to the plane. The acceleration due to gravity g can be resolved into a component $g\sin\beta$ parallel to the plane and $g\cos\beta$ perpendicular to the plane. If t be the time of flight of the projectile.



Then, $0 = u\sin(\alpha-\beta)\,.\,t - \frac{1}{2}g\cos\beta\,.\,t^2$

$\therefore\ t = \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$

The horizontal distance $AC = u\cos\alpha\,.\,t = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos\beta}$

and, the range AB along the plane $= \frac{AC}{\cos\beta} = \frac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

The projectile is at the maximum distance from the inclined plane, when its velocity is parallel to the plane. So, the component of the velocity in a direction perpendicular to the plane is zero. Thus if R be the greatest distance travelled perpendicular to the plane, then,

$0 = u^2\sin^2(\alpha-\beta) - 2(g\cos\beta)R \Rightarrow R = \frac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$

CIRCULAR MOTION

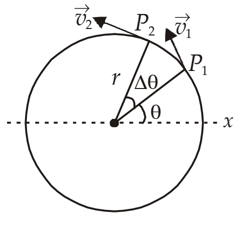
- **Uniform circular motion :** If an object moves on a circular path with a constant speed, then its motion is known as uniform circular motion in a plane. The magnitude of acceleration in such a motion remains constant but the direction changes continuously.

  Let a particle move from P_1 at time t_1 to P_2 at time t_2 and its position vector, from the centre of the circle trace an angle θ.

  If $\vec{v}_1$ and $\vec{v}_2$ are the linear velocities of the particle at P_1 and P_2 respectively, then

  $|\vec{r}_1| = |\vec{r}_2| = r$ (³ circular motion)

  $|\vec{v}_1| = |\vec{v}_2| = v$ (³ uniform circular motion)



  The figure arc $P_1P_2 = r\theta$, again arc $P_1P_2 = v(t_2 - t_1) = vt$

  $\therefore\ r\theta = vt$

  $\frac{\theta}{t} = \frac{v}{r} \Rightarrow \omega = \frac{v}{r} \Rightarrow v = r\omega$

  Again, T = period of revolution $= \frac{\text{circumference}}{\text{constant linear speed}} = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$

  and, n = frequency of revolution $= \frac{1}{T} = \frac{\omega}{2\pi} \Rightarrow \omega = 2\pi n$

  Thus a particle in a uniform circular motion has

(i) a tangential linear velocity of constant magnitude but of direction continuously changing.

(ii) a constant angular velocity about the axis of the circle.

(iii) a constant period of revolution and hence a constant frequency of revolution.

(iv) centripetal acceleration, $a = \frac{v^2}{r} = \omega^2 r = \omega v$

- **Non uniform circular motion :** Let a particle be moving (with varying speed) along a circle of radius r. In this case apart from centripetal acceleration (v^2/r) the particle will also have a tangential acceleration given by dv/dt which results because the velocity vector changes in magnitude also.

Accelerating the particle,

$$\vec{a} = \left(\frac{v^2}{r}\right)\hat{r} + \left(\frac{dv}{dt}\right)\hat{\tau}$$

$\hat{r}$ = a unit vector always pointing radially inwards

$\hat{\tau}$ = a unit vector always pointing in the tangential direction.

When $\frac{dv}{dt}$ is +ve, a_τ is directed along the velocity vector, magnitude of velocity (speed) increases with time, and when $\frac{dv}{dt}$ is –ve, a_τ is directed opposite to the velocity vector, speed decreases with time.

ILLUSTRATION

17 A point moves along a circle with a velocity $v = at$, where $a = 0.50$ m/s^2. Find the total acceleration of the point at the moment when it covers the n^{th} ($n = 0.10$) fraction of the circle after the beginning of motion.

Soln.: Velocity of point, $v = at$, Acceleration of point, $a = 0.50$ m/s^2, Distance covered by point along the circular path = nth fraction of the circle ($n = 0.1$).

Let R be the radius of the circle and S be the distance covered by point along the circle.

$$S = n \times 2\pi R$$

The tangential acceleration of a particle remains the same during the circular motion.

So, $v = at$ or $\frac{dv}{dt} = a_\tau = a$ ...(i)

and radial acceleration, $a_r = \frac{v^2}{R} = \frac{a^2t^2}{R}$ ...(ii)

From $S = \int v\,dt$

$2\pi Rn = \int at\,dt = a\int_0^t t\,dt$

or $2\pi Rn = \frac{1}{2}at^2$

So $4\pi an = \frac{a^2t^2}{R}$ ...(iii)

From eqn (ii) & (iii), we get $a_r = 4\pi an$

Hence, total acceleration $= \sqrt{a_\tau^2 + a_r^2} = \sqrt{a^2 + (4\pi an)^2} = a\sqrt{1 + 16\pi^2 n^2}$

$= 0.5\sqrt{1 + 16 \times 9.86 \times 0.01} = 0.5 \times 1.6 = 0.8$ m/s^2

18 A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 minutes. What is (i) the average speed of the taxi and (ii) the magnitude of average velocity ? Are the two equal?

Soln.: Magnitude of displacement = 10 km

Total path length = 23 km

Time taken $= 28 \text{ min} = \frac{28}{60} \text{ h} = \frac{7}{15} \text{ h}$

(i) Average speed $= \frac{\text{Total path length}}{\text{Time taken}}$

$= \frac{23 \text{ km}}{\frac{7}{15} \text{ h}} = 49.3 \text{ km h}^{-1}$

(ii) Magnitude of average velocity

$= \frac{\text{Displacement}}{\text{Time taken}} = \frac{10 \text{ km}}{\frac{7}{15} \text{ h}} = 21.43 \text{ km h}^{-1}$

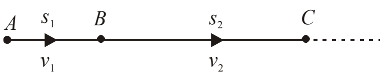
Clearly, the average speed and magnitude of the average velocity are not equal. They will be equal only for straight path.

Xtra SUPPLEMENT for competitive exams

MOTION IN A STRAIGHT LINE

- **Motion in one dimension :** The motion of a body is said to be one dimensional motion if only one out of the three coordinates specifying the position of the body changes with respect to time. In such a motion, the body moves along a straight line.
- **Distance :** The length of the actual path traversed by a body during motion in a given interval of time is called distance travelled by that body.
- Distance is a scalar quantity.
- Distance covered by a moving body can not be zero or negative.
- **Displacement :** The displacement of a body in a given interval of time is defined as the shortest distance between the two positions of the body in a particular direction during that time and is given by the vector drawn from the initial position to its final position.
- Displacement is independent of the path.
- The value of displacement can never be greater than the distance travelled.
- Speed $v = \dfrac{\text{distance travelled}}{\text{time taken}}$
- Velocity, $v = \dfrac{\text{displacement}}{\text{time taken}}$
- Speed is a scalar quantity and velocity is a vector quantity.
- Both speed and velocity has the same units and same dimensional formula.
- Average speed $\bar{v} = \dfrac{\text{total distance travelled}}{\text{total time taken}}$
- In general, if the body covers distances $s_1, s_2, \ldots s_n$ in time interval $t_1, t_2, \ldots t_n$ then

  Average speed, $\bar{v} = \dfrac{\sum_{i=1}^{n} s_i}{\sum_{i=1}^{n} t_i}$.
- **Bodies covering different distances with different speeds :** Let the body cover distance s_1 with speed v_1, a distance s_2 with speed v_2 and so on in same direction.

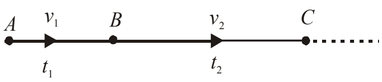


$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}} = \frac{d}{t} = \frac{s_1 + s_2 + \ldots}{t_1 + t_2 + \ldots} = \frac{s_1 + s_2 + \ldots}{\left(\frac{s_1}{v_1} + \frac{s_2}{v_2} + \ldots\right)}.$$

- Let a body travels with speeds v_1 in time interval t_1, v_2 in time interval t_2 and so on.

  Total distance travelled $= v_1t_1 + v_2t_2 + \ldots$

  Total time taken, $t = t_1 + t_2 + \ldots$



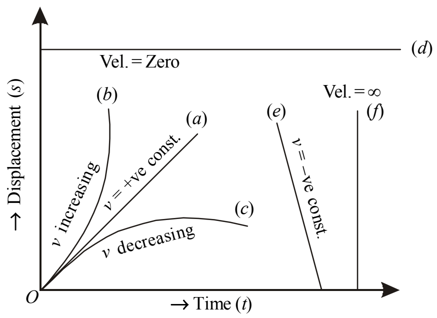
Average speed $= \dfrac{v_1t_1 + v_2t_2 + \ldots}{t_1 + t_2 + \ldots}$.

- Instantaneous velocity, $v = \lim_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$.

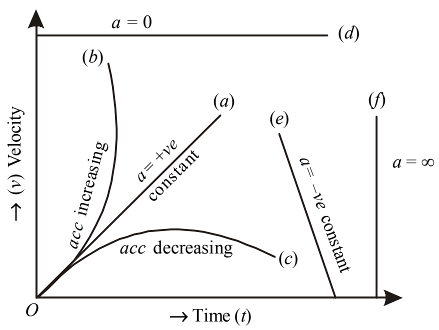
- Acceleration, $a = \dfrac{\text{change in velocity}}{\text{time taken}}$
- Average acceleration, $\bar{a} = \dfrac{\text{change in velocity}}{\text{total time taken}}$
- Instantaneous acceleration, $a = \lim_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt} = \dfrac{d^2x}{dt^2}$.
- **Displacement-time graph slope of the curve denotes velocity**

  Curve (a): Graph is a straight line. Slope is +ve. It represents constant velocity in the direction of displacement.
  Curve (b): The slope of the curve is increasing. It denotes increasing velocity. The motion is accelerated.
  Curve (c): The slope of the curve is decreasing.
  It denotes decreasing velocity. The motion is retarded.
  Curve (d): Graph is a straight line parallel to time-axis.
  Time advances but displacement is constant. Slope is zero.
  Velocity is zero. The particle is at rest.
  Curve (e): Graph is a straight line, slope is –ve. It represents constant velocity in a direction opposite to that of displacement
  Curve (f): Graph is a straight line perpendicular to time axis or parallel to displacement axis
  Slope = tan 90° = Infinity
  Velocity is infinity. This is NOT possible.
- **Velocity**

  Velocity is a vector quantity. Velocity of a body can never be greater than the speed of body.
- **Velocity time graph**

  The slope of velocity time curve denotes acceleration.
  Curve (a): Slope is +ve. The straight line graph denotes constant acceleration.
  Curve (b): The slope of the curve is increasing. It denotes increasing acceleration.
  Curve (c): The slope of the curve is decreasing. It denotes decreasing acceleration.
  Curve (d): The graph is a straight line parallel to time axis. The slope is zero. It denotes uniform velocity having zero acceleration.
  Curve (e): The slope of straight line is negative. It denotes uniform negative acceleration i.e. uniform retardation.
  Curve (f): The graph is a straight line perpendicular to time-axis or parallel to velocity axis. The slope is 90°. It denotes infinite acceleration which is not possible.
- **Area enclosed between velocity-time curve and time-axis.**
  The area represents displacement.
- **Acceleration**
  Acceleration of a body is a vector quantity.
  Curve (a): The curve is a straight line having +ve slope. It denotes uniformly (constantly) increasing acceleration.

  Curve (b): The curve is a straight line parallel to time axis. The slope is zero. It denotes constant acceleration.
  Curve (c): The curve is a straight line having –ve slope. It denotes uniformly (constantly) decreasing acceleration.
- **Area enclosed between acceleration-time curve and time axis.**
  The area represents velocity of the particle in given time.
  (d) Maximum height reached when thrown vertically upwards

  $$s \text{ (maximum)} = h = \frac{u^2}{2g}$$

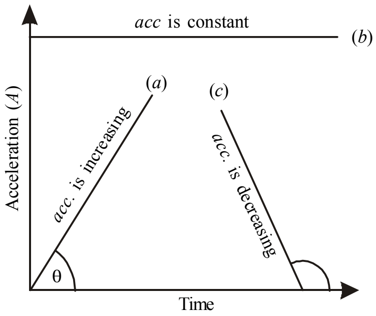
(e) Time taken to reach the maximum height = $\frac{u}{g}$

(f) Time of rise = time of fall under gravity

(g) Velocity of projection upwards = velocity of falling back to ground

(h) At highest point, velocity = 0

- Relative velocity of a body A with respect to body B, when they are moving in the same direction is given by $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$.
- Relative velocity of a body A with respect to body B when they are moving in the opposite direction is given by $\vec{v}_{AB} = \vec{v}_A + \vec{v}_B$.



- **Boat-river problem :** Let $\vec{v}_1$ = velocity of boat in still water, $\vec{v}_2$ = velocity of flow of water in river, d = width of river.

(a) *To cross the river in the shortest path* : Here it is required that the boat starting from A must reach the opposite point B along the shortest path AB. For the shortest path, the boat should be rowed upstream making an angle θ with AB such that AB gives the direction of resultant velocity.

So, $\sin\theta = \frac{v_2}{v_1}$ and $v^2 = v_1^2 - v_2^2$.

Also $t = \frac{s}{v} = \frac{s}{\sqrt{v_1^2 - v_2^2}}$.

(b) To cross the river in the shortest time : For the boat to cross the river in shortest time, the boat should be directed along AB. Let v be the resultant velocity making an angle θ with AB.

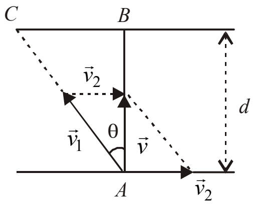
Then $\tan\theta = \frac{v_2}{v_1}$ and $v^2 = v_1^2 + v_2^2$.

∴ Time of crossing, $t = d/v_1$.

Now the boat reaches the point C rather than reaching point B. If $BC = x$, then

$$\tan\theta = \frac{v_2}{v_1} = \frac{x}{d} \quad \text{or} \quad x = d \times \left(\frac{v_2}{v_1}\right)$$

(c) If a man travels downstream in a river, then the time taken by the man to cover a distance d is $t_1 = \frac{d}{v_1 + v_2}$. If a man swims upstream in a river, then the time taken by him to cover a distance d is $t_2 = \frac{d}{v_1 - v_2}$. So $\frac{t_1}{t_2} = \frac{v_1 - v_2}{v_1 + v_2}$.

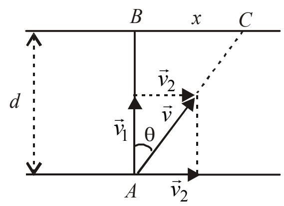


- Relative velocity of a body A with respect to body B when the two bodies moving at an angle θ is given by

$$v_{AB} = \sqrt{v_A^2 + v_B^2 + 2v_A v_B \cos(180° - \theta)} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos\theta}$$

If $\vec{v}_{AB}$ makes an angle β with the direction of $\vec{v}_A$ then,

$$\tan\beta = \frac{v_B \sin(180° - \theta)}{v_A + v_B \cos(180° - \theta)} = \frac{v_B \sin\theta}{v_A - v_B \cos\theta}$$



- If two bodies are moving at right angles to each other, then θ = 90°. Then relative velocity of A with respect to B is $v_{AB} = \sqrt{v_A^2 + v_B^2}$.
- If rain is falling vertically with a velocity $\vec{v}_r$ and a man is moving horizontally with speed $\vec{v}_m$ the man can protect himself from the rain if he holds his umbrella in the direction of relative velocity of rain with respect to man. If θ is the angle which the direction of relative velocity of rain with respect to man makes with the vertical, then tanθ = v_r/v_m.

PROJECTILE

- Any body given an initial velocity moves freely in space under the influence of gravity and is called a projectile.
- A missile shot from a gun, a bomb dropped from a plane and a ball kicked from the ground level are a few examples of projectile motion.
- The path followed by a projectile is called its *trajectory*. Trajectory of a projectile is a *parabola*.
- Projectile motion is a two dimensional motion.
- While studying the projectile motion, we have to make two assumptions.
  (i) The air resistance has no effect on the projectile motion.
  (ii) Trajectories are of short range so that free fall acceleration g remains constant in magnitude and direction.
- **For a projectile projected horizontally from a height *h* with velocity *u***

  (i) $x = ut$ (ii) $y = \frac{1}{2}gt^2$

  (iii) Equation of trajectory is $y = \frac{g}{2u^2}x^2$.

  (iv) Velocity of the projectile at any instant t is $v = \sqrt{u^2 + g^2t^2}$.

  This velocity makes angle β with the horizontal. $\tan\beta = \frac{gt}{u}$.

  (v) Time taken by the projectile to reach the ground is $\sqrt{2h/g}$.

  (vi) Time taken by the projectile to reach the ground does not depend upon the velocity of projection *i.e.* u.

  (vii) Horizontal range, $R = u\sqrt{\frac{2h}{g}}$.

  (viii) Velocity of the projectile on striking the ground = $\sqrt{u^2 + 2gh}$.
- **For a projectile projected at an angle θ with the horizontal with velocity *u***

  (i) $x = u\cos\theta t$ (ii) $y = u\sin\theta t - \frac{1}{2}gt^2$

  (iii) Equation of trajectory is $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$.

  (iv) Velocity of the projectile at any time t is

  $v = \sqrt{(u\cos\theta)^2 + (u\sin\theta - gt)^2} = \sqrt{u^2 + g^2t^2 - 2gtu\sin\theta}$.

  This velocity make angle β with horizontal. $\tan\beta = \frac{u\sin\theta - gt}{u\cos\theta}$.

  (v) Horizontal range $R = \frac{u^2\sin 2\theta}{g}$.

  (vi) For maximum horizontal range, $\theta = 45°$. $R_m = \frac{u^2}{g}$.

  (vii) Time of ascent = time of descent = $\frac{u\sin\theta}{g}$.

  (viii) Time of flight, $T = \frac{2u\sin\theta}{g}$.

  It is maximum for $\theta = 90°$.

  (x) Maximum height h_m or $H = \frac{u^2\sin^2\theta}{2g}$.

  (xi) Maximum height is also known as vertical range, attains the maximum value for $\theta = 90°$.

  (xii) Horizontal range remains the same whether the projectile is thrown at an angle θ with the horizontal or at an angle of $(90° - \theta)$ with the horizontal.

  (xiii) The horizontal range remains the same whether the projectile is thrown at angle θ with the horizontal or at an angle θ with vertical.

(xiv) When horizontal range is n times the maximum height, then : $\tan\theta = 4/n$.
(xv) When the velocity of projection of a projectile thrown at an angle θ with the horizontal is increased n times,
(a) time of ascent becomes n times
(b) time of descent becomes n times
(c) time of flight become n times
(d) maximum height is increased by a factor of n^2
(e) horizontal range is increased by a factor of n^2.
Note :

- When the horizontal range is maximum, the time of flight is $T = \frac{2u\sin 45°}{g} = \frac{\sqrt{2}u}{g}$.
- When the horizontal range is maximum, the maximum height, $H = \frac{u^2\sin^2 45°}{2g} = \frac{1}{4}\frac{u^2}{g} = \frac{R_m}{4}$.

- **Effect of resistance** : The air resistance decreases the maximum height attained and range of the projectile. It also decreases the speed with which the projectile strikes the ground.
- **Effect of variation of *g*** : Acceleration due to gravity does not remain constant when the range exceeds say 1500 km or so. Then the direction of g changes because g always points towards the centre of earth. Due to this, shape of trajectory changes from parabolic to elliptical.
- **For a projectile projected at an angle θ with the vertical with velocity *u***

(i) $x = u\sin\theta t$

(ii) $y = u\cos\theta t - \frac{1}{2}gt^2$

(iii) Equation of trajectory, $y = x\cot\theta - \frac{1}{2}\frac{gx^2}{u^2\sin^2\theta}$

(iv) Velocity of projectile at any instant t

$$v = \sqrt{(u\sin\theta)^2 + (u\cos\theta - gt)^2} = \sqrt{u^2 + g^2t^2 - 2ugt\cos\theta}$$

This velocity makes an angle β with the horizontal direction, $\tan\beta = \frac{u\cos\theta - gt}{u\sin\theta}$.

(v) Time of ascent = time of descent $= \frac{u\cos\theta}{g}$

(vi) Time of flight, $T = \frac{2u\cos\theta}{g}$.

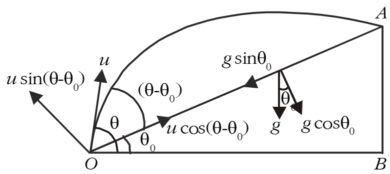
(vii) Maximum height, $H = \frac{u^2\cos^2\theta}{2g}$.

(viii) Horizontal range $R = \frac{u^2\sin 2\theta}{g}$

- **Projectile thrown on an inclined plane**

(i) The particle is thrown from a plane OA inclined at an angle θ_0 with the horizontal, with a constant velocity u in a direction making an angle θ with the horizontal.

(ii) The particle returns back on the same plane OA. Hence the net displacement of the particle in a direction normal to the plane OA is zero. Hence according to the second equation of motion
($s = ut + 1/2\ at^2$)
$0 = u\sin(\theta - \theta_0)\ t - (1/2)\ g\cos\theta_0\ t^2$.



(iii) The **time of flight** of the projectile is given by

$$t = \frac{2u\sin(\theta - \theta_0)}{g\cos\theta_0}$$

(iv) The **horizontal range** of the projectile is given by

$$OB = u\cos\theta\ t = \frac{2u^2\ \sin\ (\theta - \theta_0)\ \cos\ \theta}{g\ \cos\ \theta_0}$$

(v) The **range of the projectile at the inclined plane** is given by

$$OA = \frac{OB}{\cos\theta_0} = \frac{2u^2 \sin(\theta - \theta_0)\cos\theta}{g\cos^2\theta_0}$$

Some important facts of angular projection of projectile

S.No.	Item	Description
1.	Acceleration of projectile	It is constant throughout the motion of projectile and it acts vertically downwards.
2.	Velocity of projectile	It is different at different instants. It is maximum at the starting point O *i.e.* u and is minimum at the highest point *i.e.* $u\cos\theta$
3.	Linear momentum at the highest point	$p_H = mu\cos\theta$
4.	Linear momentum at the lowest point	$p_0 = mu$
5.	Maximum horizontal range	$R_{max} = u^2/g$. It is so when $\theta = 45°$.
6.	Horizontal range will be same	(i) if angle of projection is θ or $90° - \theta$ (ii) if angle of projection is $(45° + \theta)$ or $(45° - \theta)$
7.	Kinetic energy of projectile	It is maximum at the starting point O and is minimum at the highest point H
8.	Angular momentum of projectile at H	$L = (mu\cos\theta) \times H$

CIRCULAR MOTION

- In physics, circular motion is movement of an object with constant speed around in a circle, in a circular path or a circular orbit.
- Circular motion involves acceleration of the moving object by a centripetal force which pulls the moving object towards the centre of the circular orbit. Without this acceleration, the object would move inertially in a straight line, according to Newton's first law of motion. Circular motion is accelerated even though the speed is constant, because the velocity direction of the moving object is constantly changing.
- Examples of circular motion are: an artificial satellite orbiting the earth in geosynchronous orbit, a stone which is tied to a rope and is being swung in circles (e.g. hammer throw), a racecar turning through a curve in a racetrack, an electron moving perpendicular to a uniform magnetic field.
- A special kind of circular motion is when an object rotates around itself. This can be called **spinning motion**.
- Circular motion is characterized by an orbital radius r, a speed v, the mass m of the object which moves in a circle, and the magnitude F of the centripetal force. These quantities are all related to each other through the equations for circular motion.
- The centripetal force can be tension of the string, gravitational force, electrostatic force or Lorentzian.

  But the centrifugal force $= m\omega^2 r$ or mv^2/r in stable rotation, is equal to the centripetal force in magnitude and acts outwards.
- A centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle in uniform circular motion.
- Centrifugal force is the force acting away from the centre and is equal in magnitude to the centripetal force.
- For a safe turn the coefficient of friction between the road and the tyre should be, $\mu_s \geq \frac{v^2}{rg}$

  where v is the velocity of the vehicle, r is the radius of the circular path
- Angle of banking, $\tan\theta = \frac{v^2}{rg}$ this θ depends on the speed, v and radius of the turn r and is independent of the mass of the vehicle.

- A cyclist provides himself the necessary centripetal force by leaning inward on a horizontal track.
- The maximum permissible speed for the vehicle is much greater than the optimum value of the speed on a banked road. It is because, friction between road and the tyre of the vehicle also contributes to the required centripetal force.
- Roads are usually banked for the average speed of vehicle passing over them. If μ is the coefficient of friction between the tyres and the road and θ, the banking angle the safe value for speed limit is $v = \sqrt{\dfrac{rg(\tan\theta + \mu)}{1 - \mu\tan\theta}}$.
- In case of vertical circle the minimum velocity v, the body should possess at the top so that the string does not slack, is $\sqrt{gr}$.
- The magnitude of velocity at the lowest point with which body can safely go round the vertical circle of radius r is $\sqrt{5gr}$.
- Tension in the string at lowest point $T = 6Mg$.
- The tangent at every point of the circular motion gives the direction of motion in circular motion at that point.
- A car sometimes overturns while taking a turn. When it overturns it is the inner wheel, which leaves the ground first.
- A car when passes a convex bridge exerts a force on it which is equal to $Mg - \dfrac{Mv^2}{r}$.
- The driver of a car should brake suddenly rather than taking sharp turn to avoid accident, when he suddenly sees a broad wall in front of him.

Uniform horizontal circular motion

- The instantaneous velocity and displacement act along tangent to the circle at a point.
- The centripetal acceleration and the centripetal force act along radius towards the centre of circle.
- The centripetal force and displacement are at right angles to each other. Hence the work done by the centripetal force is zero.
- Kinetic energy of a particle performing uniform circular motion, in horizontal plane, remains constant.
- The instantaneous velocity of particle and the centripetal acceleration are at right angles to each other. Hence the magnitude of velocity does not change but the direction of velocity changes continuously. It is thus a case of **uniformly accelerated motion**.
- Centripetal acceleration is also called radial acceleration as it acts along radius of circle.
- Momentum of the particle changes continuously along with the velocity.
- The centripetal force does not increase the kinetic energy and angular momentum of the particle moving in a uniform circular path or we say angular momentum is conserved during this motion.

Non-uniform horizontal circular motion

- If the magnitude of the velocity of the particle in horizontal circular motion changes with respect to time, the motion is known as non-uniform circular motion.
- The acceleration of particle is called tangential acceleration. It acts along the tangent to the circle at a point. It changes the magnitude of linear velocity of the particle.
- Tangential acceleration $\vec{a}_T$ and angular acceleration $\vec{\alpha}$ are related as $\vec{a}_T = \vec{r} \times \vec{\alpha}$ where $\vec{r}$ denotes radius vector .
- Centripetal acceleration $\vec{a}_C$ and tangential acceleration $\vec{a}_T$ act at right angles to each other.

  $$\therefore \quad a^2 = a_C{}^2 + a_T{}^2 = \left(\frac{v^2}{r}\right)^2 + a_T^2.$$

- $\tan\phi = \dfrac{a_T}{a_C} = \dfrac{r\alpha}{v^2/r} = \dfrac{r^2\alpha}{v^2}.$

Miscellaneous Xamples

1. For any two vectors $\vec{A}$ and $\vec{B}$, if $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$, find the magnitude of $\vec{C} = \vec{A} + \vec{B}$.

Soln.: Given $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}| \Rightarrow AB\cos\theta = AB\sin\theta$

or $\tan\theta = 1 \Rightarrow \theta = 45°$

As $\vec{C} = \vec{A} + \vec{B}$

$|\vec{C}| = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 + 2AB\cos 45°}$

$= \sqrt{A^2 + B^2 + \sqrt{2}\cdot AB}$.

---

2. Show that the sum and difference of two perpendicular vectors of equal lengths are also perpendicular and of equal lengths.

Soln.: Given: $\vec{A}$ is perpendicular to $\vec{B} \Rightarrow \vec{A}\cdot\vec{B} = 0$

Further, $|\vec{A}| = |\vec{B}|$ or $A = B$ is given.

Now, $|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 - 2AB\cos 90°} = \sqrt{A^2 + B^2}$

$|\vec{A}-\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos 90°} = \sqrt{A^2 + B^2}$

$\Rightarrow |\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

and $(\vec{A}+\vec{B})\cdot(\vec{A}-\vec{B}) = \vec{A}^2 - \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} - \vec{B}^2$ $\quad (\because |A| = |B|)$

$= A^2 - B^2 = 0 \Rightarrow (\vec{A}+\vec{B})$ is perpendicular to $(\vec{A}-\vec{B})$.

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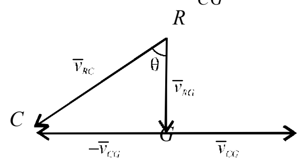
3. A car with a vertical wind shield moves along in a rainstorm at speed of 40 km h^{-1}. The raindrops fall vertically with a terminal speed of 20 ms^{-1}. At which angle the raindrops strike the wind shield?

Soln.: $\vec{v}_{RG} = \vec{v}_{RC} + \vec{v}_{CG}$

where $\vec{v}_{RG}$ is the velocity of rain w.r.t. ground $\vec{v}_{RC}$ is the velocity of rain w.r.t. car $\vec{v}_{CG}$ is the velocity of car w.r.t. ground.



$\Rightarrow \vec{v}_{RC} = \vec{v}_{RG} - \vec{v}_{CG}$

$v_{CG} = 40 \text{ kmh}^{-1} = \frac{40\times 5}{18} \text{ ms}^{-1} = \frac{200}{18} \text{ ms}^{-1}$

$v_{RG} = 20 \text{ ms}^{-1} \Rightarrow \tan\theta = \frac{v_{CG}}{v_{RG}} = \frac{200/18 \text{ ms}^{-1}}{20 \text{ ms}^{-1}} = \frac{10}{18} = \frac{5}{9}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{5}{9}\right)$

---

4. Two bodies are projected at angles θ and (90 – θ) to the horizontal with the same speed. Find the ratio of their times of flight ?

Soln.: Let t_1 and t_2 be the time of flights, then

$t_1 = \frac{2u\sin\theta}{g}$; $t_2 = \frac{2u\sin(90° - \theta)}{g} \Rightarrow \frac{t_1}{t_2} = \frac{\sin\theta}{\cos\theta}$

The ratio of their times of flight is sinθ : cosθ.

---

5. A glass marble projected horizontally from the top of a table falls at a distance x from the edge of the table. If h is the height of the table, then what is the velocity of projection ?

Soln.: If we take down as positive, then

$s_y = h,\ a_y = +g,\ u_y = 0$

Using $s_y = u_y t + \frac{1}{2}a_y t^2 \quad \Rightarrow \quad h = \frac{1}{2}gt^2$

or $t = \sqrt{\frac{2h}{g}}$ and $x = u \cdot t$ or $u = \frac{x}{t} = x \cdot \sqrt{\frac{g}{2h}}$.

6. A ball rolls off the top of a stairway horizontally with a velocity of 4.5 ms^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step. What is n equal to?

Soln.: Let the ball strike the n^{th} step.

$\Rightarrow 0.2n = \frac{1}{2}(10) \cdot t^2$

or $t^2 = \left(\frac{2 \times 0.2}{10}\right)n = 0.04\ n \quad \Rightarrow \quad t = 0.2\ n^{\frac{1}{2}}$

Horizontally the ball travels $x = u \cdot t = (4.5\ ms^{-1})(0.2n^{1/2})$ s

$= (0.9\ n^{1/2})$ m
Also, horizontal distance of nth step = $(0.3\ n)$m
$\Rightarrow (0.9\ n^{1/2})\text{m} = 0.3n$
$n^{1/2} = 3;\ n = 9$
The ball strikes the 9^{th} step

7. The equation of a projectile is $y = \sqrt{3}x - \frac{gx^2}{2}$. Find the angle of projection.

Soln.: Given: $y = \sqrt{3}\,x - \frac{gx^2}{2}$

Comparing with the equation of trajectory,

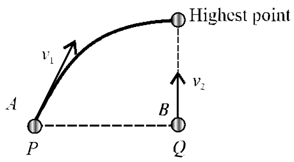
$$y = \tan\theta \cdot (x) - \frac{gx^2}{2u^2 \cos^2\theta} \Rightarrow \tan\theta = \sqrt{3} = \tan 60 = \theta = 60$$

8. A ball is projected at such an angle that the horizontal range is three times the maximum height. What is the angle of projection of the ball?

Soln.: Range = 3 (Maximum height)

$$\Rightarrow \frac{u^2 \sin 2\theta}{g} = 3\left(\frac{u^2 \sin^2\theta}{2g}\right) \quad \Rightarrow 2\ \sin\theta\ \cos\theta = \frac{3\sin^2\theta}{2} \text{ or } \tan\theta = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right).$$

9. A projectile A is thrown at an angle of 30° to the horizontal from point P. At the same time, another projectile B is thrown with velocity v_2 upwards from the point Q vertically below the highest point of P. For B to collide with A, what should be the ratio $\frac{v_2}{v_1}$?



Soln.: The time of ascent for the projectile A.

$\Rightarrow t_A = \frac{v_1 \sin 30°}{g}$

Maximum height of the projectile $A = \frac{v_1^2 \sin^2 30°}{2g}$

Height attained by projectile $B = v_2 t - \frac{1}{2}gt^2$.

For collision, both heights must be same at the time t_A,

$$\Rightarrow \frac{v_1^2 \sin^2 30°}{2g} = v_2\left(\frac{v_1 \sin 30°}{g}\right) - \frac{1}{2}g\left(\frac{v_1^2 \sin^2 30°}{g^2}\right)$$

$$\text{or } \frac{v_1^2 \sin^2 30°}{g} = \frac{v_1 \cdot v_2 \sin 30°}{g} \Rightarrow \frac{v_2}{v_1} = \sin 30° = \frac{1}{2}$$

10. A ball rolls off the top of a stairway with horizontal velocity v_0 ms^{-1}. If the steps are h metre high and w metre wide, for what value of n the ball will hit the edge of nth step ?

Soln.: The ball falls n steps $\Rightarrow nh = \frac{1}{2}gt^2$ or $t = \left(\frac{2nh}{g}\right)^{1/2}$

Edge of nth step is at a horizontal distance of nw metres. If ball's speed is v_0, horizontal distance travelled by ball = $v_0 t$

$$nw = v_0 t \Rightarrow n = \frac{v_0}{w}\left(\frac{2nh}{g}\right)^{1/2}$$

Squaring both sides, $n^2 = \frac{v_0^2}{w^2}\cdot\frac{2nh}{g}$ or $n = \frac{2hv_0^2}{gw^2}$.

---

11. On an inclined plane of inclination 30°, a ball is thrown at an angle of 60° with the horizontal from the foot of the incline with a velocity of $10\sqrt{3}$ ms^{-1}. If g = 10 ms^{-2}, then in what time will the ball hit the inclined plane ?

Soln.: Taking x-axis parallel to inclined plane and y-axis perpendicular to inclined plane,

$u_y = 10\sqrt{3}\sin(60 - 30) = 5\sqrt{3}$ ms^{-1} $\qquad a_y = -g\cos 30° = -\frac{g\sqrt{3}}{2}$

When the ball strikes back, $s_y = 0$

Using, $s_y = u_y\cdot t + \frac{1}{2}a_y\cdot t^2$

$$0 = (5\sqrt{3})t - \frac{1}{2}\frac{g\sqrt{3}}{2}\cdot t^2 \Rightarrow t = 0 \quad \text{or} \quad t = \frac{5\times 4}{g} = \frac{20}{10} = 2 \text{ s.}$$

---

12. Can two vectors of different magnitudes be combined to give a zero resultant? Can three vectors do the same?

Soln.: No, it is not possible to get a zero resultant by combining two vectors of different magnitudes. Two vectors $\vec{A}$ and $\vec{B}$ will give zero resultant, if they are equal in magnitude and opposite in directions, i.e., $\vec{A} = -\vec{B}$.

Yes, three unequal vectors can be combined to give zero resultant if the resultant of any two vectors is equal in magnitude and opposite in direction to the third vector. This is possible when the three vectors can be represented both in magnitude and direction by the three sides of a triangle taken in the same order.

---

13. Can a vector be zero if one of its components is not zero?

Soln.: A vector cannot be zero if all the components of the vector are not zero. Let a vector $\vec{P}$ be resolved along three mutually perpendicular co-ordinate axes, viz., along x, y and z axes. If the resolved components are P_x, P_y and P_z respectively, then the magnitude of $\vec{P}$ is given by

$$|\vec{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2}.$$

So $|\vec{P}|$ will be zero only if all the components become separately zero.

---

14. Find the unit vector perpendicular to each of the vectors $(3\hat{i} + \hat{j} + 2\hat{k})$ and $(2\hat{i} - 2\hat{j} + 4\hat{k})$.

Soln.: Let $\vec{A} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\qquad \vec{B} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Now, from the definition of cross product, $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Now, $\vec{A}\times\vec{B} = (3\hat{i} + \hat{j} + 2\hat{k})\times(2\hat{i} - 2\hat{j} + 4\hat{k}) = 8\hat{i} - 8\hat{j} - 8\hat{k}$

Magnitude of this vector is given by

$$|\vec{A}\times\vec{B}| = \sqrt{8^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$$

$\therefore$ Unit vector perpendicular to $\vec{A}$ and $\vec{B}$ = $\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \frac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

---

15. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, find the vector having the same magnitude as $\vec{B}$ and is parallel to $\vec{A}$.

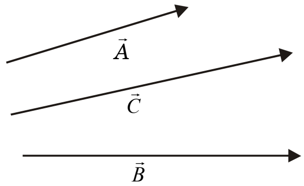
Soln.: $\vec{A} = 3\hat{i} + 4\hat{j}$

$\vec{B} = 7\hat{i} + 24\hat{j}$

Let $\vec{C}$ be the vector which is parallel to $\vec{A}$ and has the magnitude equal to $\vec{B}$.

Now, $|\vec{C}| = |\vec{B}| = \sqrt{7^2 + 24^2} = 25$

Unit vector along $\vec{A} = \hat{a} = \dfrac{3\hat{i}}{\sqrt{3^2 + 4^2}} + \dfrac{4\hat{j}}{\sqrt{3^2 + 4^2}} = \dfrac{3}{5}\hat{i} + \dfrac{4}{5}\hat{j}$



As $\vec{C}$ is parallel to $\vec{A}$, the unit vector along $\vec{C}$

$\hat{c} = \hat{a} = \dfrac{3}{5}\hat{i} + \dfrac{4}{5}\hat{j} \quad \therefore \quad \vec{C} = |\vec{C}|\,\hat{c} = 25\left(\dfrac{3}{5}\hat{i} + \dfrac{4}{5}\hat{j}\right) = 15\hat{i} + 20\hat{j}$

---

16. The resultant of two forces $\vec{P}$ and $\vec{Q}$ acting at a point is equal to $\sqrt{3}Q$ and makes an angle 30° with the direction of $\vec{P}$. Show that either $|\vec{P}| = |\vec{Q}|$ or $|\vec{P}| = |2\vec{Q}|$.

Soln.: Let the angle between $\vec{P}$ and $\vec{Q}$ be θ and that between $\vec{P}$ and $\vec{R}$ be φ, where $\vec{R}$ is the resultant of $\vec{P}$ and $\vec{Q}$.

Now, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ or $\left(\sqrt{3}Q\right)^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\therefore \cos\theta = \dfrac{2Q^2 - P^2}{2PQ}$ ...(i)

Also, $\tan\phi = \tan 30° = \dfrac{1}{\sqrt{3}} = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

or $P + Q\cos\theta = \sqrt{3}Q\sqrt{1 - \cos^2\theta}$ ...(ii)

Eliminating cosθ from equation (i) and (ii), we get

$P^4 - 5P^2Q^2 + 4Q^4 = 0$ or $(P^2 - Q^2)(P^2 - 4Q^2) = 0 \quad \therefore \quad P = \pm Q$ or $P = \pm 2Q$.

Magnitude of a vector is always positive.

$\therefore \; |\vec{P}| = |\vec{Q}|$ or $|\vec{P}| = |2\vec{Q}|$

---

17. Can four vectors which are not co-planar produce equilibrium? Give reason.

Soln.: Yes, four vectors which are not coplanar can produce equilibrium when three of them are such that their resultant is equal and opposite to the fourth vector. This is only possible when the resultant of first three vectors lies in the line of the fourth vector.

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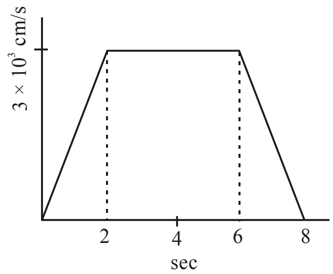
18. The velocity time graph of the motion of a car is given below. Find the distance travelled by the car in the first six seconds. What is the deceleration of the car during the last two seconds?



Soln.: The area enclosed between v-t curve give the distance travelled by the body.

$\therefore$ Distance travelled by the car in the first six seconds

= area of triangle ABC + area of rectangle $BDEC$

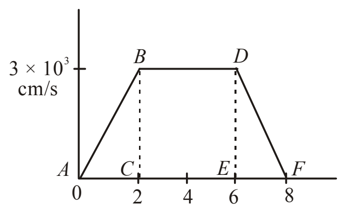
$= \dfrac{1}{2} \times 2 \times 3 \times 10^3 + (6 - 2) \times 3 \times 10^3$

$= 3 \times 10^3 + 12 \times 10^3 = 15 \times 10^3$ cm = 150 m.

From straight line equation,

$v = u + at \Rightarrow 0 = 3 \times 10^3 + a \times 2$

$a = \dfrac{-3 \times 10^3}{2} = -1.5 \times 10^3 \text{ cm/s}^2 = -15 \text{ m/s}^2$.



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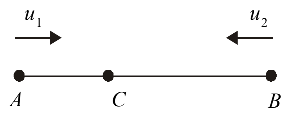
19. Two trains starting at the same instant from two stations A and B with uniform speeds u_1 and u_2 respectively take t_1 and t_2 hours from the point of crossing of each other to reach stations B and A respectively. Show that $u_1 : u_2 = \sqrt{t_2} : \sqrt{t_1}$

Soln.: Let C be the crossing point of the two trains.

Then for the train starting from A, $CB = u_1t_1$ ...(i)

Similarly, for the second trains, starting from B, $CA = u_2t_2$ ...(ii)



Also if time taken by the trains, to meet at C be t,

then, for the first train, $AC = u_1t$ ...(iii)

For the second train, $BC = u_2t$ ...(iv)

From (iii) and (iv) we get, $\frac{AC}{BC} = \frac{u_1t}{u_2t} = \frac{u_1}{u_2}$ ...(v)

Also, from (i) and (ii), we get, $\frac{AC}{BC} = \frac{u_2t_2}{u_1t_1}$ ...(vi)

Equation (v) and (vi) give, $\frac{u_1}{u_2} = \frac{u_2t_2}{u_1t_1}$ or $\frac{u_1^2}{u_2^2} = \frac{t_2}{t_1}$ or $u_1 : u_2 = \sqrt{t_2} : \sqrt{t_1}$

---

20. The relation between time t and displacement x is $t = \alpha x^2 + \beta x$, where α and β are constants, find the relation between velocity and acceleration.

Soln.: We have the relation between time t and displacement x as $t = \alpha x^2 + \beta x$

Differentiating with respect to x, we get, $\frac{dt}{dx} = 2\alpha x + \beta$

$\therefore \quad \frac{dx}{dt} = \frac{1}{2\alpha x + \beta}$ or $v = \frac{1}{2\alpha . x + \beta}$ $\left[\because \frac{dx}{dt} = \text{velocity } v\right]$

Now, acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}\left[\frac{1}{2\alpha x + \beta}\right] \Rightarrow \frac{d}{dx}\left[\frac{1}{2\alpha x + \beta}\right]\cdot\frac{dx}{dt} = -\frac{2\alpha}{(2\alpha x + \beta)^2}\cdot v = -2\alpha . v^2 . v = -2\alpha v^3$.

$\Rightarrow \; a = -2\alpha v^3$.

---

21. Rain water is falling vertically downward with a velocity v cm/s when the velocity of the wind is zero and water is collected at a certain rate in a vessel in the rain. What will be the change in the rate of collection of water when the velocity of the wind is u cm/s in a direction perpendicular to v?

Soln.: The rate of collection of rain water is proportional to the vertical velocity component of rain. The velocity of wind, being perpendicular to the velocity of rain, will not affect the vertical velocity of rain. This is because of the fact that a vector (here velocity of wind) does not have any component in a direction perpendicular to it. Hence, there will be no change in the rate of collection of water in the vessel.

---

22. A body moving in a straight line with uniform acceleration describes three successive equal distances in time intervals t_1, t_2 and t_3 respectively. Show that

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

Soln.: Let v_1, v_2 and v_3 be the initial velocities of the particle in the time intervals t_1, t_2 and t_3 respectively, and v be the final velocity of the particle in the time interval t_3. The particle moves equal distances in each time interval and let it be d.

Then, average velocity in the time interval t_1 is

$$\frac{d}{t_1} = \frac{1}{2}(v_1 + v_2) \quad \text{...(i)}$$

Similarly, $\frac{d}{t_2} = \frac{1}{2}(v_2 + v_3)$ ...(ii)

$$\frac{d}{t_3} = \frac{1}{2}(v_3 + v) \quad \text{...(iii)}$$

and average velocity in the time interval $(t_1 + t_2 + t_3)$ is

$$\frac{3d}{t_1 + t_2 + t_3} = \frac{1}{2}(v_1 + v) \quad \text{...(iv)}$$

From eqn (i), (ii), (iii) and (iv) we get,

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{1}{2d}(v_1 + v_2) - \frac{1}{2d}(v_2 + v_3) + \frac{1}{2d}(v_3 + v) = \frac{1}{2d}(v_1 + v) = \frac{3}{t_1 + t_2 + t_3}$$

EXERCISE

Multiple Choice Questions

1. The formula $v = u + at$ can be applied
(a) Always
(b) Only when acceleration direction is constant
(c) Only when the velocity of the particle is constant throughout
(d) Only if acceleration is along the direction of velocity and is constant.

2. A body travels 100 km southwest and then $50\sqrt{2}$ km in the northern direction. The total magnitude of the displacement is (in km)
(a) $100 + 50\sqrt{2}$ (b) $100 - 50\sqrt{2}$
(c) $50\sqrt{2}$ (d) 100

3. A car travels along a straight line path with speed v_1 from A to B and return back from B to A with speed v_2. The average speed of the car during its journey is given by:
(a) $\frac{v_1 + v_2}{2}$ (b) $\frac{2v_1v_2}{v_1 + v_2}$
(c) $\frac{v_1v_2}{v_1 + v_2}$ (d) $\sqrt{v_1v_2}$

4. A stone thrown upward with a speed u from the top of the tower reaches the ground with the velocity $3u$. The height of the tower is:
(a) $\frac{3u^2}{g}$ (b) $\frac{4u^2}{g}$
(c) $\frac{6u^2}{g}$ (d) $\frac{9u^2}{g}$

5. A man is walking on a road with a velocity 3 km/h. Suddenly rain starts falling. The velocity of rain is 10 km/h in vertically downward direction. The relative velocity of man with respect to rain is
(a) $\sqrt{13}$ km/h (b) $\sqrt{109}$ km/h
(c) $\sqrt{7}$ km/h (d) 13 km/h

6. The velocity of a body depends on time according to the equation $v = 20 + 0.1t^2$. The body is undergoing
(a) uniform acceleration
(b) uniform retardation
(c) non-uniform acceleration
(d) zero.

7. A particle is dropped vertically from rest from a height. The time taken by it to fall through successive distance of 1 m each will then be
(a) All equal, being $\sqrt{2/g}$ sec.
(b) In the ratio of the square roots of the integers 1, 2, 3, ...
(c) In the ratio of the difference in the square roots of the integers i.e. $\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots$
(d) In the ratio of the reciprocal of the square roots of the integers i.e. $\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \ldots$

8. Three points are located at the vertices of an equilateral triangle whose sides equal a. They all start moving simultaneously with velocity v constant in modulus, with the first point heading continually for the second, the second for the third, and the third for the first. How soon will the points converge?
(a) $\frac{a}{2v}$ (b) $\frac{2a}{3v}$
(c) $\frac{3v}{2a}$ (d) $\frac{2v}{a}$

9. A boat moves relative to water with a velocity which is $n = 2.0$ times less than the river flow velocity. At what angle to the stream direction must the boat move to minimize drifting?
(a) 60° (b) 90°
(c) 120° (d) 150°

10. A particle is travelling with a uniform acceleration. If a, b and c are the distances covered by it during x^{th}, y^{th} and z^{th} second of its motion respectively, then
(a) $ax = by = cz$
(b) $a(y - z) + b(z - x) + c(x - y) = 0$
(c) $ax + by = cz$ (d) none of these

11. A particle moves in the plane xy with constant acceleration a directed along the negative y-axis. The equation of motion of the particle has the form $y = mx - nx^2$, where m and n are positive constants. What is the velocity of the particle at the origin of coordinates?
(a) $\sqrt{\frac{a}{2n}(1 + m^2)}$ (b) $\sqrt{\frac{a}{2n}}$
(c) $\frac{a(1 - 2m)}{\sqrt{1 - n^2}}$ (d) zero

12. A particle moves in the $x-y$ plane with velocity $v_x = 8t - 2$ and $v_y = 2$. If it passes through the point $x = 14$ and $y = 4$ at $t = 2$ s, the equation of the path is,

(a) $x = y^2 - y + 2$ (b) $x = y + 2$
(c) $x = y^2 + 2$ (d) $x = y^2 + y + 2$

13. The acceleration of a particle, starting from rest, varies with time according to the relation $a = -r\omega^2 \sin \omega t$. The displacement of this particle at time t will be

(a) $-\frac{1}{2}(r\omega^2 \sin \omega t)t^2$ (b) $r\omega \sin \omega t$
(c) $r\omega \cos \omega t$ (d) $r \sin \omega t$

14. A particle starts with a velocity of 2 m/sec and moves in a straight line with a retardation of 0.1 m/sec^2. The time at which the particle is 15 m from the starting point is

(a) 40 sec (b) 30 sec
(c) 20 sec (d) 10 sec

15. Two bodies are held separated by 9.8 m vertically one above the other. They are released simultaneously to fall freely under gravity. After 2 sec, the relative distance between them is

(a) 39.2 m (b) 9.8 m
(c) 19.6 m (d) 4.9 m

16. A ball is dropped from the top of a building 100 m high. At the same instant another ball is thrown upwards with a velocity of 40 m/sec from the bottom of the building. The two balls will meet after

(a) 3 sec (b) 2 sec
(c) 2.5 sec (d) 5 sec

17. A train accelerating uniformly from rest attains a maximum speed of 40 m/sec in 20 sec, it travels at this speed for 20 sec and is brought to rest with uniform retardation in further 40 sec. Find the average velocity during this period.

(a) 30 m/sec (b) 25 m/sec
(c) 40 m/sec (d) 80/3 m/sec

18. If x denotes displacement in time t and $x = a \cos t$ then acceleration is

(a) $a \cos t$ (b) $-a \cos t$
(c) $a \sin t$ (d) $-a \sin t$

19. A body moves 6 m North, 8 m East and 10 m vertically upwards, the resultant displacement from its initial position is

(a) 20 m (b) $\frac{10}{\sqrt{2}}$ m
(c) 10 m (d) $10\sqrt{2}$ m

20. PQ is a horizontal plane of constant length. SQ is a vertical line passing through point R. A particle is kept at R and angle θ can be varied such that R lies on line SQ. The time taken by particle to come down varies, as the θ increases

(a) decreases continuously
(b) increases
(c) increases then decreases
(d) decreases then increases.

21. The displacement of a body is given to be proportional to the cube of time elapsed. The magnitude of the acceleration of the body, is

(a) constant but not zero
(b) increasing with time
(c) zero
(d) decreasing with time.

22. The ratio of magnitudes of average speed to average velocity, is

(a) always less than one
(b) always equal to one
(c) always more than one
(d) equal to or more than one.

23. A food packet is released from a helicopter rising steadily at the speed of 2 ms^{-1}. What is the velocity of the packet after 2 s?

(g = 10 ms^{-2})

(a) 18 ms^{-1} (b) 20 ms^{-1}
(c) 22 ms^{-1} (d) 98 ms^{-1}

24. A body having uniform acceleration of 10 ms^{-2} has a velocity of 100 ms^{-1}. In what time, the velocity will be doubled?

(a) 8 s (b) 10 s
(c) 12 s (d) 14 s

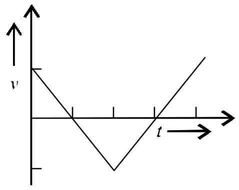
25. Which one of the following equations represents the motion of a body with finite constant acceleration? In these equations, y denotes the displacement of the body at time t and a, b and c are constants of motion.

(a) $y = at$
(b) $y = at + bt^2$
(c) $y = at + bt^2 + ct^3$
(d) $y = at^{-1} + bt$

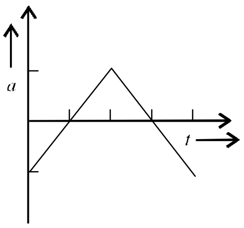
26. A pebble is dropped into a well of depth h. The splash is heard after time t. If c be the velocity of sound, then

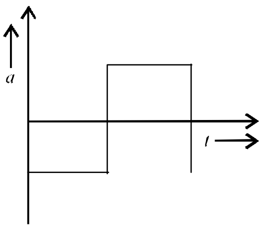
(a) $t = \frac{gh^2}{cv^2}$ (b) $t = \frac{c+v}{g}$
(c) $t = \frac{c-v}{g}$ (d) $t = \sqrt{\frac{2h}{g}} + \frac{h}{c}$

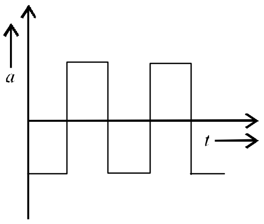
27. The graph shown in figure shows the velocity v versus time t for a body. Which of the graphs shown in figure represents the corresponding

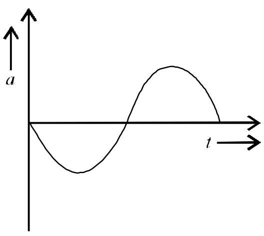


acceleration versus time graphs?

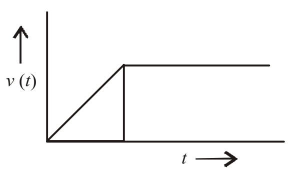
(a)


(b)


(c)


(d)


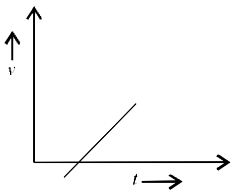
28. The graph depicts the velocity of an object at various times during its motion. The shaded portion of the graph has

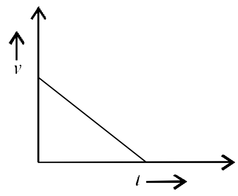


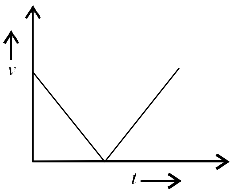
an area equal to

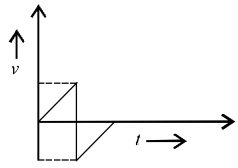
(a) the average acceleration
(b) distance moved during uniform motion
(c) distance moved during accelerated motion
(d) the average velocity.

29. Which of the following curves represents the v-t graph of an object falling on a metallic surface and bouncing back?

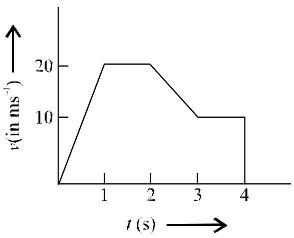
(a)


(b)


(c)


(d)


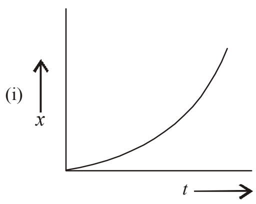
30. The variation of velocity of a particle moving along a straight line is shown in the figure. The distance travelled

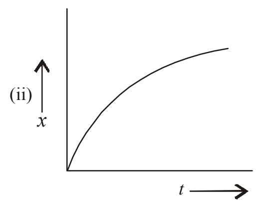


by the particle in 4 s is

(a) 25 m (b) 30 m
(c) 55 m (d) 60 m

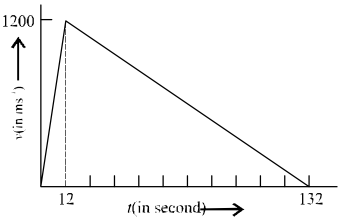
31. Figures show the displacement-time graphs of two particles moving along the x-axis. We can say that





(a) Both the particles are having a uniformly retarded motion.
(b) Both the particles are having a uniformly accelerated motion.
(c) Particle (i) is having a uniformly accelerated motion while particle (ii) is having a uniformly retarded motion.
(d) Particle (i) is having a uniformly retarded motion while particle (ii) is having a uniformly accelerated motion.

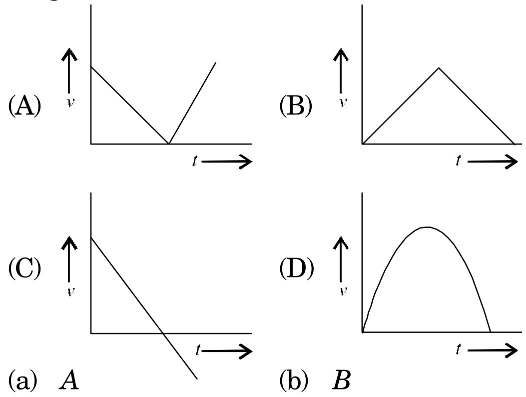
32. A rocket is fired upwards. Its engine explodes fully in 12 second. The height reached by the rocket as calculated from its velocity-time graph is



(a) 1200×66 m (b) 1200×132 m

(c) $\dfrac{1200}{12}$ m (d) 1200×12^2 m

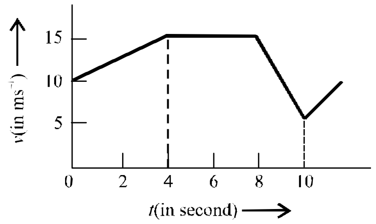
33. A ball is projected vertically upwards. Which graph in the figure represents the velocity of the ball during its flight when air resistance is ignored?



(a) A (b) B

(c) C (d) D

34. In the velocity-time graph shown in the figure, the distance travelled by the particle

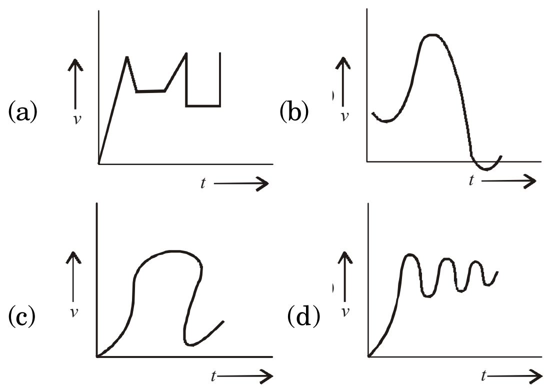


between 1.0 second and 4.0 second is nearly

(a) 39 m (b) 60 m

(c) 80 m (d) 100 m

35. The following figures show velocity v versus time t curves. But only some of these can be realised in practice. These are



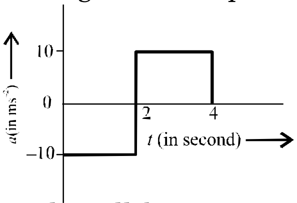
(a) (i), (ii) and (iv) only

(a) (i), (ii) and (iii) only

(c) (ii) and (iv) only

(d) all

36. A particle starts from rest at time $t = 0$ and moves on a straight line with acceleration as plotted in figure. The speed

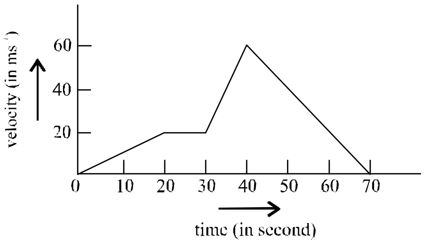


of the particle will be maximum after time.

(a) 1 s (b) 2 s

(c) 3 s (d) 4 s

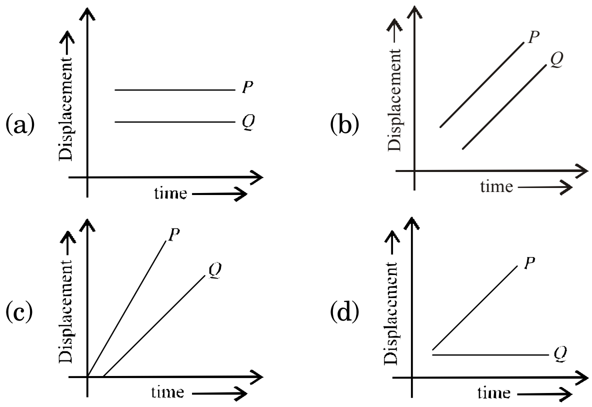
37. The velocity versus time graph of a moving particle is shown in figure. The maximum acceleration is



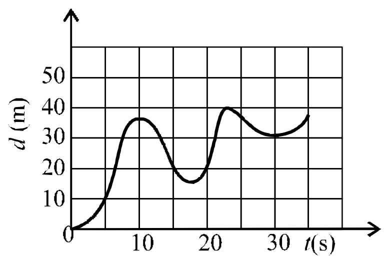
(a) 1 ms^{-2} (b) 2 ms^{-2}

(c) 3 ms^{-2} (d) 4 ms^{-2}

38. Which one of the following represents the time-displacement graph of two objects P and Q moving with zero relative speed?



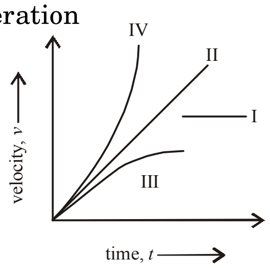
39. In the displacement d versus time t graph given below, the value of average velocity in the



time interval 0 to 20 s is (in m/s).

(a) 1.5 (b) 4
(c) 1 (d) 2

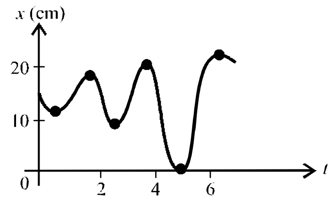
40. Five velocity-time graphs (namely I, II, III, IV and V) are shown in figure. In which case is the acceleration



uniform and positive?

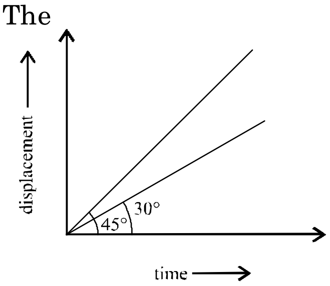
(a) I (b) II
(c) III (d) IV

41. Figure shows the position of a particle moving along the X-axis as a function of time



(a) The particle has come to rest 6 times
(b) The maximum speed is at $t = 6$ s
(c) The velocity remains positive for $t = 0$ to $t = 6$ s.
(d) The average velocity for the total period shown is negative.

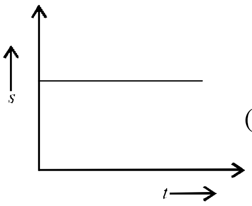
42. The displacement-time graphs of two moving particles make angles of 30° and 45° with the x-axis. The

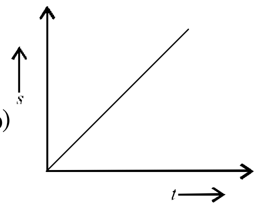


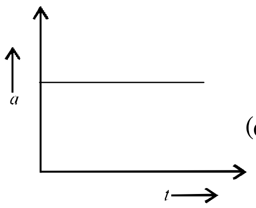
ratio of the two velocities is

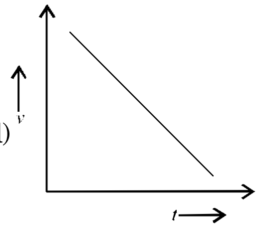
(a) $\sqrt{3}:1$ (b) 1 : 1
(c) 1 : 2 (d) $1:\sqrt{3}$

43. Which of the following represents uniform speed?

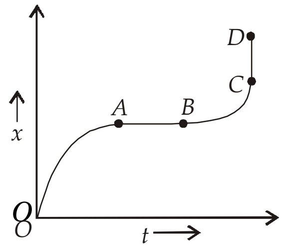
(a)


(b)


(c)


(d)


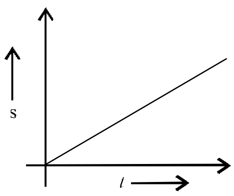
44. Figure shows the graphical variation of displacement with time for the case of a particle moving along a straight line. The accelerations of the particle

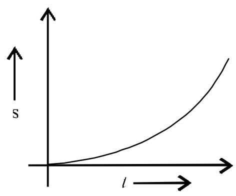


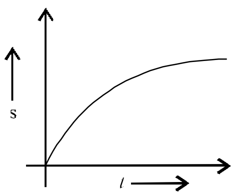
during the intervals OA, AB, BC and CD are respectively.

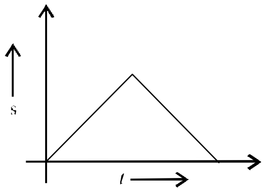
	OA	AB	BC	CD
(a)	–	0	+	0
(b)	+	0	+	+
(c)	–	0	–	0
(d)	+	0	–	+

45. From a high tower at time $t = 0$, one stone is dropped from rest and simultaneously another stone is projected vertically up with an initial velocity. The graph of the distance 's' between the two stones, before either hits the ground, plotted against time 't' will be as

(a)


(b)


(c)


(d)


46. A particle starting from rest with uniform acceleration travels a distance x in first 2 second and a distance y in next 2 second, then

(a) $y = 2x$ (b) $y = 3x$
(c) $y = 4x$ (d) $y = x$

47. A ball dropped from the 9th storey of a multi-storeyed building reaches the ground in 3 second. In the first second of its free fall, it passes through n storeys, where n is equal to
(Take $g = 10$ m s^{-2})

(a) 1 (b) 2
(c) 3 (d) 4

48. A balloonist releases a ballast bag from a balloon rising at 40 ms^{-1} at a time when the balloon is 100 m above the ground. If g = 10 ms^{-2}, then the bag reaches the ground in
(a) 16 s (b) 18 s
(c) 10 s (d) 20 s

49. A train is moving east at a speed of 5 ms^{-1}. A bullet fired westwards with a velocity of 10 ms^{-1} crosses the train in 8 s. The length of the train is
(a) 120 m (b) 60 m
(c) 30 m (d) 15 m

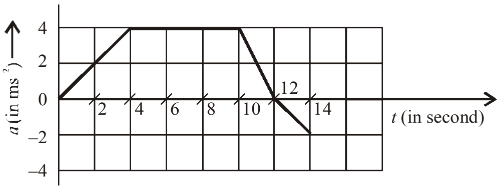
50. Two bodies of masses m_1 and m_2 fall from heights h_1 and h_2 respectively. The ratio of their velocities, when they hit the ground is

(a) $\frac{h_1}{h_2}$ (b) $\sqrt{\frac{h_1}{h_2}}$

(c) $\frac{m_1 h_1}{m_2 h_2}$ (d) $\frac{h_1^2}{h_2^2}$

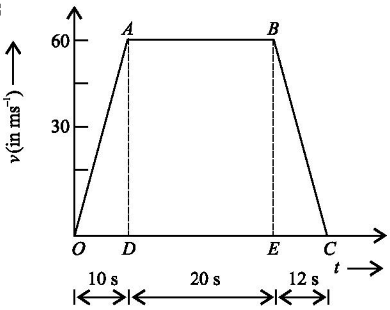
51. P, Q and R are three balloons ascending with velocities u, $4u$ and $8u$ respectively. If stones of the same mass be dropped from each, when they are at the same height then
(a) stone from P reaches the ground first
(b) stone from Q reaches the ground first
(c) stone from R reaches the ground first
(d) all reach at the same time.

52. A graph of acceleration versus time of a particle starting from rest at t = 0 is as shown in figure. The speed of the particle at t = 14 second is



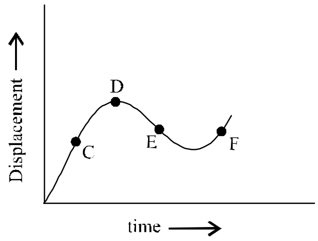
(a) 2 ms^{-1} (b) 34 ms^{-1}
(c) 20 ms^{-1} (d) 42 ms^{-1}

53. The velocity-time graph of a moving train is depicted in figure. The average velocity in time OD is



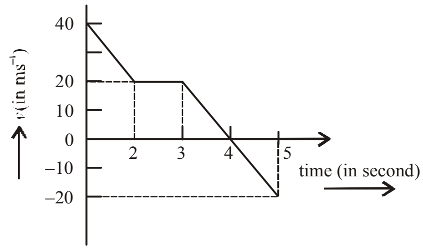
(a) 30 m s^{-1} (b) 60 m s^{-1}
(c) 45 m s^{-1} (d) 23 m s^{-1}

54. The displacement-time graph of a moving particle is shown below. The instantaneous velocity of the particle is negative at the point.



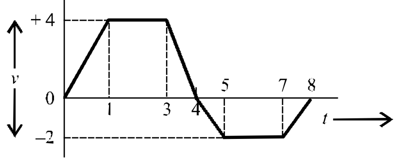
(a) C (b) D
(c) E (d) F

55. In the given v-t graph, the distance travelled by the body in 5 second will be



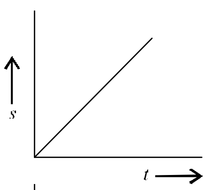
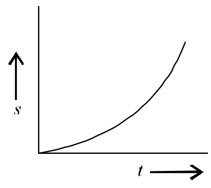
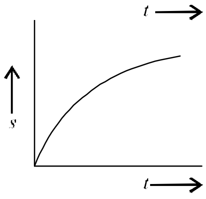
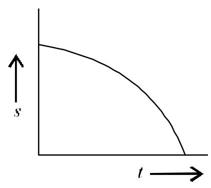
(a) 20 m (b) 40 m
(c) 80 m (d) 100 m

56. The velocity-time graph of a particle in linear motion is shown in figure. Both v and t are in SI units. What is the displacement of the particle from the origin after 8 second?

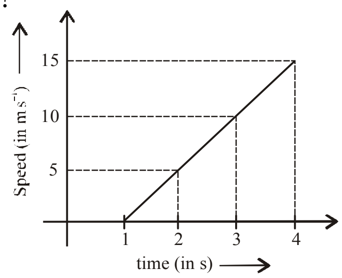


(a) 6 m (b) 8 m
(c) 16 m (d) 18 m

57. Which of the following graphs represents the distance-time variation of a body released from the top of a building?

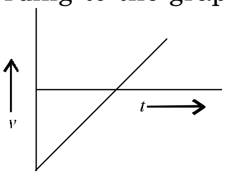
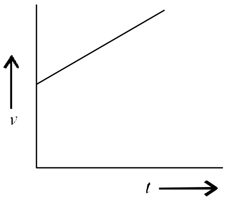
(a)

(b)

(c)

(d)


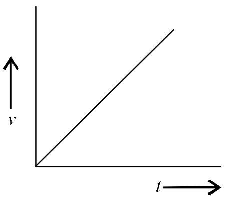
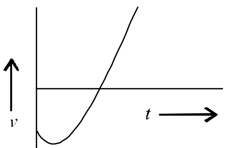
58. The speed versus time graph of a body is shown in figure. Which of the following statements is correct?



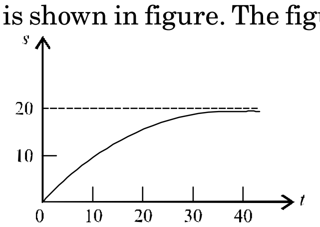
(a) The body is moving with uniform acceleration of 6.67 ms^{-2} at all the times.
(b) The body is at rest for 1 s and has a uniform acceleration of 6.67 ms^{-2}.
(c) The body is at rest for one second and has a uniform acceleration of 5 ms^{-2} afterwards.
(d) The body is at rest for 1 s and has a uniform retardation of 6.67 ms^{-2} afterwards

59. A particle moves along X-axis in such a way that its x-coordinate varies with time t according to the equation $x = (6 - 4t + 6t^2)$ metre. The velocity of the particle will vary with time according to the graph

(a)  (b) 

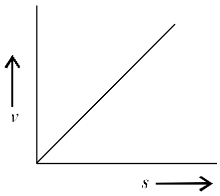
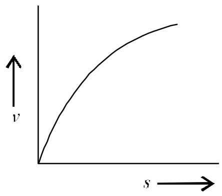
(c)  (d) 

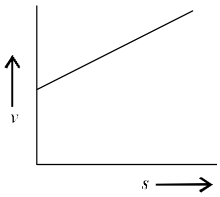
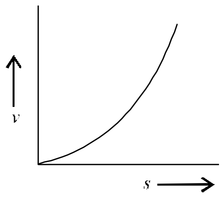
60. The displacements of a particle as a function of time t is shown in figure. The figure indicates



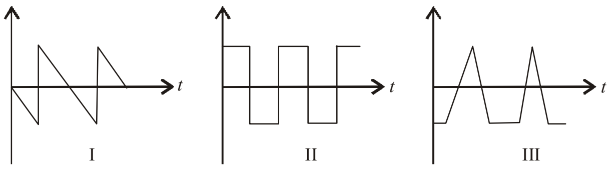
(a) the velocity of the particle is constant throughout
(b) the particle starts with certain velocity, but the motion is retarded and finally the particle stops
(c) the particle starts with a constant velocity, the motion is acceleration and finally the particle moves with another constant velocity.
(d) the acceleration of the paticle is constant throughout.

61. A particle starts from rest and moves along a straight line with constant acceleration. The variation of velocity v with displacement s is

(a)  (b) 

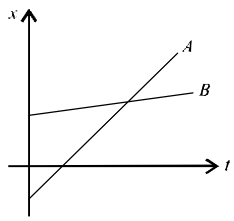
(c)  (d) 

62. Which of the graphs below correctly shows how the acceleration and velocity of a perfectly elastic ball bouncing on a horizontal surface varying with time?



	(a)	(b)	(c)	(d)
Acceleration	II	III	III	I
Velocity	I	I	II	II

63. Figure shows the time-displacement graph of the particles A and B. Which of the following statements is correct?

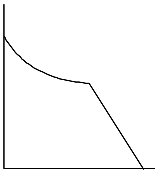
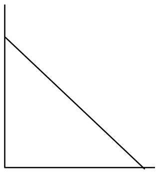
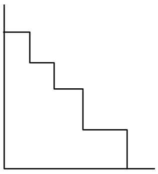
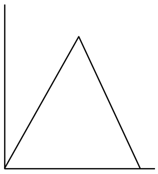


(a) Both A and B move with uniform equal speed.
(b) A is accelerated, B is retarded.
(c) Both A and B move with uniform speed. The speed of B is more than the speed of A.
(d) Both A and B move with uniform speeds but the speed of A is more than the speed of B.

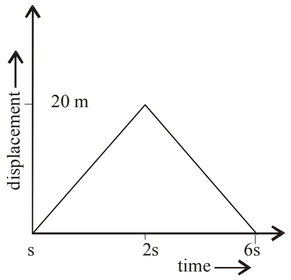
64. A football is rolling down a hill of unknown shape. The speed of the football at different times is noted as given below:

Time	Instantaneous speed
0 s	0 ms^{-1}
1 s	4 ms^{-1}
2 s	8 ms^{-1}
3 s	12 ms^{-1}
4 s	16 ms^{-1}

The correct shape of the hill is depicted in

(a) 
(b) 
(c) 
(d) 

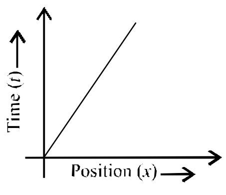
65. For the displacement-time graph shown in figure the ratio of the magnitudes of the speeds

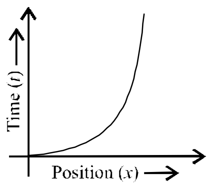


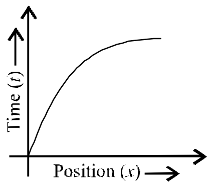
during the first two seconds and the next four seconds is

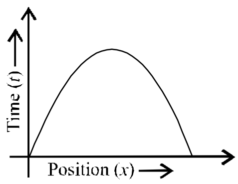
(a) 1 : 1 (b) 2 : 1
(c) 1 : 2 (d) 3 : 2

66. Which of the following represents an impossible situation?

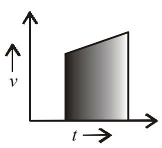
(a) 

(b) 

(c) 

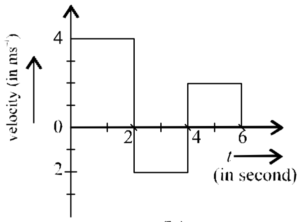
(d) 

67. What does the shaded portion in figure represent?



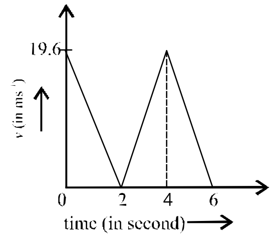
(a) momentum (b) acceleration
(c) distance (d) velocity.

68. The velocity-time graph of a body moving in a straight line is shown in figure. The displacement and distance travelled by the body in 6 second are respectively.



(a) 8 m, 16 m (b) 16 m, 8 m
(c) 16 m, 16 m (d) 8 m, 8 m

69. The velocity-time graph of a particle is as shown in figure.



(a) It moves with a constant acceleration throughout
(b) It moves with an acceleration of constant magnitude but changing direction at the end of every two second
(c) The displacement of the particle is zero
(d) The velocity becomes zero at $t = 4$ second.

70. A body released from the top of a tower falls through half the height of the tower in 2 s. In what time shall the body fall through the height of the tower?

(a) 4 s (b) 3.26 s
(c) 3.48 s (d) 2.828 s

71. Two bodies begin a free fall from the same height at a time interval of N s. If vertical separation between the two bodies is 1 after n second from the start of the first body, then n is equal to

(a) $\sqrt{nN}$ (b) $\frac{1}{gN}$
(c) $\frac{1}{gN}+\frac{N}{2}$ (d) $\frac{1}{gN}-\frac{N}{4}$

72. What is the angle between $(\vec{A}-\vec{B})$ and $(\vec{A}\times\vec{B})$?

(a) 0° (b) $\frac{\pi}{2}$ radian
(c) π radian (d) $\frac{3\pi}{2}$ radian

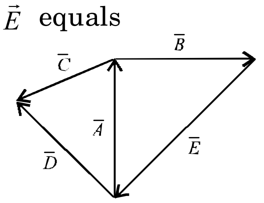
73. A boat which has a speed of 5 km h^{-1} in still water crosses a river of width 1 km along the shortest possible path in 15 minutes. The velocity of the river water is

(a) 1 $kmhr^{-1}$ (b) 3 $kmhr^{-1}$
(c) 4 $kmhr^{-1}$ (d) $\sqrt{41}$ $kmhr^{-1}$

74. If $|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is

(a) 0° (b) 60°
(c) 90° (d) 120°

75. In figure $\vec{E}$ equals



(a) $\vec{A}$ (b) $-\vec{A}$
(c) $\vec{A}+\vec{B}$ (d) $-(\vec{A}+\vec{B})$

76. The ratio of maximum and minimum magnitudes of the resultant of two vectors $\vec{a}$ and $\vec{b}$ is 3 : 1, Now, $|\vec{a}|$ =

(a) $|\vec{b}|$ (b) $2|\vec{b}|$
(c) $3|\vec{b}|$ (d) $4|\vec{b}|$

77. The resultant of two forces $3P$ and $2P$ is R. If the first force is doubled, then the resultant is also doubled. The angle between the two forces is

(a) 120° (b) 60°
(c) 180° (d) 90°

78. If $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{r} = 4\hat{j} - 3\hat{k}$ then the magnitude of $\vec{v}$ is

(a) $\sqrt{29}$ units (b) $\sqrt{31}$ units
(c) $\sqrt{37}$ units (d) $\sqrt{41}$ units

79. A force $\vec{F} = 2\hat{i} + 2\hat{j}$ newton displaces a particle through $\vec{s} = 2\hat{i} + 2\hat{k}$ meter in 16 second. The power developed by $\vec{F}$ is

(a) 0.25 Js^{-1} (b) 25 Js^{-1}
(c) 225 Js^{-1} (d) 450 Js^{-1}

80. The area of a parallelogram formed from the vectors $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$ as adjacent sides is

(a) $8\sqrt{3}$ units (b) 64 units
(c) 32 units (d) $4\sqrt{6}$ units

81. Given: $\vec{A} = 3\hat{i} - 4\hat{j} - 2\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 5\hat{k}$. The angle which $\vec{A}+\vec{B}$ makes with Y-axis is

(a) 0° (b) 45°
(c) 60° (d) 90°

82. A projectile fired with initial velocity v at some angle θ, has a range R. If the initial velocity be doubled at the same angle of projection, then the range will be

(a) $R/2$ (b) R
(c) $2R$ (d) $4R$.

83. One stone is projected horizontally from a 20 m high cliff with an initial speed of 10 ms^{-1}. A second stone is simultaneously dropped from that cliff. Which of the following is true?

(a) Both strike the ground with the same velocity
(b) The stone with initial speed 10 ms^{-1} reaches the ground first
(c) Both the stone reach ground at the same time
(d) One cannot say without knowing the height of the building.

84. Two stones are projected from the same point with same speed making angles 45° + θ and 45° − θ with the horizontal respectively. If θ ≤ 45°, then the horizontal ranges of the two stones are in the ratio of

(a) 1 : 1 (b) 1 : 2
(c) 1 : 3 (d) 1 : 4

85. Four bodies are projected with same speed at angles 30°, 45°, 55° and 65° with the horizontal. The horizontal range will be largest for the one projected at an angle of

(a) 30° (b) 45°
(c) 55° (d) 65°

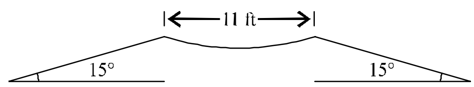
86. The maximum height attained by an oblique projectile does not depend upon

(a) velocity of projection
(b) acceleration due to gravity
(c) angle of projection
(d) mass of projectile.

87. Two balls are projected from the same point in directions at 30° and 60° with the horizontal. Both the balls attain the same height. The ratio of their velocities of projection is

(a) $\sqrt{3}$: 2 (b) $\sqrt{3}$: 3
(c) $\sqrt{3}$: 5 (d) $\sqrt{3}$: 1

88. Figure shows a 11 feet wide ditch with the approach roads at an angle of 15° with the horizontal. With what minimum speed should a motor bike be moving on the road so that it safely crosses the ditch? Assume that the length of the bike is 5 feet and it leaves the road when the front part runs out of the approach road. Take g = 32 feet/s^2.



(a) 32 feet s^{-1} (b) 16 feet s^{-1}
(c) 8 feet s^{-1} (d) 4 feet s^{-1}

89. An aeroplane flying horizontally with a velocity of 216 km h^{-1} drops a food packet while flying at a height of 490 m. The total horizontal distance travelled by the packet is
(a) 600 m (b) 490 m
(c) 216 m (d) 490 × 216 m

90. The x and y coordinates of a particle at any time t are given by $x = 2t + 4t^2$ and $y = 5t$, where x and y are in metre and t in second. The acceleration of the particle at $t = 5$ s is
(a) 40 m s^{-2} (b) 20 m s^{-2}
(c) 8 m s^{-2} (d) zero

91. A cricket ball is hit with a velocity 25 ms^{-1}, 60° above the horizontal. How far above the ground, ball passes over a fielder 50 m from the bat (consider the ball is struck very close to the ground). (Take $\sqrt{3}$ = 1.7 and g = 10 m s^{-2})
(a) 6.5 m (b) 7 m
(c) 5 m (d) 10 m.

92. A bomb is dropped from an aeroplane flying horizontally with a velocity 469 m s^{-1} at an altitude of 980 m. The bomb will hit the ground after a time
(a) 2 s (b) $\sqrt{2}$ s
(c) $5\sqrt{2}$ s (d) $10\sqrt{2}$ s.

93. Two projectiles, one fired from earth with 5 m s^{-1} and the other fired from a planet with 3 m s^{-1} trace identical trajectories. If acceleration due to gravity on earth is 9.8 m s^{-2}, then the acceleration due to gravity on the planet is
(a) 1.5 m s^{-2} (b) 3.5 m s^{-2}
(c) 7.5 m s^{-2} (d) 9.5 m s^{-2}

94. An arrow is shot into air. Its range is 200 m and its time of flight is 5 second.
If g = 10ms^{-2}, then the horizontal component of the velocity of arrow is
(a) 12.5 m s^{-1} (b) 25 m s^{-1}
(c) 31.25 m s^{-1} (d) 40 m s^{-1}

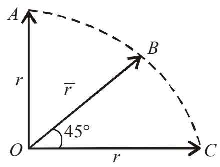
95. A large number of bullets are fired in all directions with the same speed v. What is the maximum area on the ground on which these bullets will spread?
(a) $\frac{\pi v^2}{g}$ (b) $\frac{\pi v^4}{g^2}$
(c) $\frac{\pi^2 v^4}{g^2}$ (d) $\frac{\pi^2 v^2}{g^2}$

96. Two projectiles A and B are thrown with velocities v and $\frac{v}{2}$ respectively. They have the same range. If B is thrown at an angle of 15° to the horizontal, A must have been thrown at an angle
(a) $\sin^{-1}\left(\frac{1}{16}\right)$ (b) $\sin^{-1}\left(\frac{1}{4}\right)$
(c) $2\sin^{-1}\left(\frac{1}{4}\right)$ (d) $\frac{1}{2}\sin^{-1}\left(\frac{1}{8}\right)$

97. Two non-zero vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$?
(a) 0° (b) 90°
(c) 60° (d) 180°

98. Rain is falling vertically downwards with a speed of 4 km h^{-1}. A girl moves on a straight road with a velocity of 3 km h^{-1}. The apparent velocity of rain with respect to the girl is
(a) 3 km h^{-1} (b) 4 km h^{-1}
(c) 5 km h^{-1} (d) 7 km h^{-1}

99. The resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ shown in figure is



(a) r (b) $2r$
(c) $r(1+\sqrt{2})$ (d) $r(\sqrt{2}-1)$

100. The following four forces act simultaneously on a particle at rest at the origin of the co-ordinate system.

$\vec{F}_1 = 2\hat{i} - 3\hat{j} - 2\hat{k}$, $\vec{F}_2 = 5\hat{i} + 8\hat{j} + 6\hat{k}$

$\vec{F}_3 = -4\hat{i} - 5\hat{j} + 5\hat{k}$, $\vec{F}_4 = -3\hat{i} + 4\hat{j} - 7\hat{k}$

The particle will move in
(a) XY plane (b) YZ plane
(c) ZX plane (d) space.

101. Angle between two vectors $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$ is
(a) 30° (b) 60°
(c) 45° (d) 90°

102. The adjacent sides of a parallelogram are represented by $\hat{i} + 4\hat{j}$ and $7\hat{i} + 3\hat{j}$. The area of the parallelogram is
(a) 10 units (b) 15 units
(c) 25 units (d) 30 units

103. The area of the parallelogram whose diagonals are represented by the vectors $3\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$ is

(a) 28 units (b) 8.66 units
(c) 21 units (d) 18 units

104. The position vectors of the head and tail of radius vector are $2\hat{i} + \hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + \hat{k}$. The linear momentum is $2\hat{i} + 3\hat{j} + \hat{k}$. The angular momentum is

(a) $4\hat{i} - 8\hat{k}$ (b) $2\hat{i} + \hat{j} + \hat{k}$
(c) $2\hat{i} - 3\hat{j} + \hat{k}$ (d) $2\hat{i} + 3\hat{j} + \hat{k}$

105. The area of the triangle formed by the adjacent sides with $\vec{A} = -3\hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$ is

(a) $\frac{\sqrt{165}}{2}$ units (b) $\frac{\sqrt{137}}{2}$ units
(c) $\sqrt{165}$ units (d) $\sqrt{137}$ units

106. The time of flight of a projectile on an upward inclined plane depends upon

(a) angle of inclination of the plane
(b) angle of projection
(c) the value of acceleration due to gravity
(d) all of these.

107. Two bullets are fired horizontally, simultaneously and with different velocities from the same place. Which bullet will hit the ground earlier?

(a) It would depend upon the weights of the bullets.
(b) The slower one
(c) The faster one
(d) Both will reach simultaneously

108. A projectile rises to a height of 10 m above and then falls at a distance 30 m away from the point of projection. Its vertical displacement is

(a) 10 m (b) 20 m
(c) 30 m (d) zero

109. An aeroplane flying horizontally with a speed of 360 km h^{-1} releases a bomb at a height of 490 m from the ground. When will the bomb strike the ground?

(a) 8 s (b) 6 s
(c) 7 s (d) 10 s

110. A body is thrown with a velocity of 10 ms^{-1} at angle of 60° with the horizontal. Its velocity at the highest point is

(a) 7 m s^{-1} (b) 9 m s^{-1}
(c) 18.7 m s^{-1} (d) 5 m s^{-1}.

111. In the case of an oblique projectile, the velocity is perpendicular to acceleration

(a) once only (b) twice
(c) thrice (d) four times

112. Two particles move in a uniform gravitational field with an acceleration g. At the initial moment, the particles were located at one point and move with velocity v_1 = 4 m s^{-1} and v_2 = 1 m s^{-1} horizontally in opposite directions. The distance between the particles at the moment when their velocity vectors become mutually perpendicular is (Take g = 10 ms^{-2}).

(a) 1 m (b) 2 m
(c) 3 m (d) 5 m.

113. A body is projected at 30° angle with the horizontal with velocity 30 m s^{-1}. What is the angle with the horizontal after 1.5 s? (Take g = 10 m s^{-2}).

(a) 0° (b) 30°
(c) 60° (d) 90°

114. A monkey can jump a maximum horizontal distance of 20 m. Then the velocity of the monkey is

(a) 10 m s^{-1} (b) 14 m s^{-1}
(c) 20 m s^{-1} (d) 24 m s^{-1}

115. A projectile is projected with kinetic energy of 800 J. If it has the maximum possible horizontal range, then its kinetic energy at the highest point will be

(a) 800 J (b) 400 J
(c) 200 J (d) 100 J.

116. A projectile is thrown at angle β with vertical. It reaches a maximum height H. The time taken to reach the highest point of its path is

(a) $\sqrt{\frac{H}{g}}$ (b) $\sqrt{\frac{2H}{g}}$
(c) $\sqrt{\frac{H}{2g}}$ (d) $\sqrt{\frac{2H}{g\cos\beta}}$

117. A plane flying horizontally at 98 ms^{-1} releases an object which reaches the ground in 10 second. The angle made by the velocity of the object with the horizontal at the time of hitting the ground is

(a) 30° (b) 45°
(c) 60° (d) 75°

118. A projectile of mass m is thrown with a velocity v making an angle of 45° with the horizontal. The change in momentum from departure to arrival along vertical direction, is

(a) 2 mv (b) $\sqrt{2}\ mv$

(c) mv (d) $\frac{mv}{\sqrt{2}}$

119. The range of a projectile when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle of 45° with horizontal with the same velocity?

(a) 1.5 km (b) 3.0 km

(c) 6.0 km (d) 0.75 km

120. If the value of acceleration due to gravity is 10 ms^{-2} and the time of flight is 5 second, then the maximum height reached by the projectile is nearly

(a) 31 m (b) 69 m

(c) 46 m (d) 79 m

121. A ball is projected horizontally at 20 ms^{-1}. The approximate speed of the ball after 5 second is (Take g = 10 ms^{-2})

(a) 24 m s^{-1} (b) 34 m s^{-1}

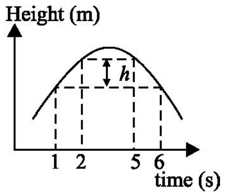
(c) 44 m s^{-1} (d) 54 m s^{-1}

122. A 210 metre long train is moving due north at a speed of 25 m/s. A small bird is flying due south a little above the train with speed 5 m/s. The time taken by the bird to cross the train is

(a) 6 s (b) 7 s

(c) 9 s (d) 10 s

123. A ball is thrown upwards. Its height varies with time as shown in figure. If the acceleration due to gravity is 7.5 m/s^2, then the height h is



(a) 10 m (b) 15 m

(c) 20 m (d) 25 m

124. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min is equal to

(a) 5 km h^{-1} (b) 25/4 km h^{-1}

(c) 30/4 km h^{-1} (d) 45/8 km h^{-1}

125. A river 4.0 miles wide is flowing at the rate of 2 miles/hr. The minimum time taken by a boat to cross the river with a speed v = 4 miles/hr (in still water) is approximately

(a) 1 hr and 9 minutes

(b) 2 hr and 7 minutes

(c) 1 hr and 12 minutes

(d) 2 hr and 25 minutes

126. A point object transverses half the distance with velocity v_0. The remaining part of the distance was covered with velocity v_1 for the half the time and with velocity v_2 for the rest half. The average velocity of the object for the whole journey is

(a) $2v_1(v_0 + v_2) / (v_0 + 2v_1 + 2v_2)$

(b) $2v_0(v_0 + v_1) / (v_0 + v_1 + v_2)$

(c) $2v_0(v_1 + v_2) / (v_1 + v_2 + 2v_0)$

(d) $2v_2(v_0 + v_1) / (v_1 + 2v_2 + v_2)$

127. An elevator is accelerating upward at a rate of 6 ft/sec^2 when a bolt from its ceiling falls to the floor of the lift (Distance = 9.5 feet). The time taken (in seconds) by the falling bolt to hit the floor is (take g = 32 ft/sec^2)

(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $2\sqrt{2}$ (d) $\frac{1}{2\sqrt{2}}$

128. A river 2 km wide is flowing at the rate of 2 km/hr. A boatman, can row the boat at a speed of 4 km/hr in still water, goes a distance of 2 km upstream and then comes back. The time taken by him to complete his journey is

(a) 60 minutes (b) 70 minutes

(c) 80 minutes (d) 90 minutes

129. The displacement x of a particle varies with time t as $x = ae^{-\alpha t} + be^{\beta t}$, where a, b, α and β are positive constants. The velocity of the particle will

(a) be independent of α and β

(b) go on increasing with time

(c) drop to zero when $\alpha = \beta$

(d) go on decreasing with time.

130. Four persons K, L, M and N are initially at the corners of a square of side of length d. If every person starts moving with the same speed v such that K is always headed towards L, L towards M, M is headed directly towards N and N towards K, then the four persons will meet after

(a) d/v seconds (b) $\sqrt{2}d/v$ seconds

(c) $d/\sqrt{2}v$ seconds (d) $d/2v$ seconds.

131. A juggler keeps n balls going with one hand, so that at any instant, $(n - 1)$ balls are in air and one ball in the hand. If each ball rises to a height of x metres, the time for each ball to stay in his hand is

(a) $\frac{1}{n-1}\sqrt{\frac{2x}{g}}$ (b) $\frac{2}{n-1}\sqrt{\frac{2x}{g}}$

(c) $\frac{2}{n}\sqrt{\frac{2x}{g}}$ (d) $\frac{1}{n}\sqrt{\frac{2x}{g}}$

132. A body starts from rest with uniform acceleration. The velocity of the body after t second is v. The displacement of the body in last three seconds is

(a) $\frac{3v}{2}(t-3)$ (b) $\frac{3v}{2}(t+3)$

(c) $3v\left(1-\frac{3}{2t}\right)$ (d) $3v\left(1+\frac{3}{2t}\right)$

133. A block slides down a smooth inclined plane to the ground when released at the top, in time t sec. Another block is dropped vertically from the same point in the absence of the inclined plane and reaches the ground in $t/2$ sec. Then the angle of the inclination of the plane is

(a) 30° (b) 45°
(c) 60° (d) 75°.

134. Which of the following does not effect the maximum height attained by projectile.

(a) Acceleration of the projectile
(b) Magnitude of the initial velocity
(c) Mass of projectile
(d) Angle of projection.

135. A car is moving with speed 30 m/s on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/s^2. What is the acceleration of the car?

(a) 2 m/s^2 (b) 2.7 m/s^2
(c) 1.8 m/s^2 (d) 9.8 m/s^2

136. When air resistance is taken into account while dealing with the motion of the projectile which of the following properties of the projectile, shows an increase,

(a) range
(b) maximum height
(c) speed at which it strikes the ground
(d) the angle at which the projectile strikes the ground.

137. A small block slides down from the top of a hemisphere of radius R. It is assumed that there is no friction between the block and the hemisphere. At what height h the block will lose contact with the surface of the sphere

(a) $2R/3$ (b) $R/2$
(c) $R/3$ (d) $R/4$

138. If water and mercury are rotated in a test tube

(a) water will be forced to the outer part
(b) mercury will be forced to the outer part
(c) water and mercury will be throughly mixup
(d) water will be at the top and mercury at the bottom.

139. For traffic moving at 60 km/hour along a circular track of radius 0.1 km, the correct angle of banking is

(a) $\tan^{-1}\frac{(60)^2}{0.1}$ (b) $\tan^{-1}\frac{(50/3)^2}{100\times 9.8}$

(c) $\tan^{-1}\left[\frac{100\times 9.8}{(50/3)^2}\right]$ (d) $\tan^{-1}(60 \times 0.1 \times 9.8)$

140. A body is projected at an angle α to the horizontal so as to clear two walls of equal height h at a distance $2h$ from each other. The horizontal range of the projectile is:

(a) $2h\cos(\alpha/2)$ (b) $\frac{h}{2}\sin^2\alpha$

(c) $2h\sin 2\alpha$ (d) $2h\cot(\alpha/2)$

141. A cannon and a target are 5.10 km apart and located at the same level. How soon will the shell launched with the initial velocity 240 m/s reach the target in the absence of air drag?

(a) 0.71 min (b) 0.41 min
(c) both a & b (d) none of these

142. A car is moving in a circular horizontal track of radius 10 m with a constant speed of 10 m/sec. A plumb bob is suspended from the roof of the car by a light rigid rod of length 1.00 m. The angle made by the rod with vertical is:

(a) zero (b) 30°
(c) 45° (d) 60°

143. A body is projected horizontally from a point above the ground and motion of the body is described by the equation $x = 2t, y = 5t^2$ where x, and y are horizontal and vertical coordinates in metre after time t. The initial velocity of the body will be

(a) $\sqrt{29}$ m/s horizontal
(b) 5 m/s horizontal
(c) 2 m/s vertical
(d) 2 m/s horizontal

144. A projectile can have the same range for two angles of projection. If h_1 and h_2 are maximum heights when the range in the two cases is R, then the relation between R, h_1 and h_2 is

(a) $R = 4\sqrt{h_1 h_2}$ (b) $R = 2\sqrt{h_1 h_2}$

(c) $R = \sqrt{h_1 h_2}$ (d) none of these

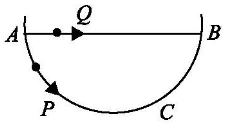
145. A bullet is fired with a speed of 1500 m/s in order to hit a target 100 m away. If g = 10 m/s^2. The gun should be aimed

(a) 15 cm above the target
(b) 10 cm above the target
(c) 2.2 cm above the target
(d) directly towards the target

146. A wheel rotates with constant acceleration of 2.0 rad/s^2, if the wheel starts from rest the number of revolutions it makes in the first ten seconds will be approximately.
(a) 32 (b) 24
(c) 16 (d) 8

147. An aeroplane is flying in a horizontal direction with a velocity u and at a height of 2000 m. When it is vertically above a point A on the ground it releases a bomb which strikes the ground at point B. If AB = 3 km and g = 10 m/s^2. The value of u is
(a) 300 km/hr (b) 150 km/hr
(c) 540 km/hr (d) 54 km/hr

148. A particle P is sliding down a frictionless hemispherical bowl. It passes the point A at t = 0. At this instant of time, the horizontal component of its velocity is v. A bead Q of the same mass as P is ejected from A at t = 0 along the horizontal string AB, with the speed v. Friction between the bead and the string may be neglected. Let t_P and t_Q be the respective times taken by P and Q to reach the point B. Then



(a) $t_P < t_Q$
(b) $t_P = t_Q$
(c) $t_P > t_Q$
(d) $\frac{t_P}{t_Q} = \frac{\text{length of arc } ACB}{\text{length of chord } AB}$.

149. The angular velocity of a particle rotating in a circular orbit 100 times per minute is
(a) 10.47 radian/sec (b) 10.0 degree/sec
(c) 40.32 radian/sec (d) 20.57 radian/sec

150. A cart is moving horizontally along a straight line with constant speed 30 m/s. A projectile is to be fired from the moving cart in such a way that it will return to the cart after the cart has moved 80 m. At what speed (relative to the cart) and at what angle (to the horizontal) must the projectile be fired ? (Take g = m/s^2)
(a) $10\sqrt{8}$ m/s at 45°
(b) $10\sqrt{8}$ m/s at $\cos^{-1}\left(\frac{3}{\sqrt{8}}\right)$
(c) $\frac{40}{3}$ m/s at 90°
(d) none of these.

151. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60° (b) yes, 30°
(c) no (d) yes, 45°.

152. A cannon at the ground fires a shell at speed of 50 km/hr. If the angle of projection is 30°, the range of the shell (take g = 10 m/s^2) is, in km
(a) $2.5\sqrt{3}$ (b) $62.5\sqrt{3}$
(c) $125\sqrt{3}$ (d) $9.65\sqrt{3} \times 10^{-3}$

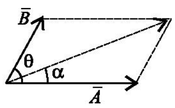
153. A body of mass m is projected to describe a parabolic path in vertical plane with velocity $(3\hat{i} + 8\hat{j})$ m/s. The time of flight of the body is about
(a) 3.2 sec (b) 1.6 sec
(c) 6.4 sec (d) 1.2 sec

154. The moving fan has a constant angular acceleration of π/6 rad/s^2. If it starts from rest, the number of revolution it makes in one minute is
(a) 1600 (b) 500
(c) 350 (d) 150

155. A projectile is thrown obliquely into the air from origin of a reference frame with y-axis vertically upward and x-axis perpendicular to y then equation of motion has the form
(a) $y = ax + bx^2$ (b) $y = ax^2 + bx^3$
(c) $y = a + bx$ (d) $y = (a + b)x$
Here a and b are constants.

156. Rain is falling vertically with a speed of 35 ms^{-1}. Wind starts blowing after some time with a speed of 12 ms^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella, with the vertical?
(a) $\sin^{-1}\frac{12}{35}$ (b) $\cos^{-1}\frac{12}{35}$
(c) $\tan^{-1}\frac{12}{35}$ (d) $\cot^{-1}\frac{12}{35}$

157. The magnitude and direction of the resultant of two vectors $\bar{A}$ and $\bar{B}$ in terms of their magnitudes and angle θ between them is
(a) $R = \sqrt{A^2 + B^2 - 2AB\cos B}$,
$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$



(b) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$,

$\tan\alpha = \dfrac{B\cos\theta}{A + B\sin\theta}$

(c) $R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$,

$\tan\alpha = \dfrac{A\sin\theta}{B + A\cos\theta}$

(d) $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$,

$\tan\alpha = \dfrac{A\sin\theta}{B + A\cos\theta}$

158. A motorboat is racing towards north at 25 kmph and the water current in that region is 10 kmph in the direction of 60° east of south. The resultant speed of the boat is

(a) 22 kmph (b) 31 kmph
(c) 34 kmph (d) 41 kmph

159. The position of a particle is given by

$\bar{r} = 3t\,\hat{i} + 2t^2\,\hat{j} + 5\,\hat{k}$

where t is in seconds and the coefficients have proper units for $\bar{r}$ to be in metres. The direction of $V(t)$ at t = 1s is

(a) 37° with the x-axis
(b) 53° with the x-axis
(c) 45° with the y-axis
(d) 60° with the y-axis

160. A particle starts from origin at $t = 0$ with a velocity $5\,\hat{i}$ m/s and moves in x - y plane under action of a force which produces a constant acceleration of $(3\hat{i} + 2\hat{j})$ m/s^2. The y-coordinate of the particle, when its x-coordinate is 84 m is

(a) 12 m (b) 24 m
(c) 36 m (d) 48 m

161. Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 ms^{-1} in east to west direction. The direction in which she should hold her umbrella is

(a) $\theta = \tan^{-1}\left(\dfrac{12}{35}\right)$ with vertical towards the east

(b) $\theta = \tan^{-1}\left(\dfrac{12}{35}\right)$ with vertical towards the west

(c) $\theta = \tan^{-1}\left(\dfrac{12}{37}\right)$ with vertical towards the north

(d) $\theta = \tan^{-1}\left(\dfrac{12}{37}\right)$ with vertical towards the south

162. For elevations which exceed or fall short of θ by equal amounts, the ranges are equal hence θ equals

(a) 30° (b) 45°
(c) 60°
(d) there is no such angle.

163. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an intial speed of 15 ms^{-1}. The speed with which it hits the ground is

(a) 99 ms^{-1} (b) 101 ms^{-1}
(c) 103 ms^{-1} (d) 105 ms^{-1}

164. A batsman hits a ball from the ground level with a speed of 28 ms^{-1} in a direction 30° above the horizontal. How far from the batsman does the ball land for the first time?

(a) 45 m (b) 53 m
(c) 59 m (d) 69 m

165. Pick out the only vector quantity in the following list :

(a) pressure
(b) impulse
(c) gravitational potential
(d) coefficient of friction.

166. On an open ground, a motorist follows a truck that turns to his left by an angle of 60° after every 500 m. Starting from a given turn counted as one, what is the displacement of the motorist at the fourth turn?

(a) 500 m (b) $500\sqrt{3}$ m
(c) 1000 m (d) $1000\sqrt{3}$ m

167. A man can swim with a speed of 4 kmph in still water. He crosses a river 1km wide that flows steadily at 3 kmph. If he makes his strokes normal to the river current, how far down the river does he go when he reaches the other bank?

(a) 500 m (b) 600 m
(c) 750 m (d) 850 m

168. In a harbour, wind is blowing at the speed of 72 kmph and the flag on the mast of a boat anchored in the harbour flutters along the N-E direction. If the boat starts moving at a speed of 51 km/h to the north, what is the direction of the flag on the mast of the boat?

(a) east (b) 37° north of east
(c) 37° south of east (d) south-west.

169. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 ms^{-1} can go without hitting the ceiling.

(a) 108 m (b) 120 m
(c) 150 m (d) 162 m

170. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How much high above the ground can the cricketer throw the same ball?
(a) 50 m (b) 60 m
(c) 75 m (d) 80 m

171. The position of a particle is given by
$\bar{r} = (3t\hat{i} - 2t^2\hat{j} + 4\hat{k})$ m
Where t is in seconds and the coefficients have the proper units for $\bar{r}$ to be in metres. The acceleration of the particle in m/s^2 at t = 2 s is
(a) $2\hat{j}$ (b) $3\hat{i} - 2\hat{j}$
(c) $-2\hat{j}$ (d) $-4\hat{j}$

172. A particle starts from the origin at t = 0 sec with a velocity of $10\hat{j}$ m/s and moves in the x-y plane with a constant acceleration of $(8\hat{i} + 2\hat{j})$ ms^{-2}. What is the speed of the particle when its x-coordinate is 16 m.
(a) 18 m s^{-1} (b) 21 m s^{-1}
(c) 25 m s^{-1} (d) 32 m s^{-1}

173. What are the components of a vector $\hat{A} = 2\hat{i} + 3\hat{j}$ along the directions of $(\hat{i} + \hat{j})$ and $(\hat{i} - \hat{j})$
(a) $\left(2, \frac{1}{2}\right)$ (b) $\left(\frac{5}{2}, \frac{-1}{2}\right)$
(c) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{-5}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

174. For any arbitrary motion, which of the following relations are true
(a) $\bar{v}_{\text{average}} = \left(\frac{1}{2}\right)(\bar{v}(t_1) + \bar{v}(t_2))$
(b) $\bar{v}_{\text{Average}} = \frac{\bar{r}(t_2) - \bar{r}(t_1)}{(t_2 - t_1)}$
(c) $\bar{v}(t) = \bar{v}(0) + \bar{a}t$
(d) $\bar{v}(t) = \bar{r}(0) + \bar{v}(0) \cdot t + \frac{1}{2}\bar{a}t^2$

175. A scalar quantity is one that
(a) is conserved in a process
(b) can never take negative values
(c) does not vary from one point to another in space
(d) has the same value for observers with different orientations of axes.

176. A bullet fired at an angle of 30° with the horizontal hits the ground 3 km away. By adjusting its angle of projection, what is the farthest distance can one hope to hit.
(a) 3.46 km (b) 3.82 km
(c) 4.12 km (d) 4.46 km

177. A fighter plane flying horizontally at an altitude of 1.5 km with speed of 720 km/h passes directly overhead an anti-aircraft gun. At what angle from the vertical should the gum be fired for the shell with muzzle speed 600 ms^{-1} to hit the plane?
(a) $\sin^{-1}\frac{1}{3}$ (b) $\sin^{-1}\frac{2}{3}$
(c) $\cos^{-1}\frac{1}{3}$ (d) $\cos^{-1}\frac{2}{3}$

178. In the above question, at what minimum altitude should the pilot fly the plane to avoid being hit.
(a) 8 km (b) 10 km
(c) 12 km (d) 16 km

179. For a projection, the angle between the velocity and x-axis as a function of time is given by $\theta =$
(a) $\sin^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$ (b) $\tan^{-1}\left(\frac{v_{oy} - gt}{v_{ox}}\right)$
(c) $\sin^{-1}\left(\frac{v_{oy} + gt}{v_{ox}}\right)$ (d) $\tan^{-1}\left(\frac{v_{oy}}{v_{ox}}\right)$

180. The projection angle θ_o for a projectile launched from the origin is given by $\theta_o =$
(a) $\tan^{-1}\left(\frac{h_m}{R}\right)$ (b) $\tan^{-1}\left(\frac{2h_m}{R}\right)$
(c) $\tan^{-1}\left(\frac{4h_m}{R}\right)$ (d) $\tan^{-1}\left(\frac{8h_m}{R}\right)$

181. The position of an object moving along x-axis is given by $x = a + bt^2$ where a = 8.5 m and b = 2.5 ms^{-2} and t is measured in seconds. Its velocity at t = 2 s and the average velocity between t = 2 s and 4 s is
(a) 2 ms^{-1}, 5 ms^{-1} (b) 5 ms^{-1}, 10 ms^{-1}
(c) 10 ms^{-1}, 15 ms^{-1} (d) 10 ms^{-1}, 20 ms^{-1}

182. A ball is thrown vertically upwards with a velocity of 20 ms^{-1} from the top of a multistorey building. The height of the point from where the ball is thrown is 25 m from the ground. The height to which the ball rises from the ground and the time taken before the ball hits the ground are (Take g = 10 ms^{-2})
(a) 20 m, 2 s (b) 40 m, 3 s
(c) 40 m, 5 s (d) 45 m, 5 s

183. The distances transversed, during equal intervals of time, by a body falling from rest, stand to one another in the same ratio as

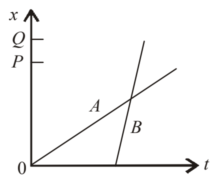
(a) 1 : 2 : 3 : 4 = ...
(b) 1 : 3 : 5 : 7 = ...
(c) 1 : 4 : 9 : 16 = ...
(d) 1 : 1 : 2 : 3 = ...

184. Two parallel rail-tracks run north-south. Train A moves north with a speed of 54 km h^{-1} and train B moves south with a speed of 90 km h^{-1}. The velocity of a monkey running on the roof of train A against its motion (with a velocity of 18 km h^{-1} w.r.t. train A) as observed by a man travelling in train B is
(a) 20 ms^{-1} (b) 25 ms^{-1}
(c) 30 ms^{-1} (d) 35 ms^{-1}

185. Mark the wrong statement in the following for one dimensional motion
(a) The path length traversed by an object between two points is in general not the same as the magnitude of displacement.
(b) Average speed can be greater than or equal to the magnitude of average velocity.
(c) Average speed can be less than magnitude of average velocity
(d) In one dimension, average speed and magnitude of average velocity are equal only if the object does not change its direction during the course of motion

186. Mark the incorrect statement in the following
(a) The sign of acceleration tells us whether the particle's speed is increasing or decreasing
(b) The zero velocity of a particle's speed is increasing or decreasing
(c) The zero velocity of a particle does not necessarily imply zero acceleration at that instant
(d) The kinematic equations ($v = u + at$, $s = ut + \frac{1}{2}at^2$, $v^2 = u^2 + 2as$) are true only for motion in which the magnitude and the direction of acceleration are constant during the course of motion.

187. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in the graph. Choose the correct entry below:



(a) B lives closer to school than A
(b) B starts from the school earlier than A
(c) A walks faster than B
(d) B overtakes A on the road.

188. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward followed again by 5 steps forward and 3 steps backward and so on. Each step is 1 m long and requires 1 s. The time the drunkard takes to fall in a pit 13 m away is
(a) 13 s (b) 32 s
(c) 37 s (d) 49 s

189. A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 kmh^{-1} relative to the jet plane. The speed of the products of combustion with respect to an observer on the ground is
(a) 500 kmh^{-1} (b) 1000 kmh^{-1}
(c) 1500 kmh^{-1} (d) 2000 kmh^{-1}

190. A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200 m. The retardation of the car and the time for it to stop are
(a) 3.06 ms^{-2}, 11.4s (b) 3.5 ms^{-2}, 12.2 s
(c) 4.0 ms^{-2}, 16 s (d) 4.2 ms^{-2}, 15.4 s

191. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction with A ahead of B. The driver of B decides to overtake A and accelerates by 1 ms^{-2}. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?
(a) 500 m (b) 750 m
(c) 1050 m (d) 1250 m

192. On a two-lane road, car A is travelling with a speed of 36 kmh^{-1}. Two cars B and C approach car A in opposite directions, with a speed of 54 kmh^{-1}. At a certain instant, when the distance of AB is equal to AC, both being 1 km, B decides to overtake A before C does. The minimum required acceleration of car B to avoid an accident is
(a) 1 m/s^2 (b) 1.5 m/s^2
(c) 2 m/s^2 (d) 3 m/s^2

193. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T minutes. A man cycling with a speed of 20 kmh^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. The period T of bus service is
(a) 4.5 min (b) 9 min
(c) 12 min (d) 24 min

194. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}. Taking g = 9.8 ms^{-2}, the time taken by the ball to return to the player's hand is

(a) 3 s (b) 6 s
(c) 9 s (d) 12 s

195. Mark the true statement

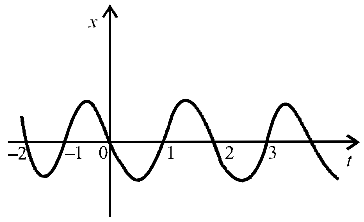
A particle in one-dimensional motion

(a) with zero speed at an instant may have non-zero acceleration at that instant
(b) with non-zero speed may have non-zero velocity
(c) with constant speed, may not have zero acceleration
(d) with positive value of acceleration must be speeding up

196. A police van moving on a highway with a speed of 30 kmh^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 kmh^{-1}. If the muzzle speed of the bullet is 150 ms^{-1}, with what speed does the bullet hit the thief's car?

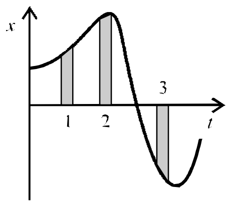
(a) 90 ms^{-1} (b) 105 ms^{-1}
(c) 110 ms^{-1} (d) 120 ms^{-1}

197. Figure gives the x-t plot of a particle executing one dimensional simple harmonic motion. Mark the correct combination. The signs of position, velocity and acceleration variables at



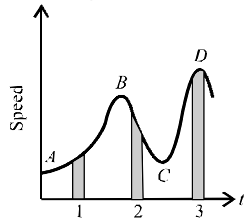
(a) t = 0.3 s, x < 0, v < 0, a < 0
(b) t = 1.2 s, x > 0, v > 0, a > 0
(c) t = –1.2 s, x > 0, v > 0, a < 0
(d) t = –0.3 s, x > 0, v < 0, a < 0

198. Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. Let v_1, v_2, v_3 be the average speed in each of the intervals respectively, then



(a) $v_1 < v_2 > v_3$ (b) $v_2 > v_1 > v_3$
(c) $v_3 > v_1 > v_2$ (d) $v_2 > v_1 < v_3$

199. Figure gives speed time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. Mark the incorrect statement. Among the three intervals,



(a) The average acceleration in the second interval is greatest
(b) The average speed is greatest in the third interval
(c) The acceleration at A, B, C, D is zero.
(d) The distance travelled in the second interval is greatest.

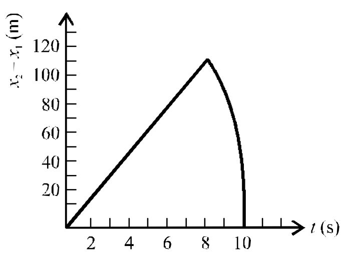
200. A boy standing on a stationary lift (one from above) throws a ball upwards with the maximum initial speed he can, equal to 49 ms^{-1}. t_1 is the time the ball takes to return to his hands. If the lift starts moving up with a uniform speed of 5 ms^{-1} and the boy again throws the ball up with the maximum speed he can, now the ball returns to his hand in t_2 seconds. (Take g = 9.8 ms^{-2})

(a) t_1 = 10 s, t_2 = 10 s
(b) t_1 = 10 s, t_2 = 8 s
(c) t_1 = 8 s, t_2 = 10 s
(d) t_1 = 9 s, t_2 = 6 s

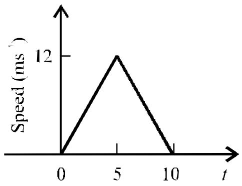
201. On a long horizontally moving belt, a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. The time taken by the child in running against the motion of the belt, between his parents is

(a) 10 s (b) 20 s
(c) 25 s (d) 30 s

202. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Taking g = 10 ms^{-2}, the graph of relative position of the second stone with respect to the first has been shown. The equation of the curved part is

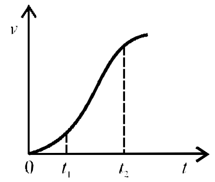


(a) $100 + 15t - t^2$ (b) $200 + 30t - 5t^2$
(c) $200 - 15t + 5t^2$ (d) $100 - 30t + 5t^2$

203. 

The speed time graph of a particle moving along a fixed direction is shown. The distance traversed by the particle in 10 s is
(a) 20 m (b) 30 m
(c) 40 m (d) 60 m

204. The velocity-time graph of a particle in one-dimensional motion is shown.
Which of the following formulae are correct for describing the motion of the particle over the time interval t_1 to t_2?



(a) $x(t_2) = x(t_1) + v(t_1)\cdot(t_2 - t_1) + \frac{1}{2}a(t_2 - t_1)^2$
(b) $v(t_2) = v(t_1) + a(t_2 - t_1)$
(c) $v_{\text{average}} = \frac{x(t_2)+x(t_1)}{2(t_2-t_1)}$
(d) $a_{\text{average}} = \frac{(v(t_2)-v(t_1))}{(t_2-t_1)}$

Assertion & Reason

Directions: *Each of these question contains statement-1 (Assertion) and Statement-2 (Reason). Each question has four choices. You have to select the correct choice.*
(a) if both statement-1 and statement-2 are true and statement-2 is the correct explanation of statement-1
(b) if both statement-1 and statement-2 are true but statement-2 is not the correct explanation of statement-1
(c) if statement-1 is true but statement-2 is false
(d) if statement-1 is false and statement-2 is true

1. **Statement-1 :** For straight line motion, the velocity and acceleration of an object are always along the same straight line.
Statement-2 : Only the magnitude of velocity changes due to acceleration in straight line motion.

2. **Statement-1 :** The magnitude of average velocity is the same as the average speed.
Statement-2 : The magnitude of instantaneous velocity is the magnitude of instantaneous speed.

3. **Statement-1 :** A body may be accelerated even when it is moving at uniform speed.
Statement-2 : When direction of motion of the body is changing then body may have acceleration.

4. **Statement-1 :** Two balls of different masses are thrown vertically upward with same speed. They will pass through their point of projection in the downward direction with the same speed.
Statement-2 : The maximum height and downward velocity attained at the point of projection are independent of the mass of the ball.

5. **Statement-1 :** The position-time graph of a body may be a straight line parallel to time axis.
Statement-2 : It is possible that position of a body does not change with time.

6. **Statement-1 :** The average speed of an object may be equal to arithmetic mean of individual speed.
Statement-2 : Average speed is equal to total distance travelled divided by total time taken.

7. **Statement-1 :** The average and instantaneous velocities have same value in a uniform motion.
Statement-2 : In uniform motion, the velocity of an object increases uniformly.

8. **Statement-1 :** A body falling freely may do so with constant velocity.
Statement-2 : The body falls freely, when acceleration of a body is equal to acceleration due to gravity.

9. **Statement-1 :** Displacement of a body is the signed sum of the area under velocity - time graph.
Statement-2 : Displacement is a vector quantity.

10. **Statement-1 :** In projectile motion, the angle between the instantaneous velocity and acceleration at the highest point is 180º.
Statement-2 : At the highest point, velocity of projectile will be in horizontal direction only.

11. **Statement-1 :** Two particle of different mass, projected with same velocity, the maximum height attained by both the particle will be same.
Statement-2 : The maximum height of projectile is independent of particle mass.

12. **Statement-1 :** The maximum horizontal range of projectile is proportional to square of velocity.
Statement-2 : The maximum horizontal range of projectile is equal to maximum height attained by projectile.

13. **Statement-1 :** The trajectory of projectile is quadratic in x and linear in y.
Statement-2 : y component of trajectory is independent of x-component.

14. **Statement-1 :** When a body is dropped or thrown horizontally from the same height, it would reach the ground at the same time.
Statement-2 : Horizontal velocity has no effect on the vertical direction.

15. **Statement-1 :** When the velocity of projection of a body is made n times, its time of flight becomes n times.
Statement-2 : Range of projectile does not depend on the initial velocity of a body.

16. **Statement-1 :** When range of a projectile is maximum, its angle of projection may be 45° or 135°.
Statement-2 : Whether θ is 45° or 135°, value of range remains the same, only the sign changes.

17. **Statement-1 :** For projection angle $\tan^{-1}(4)$, the horizontal range and the maximum height of a projectile are equal.
Statement-2 : The maximum range of projectile is directly proportional to square of velocity and inversely proportional to acceleration due to gravity.

18. **Statement-1 :** The maximum height of a projectile is 25 percent of maximum range for $\theta = 45°$.
Statement-2 : The maximum height is independent of initial velocity of projectile.

19. **Statement-1 :** In order to hit a target, a man should point his rifle in the same direction as target.
Statement-2 : The horizontal range of the bullet is dependent on the angle of projectile with horizontal direction.

20. **Statement-1 :** If both the speed of a body and radius of its circular path are doubled, then centripetal force also gets doubled.
Statement-2 : Centripetal force is directly proportional to both speed of a body and radius of circular path.

21. **Statement-1 :** When an automobile while going too fast around a curve overturns, its inner wheels leave the ground first.
Statement-2 : For a safe turn the velocity of automobile should be less than the value of safe limit velocity.

22. **Statement-1 :** A safe turn by a cyclist should neither be fast nor sharp.
Statement-2 : The bending angle from the vertical would decrease with increase in velocity.

23. **Statement-1 :** When velocity is zero, acceleration is zero.
Statement-2 : Rate of change of velocity is acceleration.

24. **Statement-1 :** If $a = -2t$ for a particle moving in a straight line starting with an initial velocity 4 m/s from the origin, then distance travelled by it in 2 seconds is same as displacement.
Statement-2 : Velocity changes direction after 2 s only.

25. **Statement-1 :** A person can swim in still water with a velocity less than river flow. If he starts swimming perpendicularly to the river velocity observed by stationary frame, then drift of the person is minimum.
Statement-2 : The time spent in river will be minimum, when swimmer swims in the direction perpendicular to the velocity of river.

26. **Statement-1 :** Two particles are projected under gravity with same speed at two different angles $(45° + \theta)$ and $(45° - \theta)$. Their range are same.
Statement-2 : The angle $(45° + \theta)$ and $(45 - \theta)$ are complementary angles.

27. **Statement-1 :** A particle is projected from origin under gravity with the angle of projection $\theta = \tan^{-1}(2)$. The radius vector locating highest point is at an angle of elevation $\frac{\pi}{4}$.
Statement-2 : The ratio of average velocity to the velocity of projection is greater than unity for the particle projected under gravity at some angle from horizontal.

QUESTIONS FROM PREVIOUS YEARS AIEEE/JEE MAIN

1. From a building two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then
(a) $v_B > v_A$ (b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses. **(2002)**

2. A car moving with a speed of 50 km/hr, can be stopped by brakes after at least 6 m. If the same car is moving at a speed of 100 km/hr, the minimum stopping distance is
(a) 12 m (b) 18 m
(c) 24 m (d) 6 m. **(2003)**

3. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground ?
[g = 10 m/s^2, sin30° = 1/2, cos30° = $\sqrt{3}/2$]
(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m. **(2003)**

4. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $T/3$ second?
(a) $h/9$ metre from the ground
(b) $7h/9$ metre from the ground
(c) $8h/9$ metre from the ground
(d) $17h/18$ metre from the ground. **(2004)**

5. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant a person starts running with a constant speed $v_0/2$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection?
(a) yes, 60° (b) yes, 30°
(c) no (d) yes, 45°. **(2004)**

6. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e.* 120 km/h, the stopping distance will be
(a) 20 m (b) 40 m
(c) 60 m (d) 80 m. **(2004)**

7. A particle is moving eastwards with a velocity of 5 m/s. In 10 s the velocity changes to 5 m/s northwards. The average acceleration in this time is
(a) zero
(b) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-west
(c) $\frac{1}{\sqrt{2}}$ ms^{-2} towards north-east
(d) $\frac{1}{2}$ ms^{-2} towards north **(2005)**

8. The relation between time t and distance x is $t = ax^2 + bx$ where a and b are constants. The acceleration is
(a) $-2av^3$ (b) $2av^2$
(c) $-2av^2$ (d) $2bv^3$ **(2005)**

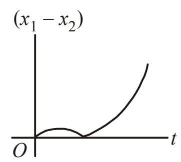
9. A car, starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance traversed is $15s$, then
(a) $s = \frac{1}{2}ft^2$ (b) $s = \frac{1}{4}ft^2$
(c) $s = ft$ (d) $s = \frac{1}{6}ft^2$ **(2005)**

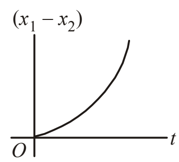
10. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 m/s^2. He reaches the ground with a speed of 3 m/s. At what height, did he bail out?
(a) 293 m (b) 111 m
(c) 91 m (d) 182 m **(2005)**

11. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as
(a) t^3 (b) t^2
(c) t (d) $t^{1/2}$. **(2006)**

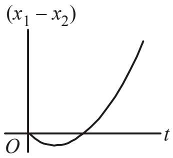
12. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time ($t = 1$) is
(a) $v_0 + g/2 + f$ (b) $v_0 + 2g + 3f$
(c) $v_0 + g/2 + f/3$ (d) $v_0 + g + f$ **(2007)**

13. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by x_2 (t) after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time t?

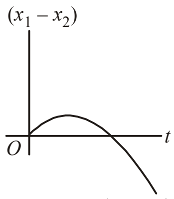
(a) 

(b) 

(c)



(d)

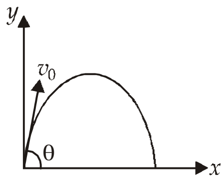


(2008)

14. A particle has an initial velocity $3\hat{i} + 4\hat{j}$ and an acceleration of $0.4\hat{i} + 0.3\hat{j}$. Its speed after 10 s is

(a) 10 units (b) $7\sqrt{2}$ units
(c) 7 units (d) 8.5 units **(2009)**

15. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(a) $\frac{1}{2} mgv_0t^2 \cos\theta\,\hat{i}$
(b) $-mgv_0t^2 \cos\theta\,\hat{j}$
(c) $mgv_0t \cos\theta\,\hat{k}$
(d) $-\frac{1}{2} mg\, v_0t^2 \cos\theta\,\hat{k}$

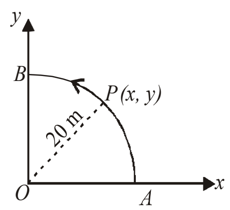
where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. **(2010)**

16. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is
(a) $y^2 = x^2$ + const. (b) $y = x^2$ + const.
(c) $y^2 = x$ + const. (d) xy = const. **(2010)**

17. For a particle in uniform circular motion, the acceleration at a point $P(R, \theta)$ on the circle of radius R is (Here θ is measured from the x-axis)

(a) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$ (b) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$
(c) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$ (d) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$
(2010)

18. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when t = 2 s is nearly



(a) 14 m s^{-2} (b) 13 m s^{-2}
(c) 12 m s^{-2} (d) 7.2 m s^{-2}
(2010)

19. An object moving with a speed of 6.25 m s^{-1}, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$ where v is the instantaneous speed. The time taken by the object, to come to rest, would be
(a) 1 s (b) 2 s
(c) 4 s (d) 8 s **(2011)**

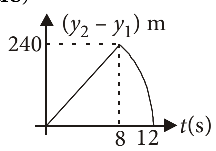
20. A boy can throw a stone up to a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone up to will be
(a) 10 m (b) $10\sqrt{2}$ m
(c) 20 m (d) $20\sqrt{2}$ m **(2012)**

21. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical.
If g = 10 m/s^2, the equation of its trajectory is
(a) $4y = 2x - 25x^2$ (b) $y = x - 5x^2$
(c) $y = 2x - 5x^2$ (d) $4y = 2x - 5x^2$
(JEE Main 2013)

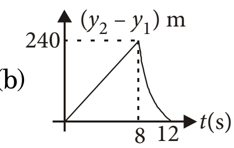
22. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is
(a) $gH = (n - 2)u^2$ (b) $2\,gH = n^2u^2$
(c) $gH = (n - 2)^2u^2$ (d) $2\,gH = nu^2(n - 2)$
(JEE Main 2014)

23. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)
(The figures are schematic and not drawn to scale)

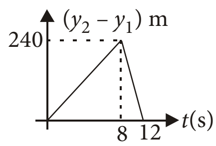
(a)



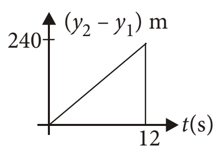
(b)



(c)



(d)



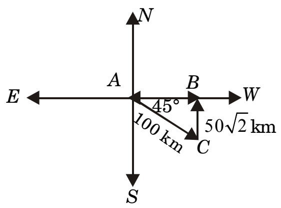
(JEE Main 2015)

☺☺☺

HINTS & SOLUTIONS

Multiple Choice Questions

1. **(d):** Since velocity and acceleration are vector quantities, therefore given equation is a vector equation, $\vec{v} = \vec{u} + \vec{a}t$. This equation is valid when acceleration direction is same as velocity.
Acceleration is the rate of change of velocity, *i.e.* $a = \frac{dv}{dt} \Rightarrow v = \int a\,dt$, in order to get the given equation acceleration should be independent of time; *i.e.* acceleration is constant.
i.e., given equation can be applied only if acceleration is along the direction of velocity and is constant.

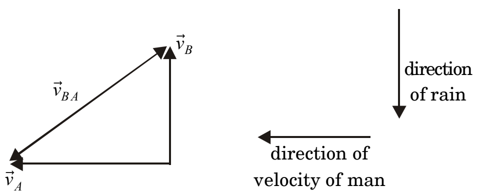


2. **(c):** Displacement is the minimum distance between initial and final position.
If body starts from A along southwest direction and finally reach at point B in northern direction; displacement of body $= AB$

$\therefore \; AB = \sqrt{(100)^2 - (50\sqrt{2})^2} = 50\sqrt{2}$ km

3. **(b):** Average speed $= \frac{\text{Total distance}}{\text{Total time}}$
Let distance between A and B be s. Then
Average speed $= \frac{2s}{\frac{s}{v_1} + \frac{s}{v_2}} = \frac{2v_1v_2}{v_1 + v_2}$

4. **(b):** In order to solve such type of problems first we choose a suitable equation of motion out of three with the help of given data.
Here, initial velocity $= u$, final velocity $v = 3u$
$h = ?$, we can use third equation of motion,
$v^2 = u^2 + 2gh$ or $9u^2 = u^2 + 2gh$



$h = \frac{8u^2}{2g} = \frac{4u^2}{g}$

5. **(b):** When two bodies A and B are in relative motion, the relative velocity of body A with respect to B can be obtained by imposing equal and opposite velocity of B on both A and B, so that B is brought to rest.
Taking velocity of man as $\vec{v}_A$ and velocity of rain as $\vec{v}_B$.
The relative velocity of man w.r.t. rain
$= \sqrt{v_A^2 + v_B^2} = \sqrt{(3)^2 + (10)^2} = \sqrt{109}$ km/h.

6. **(c):** Acceleration is the time rate of change of velocity, i.e. $a = \frac{dv}{dt}$
Given: $v = 20 + 0.1t^2$
$\frac{dv}{dt} = 0 + 0.1 \times 2t = 0.2t$
$\therefore \; a = 0.2t$
Since acceleration is time dependent, therefore body is in non-uniform accelerated motion.

7. **(c) :** In the free fall of a particle acceleration due to gravity act on it, therefore its velocity increases as height decreases. Due to this increasing velocity the time taken for each 1m will be different.

Using the relation, $s = ut + \frac{1}{2}at^2$, time to fall a distance s, when $u = 0$ and $a = g$ will be $t = \sqrt{\frac{2s}{g}}$.
Let t_1, t_2, t_3, ...be the time taken to fall 1m, 2m, 3m, ...respectively. Then,

$t_1 = \sqrt{\frac{2 \times 1}{g}}; t_2 = \sqrt{\frac{2 \times 2}{g}}; t_3 = \sqrt{\frac{2 \times 3}{g}}$

So the time taken to fall 1 m $= t_1 - 0 = \sqrt{\frac{2}{g}}(\sqrt{1} - 0)$
Time taken to fall 2nd metre
$= (t_2 - t_1) = \sqrt{\frac{2}{g}}(\sqrt{2} - \sqrt{1})$

Time taken to fall 3rd metre

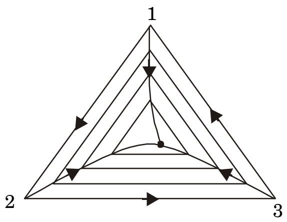
$$= (t_3 - t_1) = \sqrt{\frac{2}{g}}(\sqrt{3} - \sqrt{2})$$

$\therefore$ ratio of successive 1 m distance

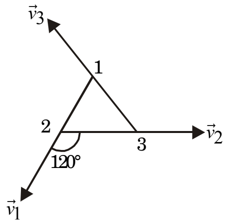
$$= \sqrt{1} : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) : \cdots$$

8. **(b) :** Since three points start moving simultaneously with same speed towards each other as specified, they are always located at the vertices of equilateral triangles of varying side and finally meet at the centroid of the equilateral triangle whose side length is a, in a time interval say t.

Suppose at an instant dt, the velocity of point 1 is $\vec{v}_1$, and velocity of point 2 is $\vec{v}_2$ with direction as shown in figure.



Component of $\vec{v}_1$ in the direction of



$$\vec{v}_2 = |\vec{v}_1| = v\cos\frac{2\pi}{3}$$

and $|\vec{v}_2| = v$

$\therefore$ Approaching velocity of 1 w.r.t. 2

$$= |\vec{v}_1| - |\vec{v}_2| = v - v\cos\frac{2\pi}{3}$$

= app. vel. of 2 w.r.t. 3

= app. vel. of 3 w.r.t. 1

If at this instant side of equilateral triangle is dl. Then the rate by which 1 approaches 2, 2 approaches 3 and 3 approaches 1, becomes

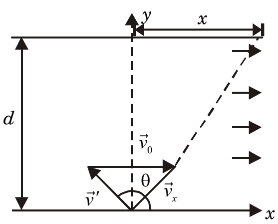
$$-\frac{dl}{dt} = v - v\cos\frac{2\pi}{3} = v + \frac{v}{2} = \frac{3v}{2}$$

On integrating:

$$-\int_a^0 dl = \frac{3v}{2}\int_0^t dt \quad \text{or} \quad a = \frac{3v}{2}t \Rightarrow t = \frac{2a}{3v}$$

9. **(c) :** Let v_0 be the stream velocity and v' the velocity of boat with respect to water.

As $\frac{v_0}{v'} = n = 2 > 0$, the drifting of boat is must.



Let $\vec{v}'$ makes an angle θ with the flow direction.

$\therefore$ Drifting velocity, $v_x = v' \cos\theta + v_0$

$$v_x = \frac{v_0}{n}\cos\theta + v_0$$

The time taken to cross the river,

$$t = \frac{d}{v'\sin\theta} = \frac{d}{\frac{v_0}{n}\sin\theta}$$

In this time interval, the drifting of the river,

$$x = v_x t = \left(\frac{v_0}{n}\cos\theta + v_0\right)\frac{d}{\frac{v_0}{n}\sin\theta}$$

$$= (\cot\theta + n\,\text{cosec}\,\theta)d$$

For minimum drifting, $\frac{dx}{d\theta} = 0$

or, $(-\,\text{cosec}^2\theta - n\,\text{cosec}\,\theta \,.\, \cot\theta) = 0$

$\text{cosec}\,\theta + n\cot\theta = 0$

$1 + n\cos\theta = 0$

$$\cos\theta = -\frac{1}{n} = -\frac{1}{2} \quad \Rightarrow \quad \theta = 120°$$

10. **(b):** Distance travelled by a particle in the nth second of its motion is:

$$s_n = u + \frac{1}{2}A(2n - 1)$$

where A is the acceleration.

Now, for the given problem,

$$a = u + \frac{1}{2}A(2x - 1) \quad \text{...(i)}$$

$$b = u + \frac{1}{2}A(2y - 1) \quad \text{...(ii)}$$

$$c = u + \frac{1}{2}A(2z - 1) \quad \text{...(iii)}$$

Equation (i) can be rewritten as,

$$a = Ax + \left(u - \frac{A}{2}\right)$$

Multiplying by y and z successively,

$$ay = Axy + \left(u - \frac{A}{2}\right)y \text{ and } az = Axz + \left(u - \frac{A}{2}\right)z$$

Substracting,

$$a(y - z) = A(xy - xz) + \left(u - \frac{A}{2}\right)(y - z) \quad \text{...(iv)}$$

Similarly, rearranging equation (ii), multiplying successively by z and x and substracting,

$$b(z - x) = A(yz - yx) + \left(u - \frac{A}{2}\right)(z - x) \quad \text{...(v)}$$

Similarly,

$$c(x - y) = A(zx - yz) + \left(u - \frac{A}{2}\right)(x - y) \quad \text{...(vi)}$$

Adding equations (iv), (v) and (vi)

$$a(y - z) + b(z - x) + c(x - y)$$
$$= A[(yz - xy) + (zx - yz) + (xy - xz)]$$
$$+ \left(u - \frac{A}{2}\right)[(y - z) + (z - x) + (x - y)] = 0$$

$\therefore$ $a(y - z) + b(z - x) + c(x - y) = 0$

11. **(a) :** In order to find out the velocity of particle at origin, first we differentiate the given equation of trajectory and then do substitution of coordinates of origin.

According to problem, acceleration of particle is directed along negative y-axis.

Therefore, $\vec{a} = a(-\hat{j})$

i.e., acceleration towards x-axis is zero.

$$\left.\begin{aligned} a_x &= \frac{dv_x}{dt} = 0 \\ \text{and,}\quad a_y &= \frac{dv_y}{dt} = -a \end{aligned}\right\} \qquad \text{...(i)}$$

The given equation of trajectory is,

$y = mx - nx^2$

Differentiating w.r.t. time,

$$\frac{dy}{dt} = m\frac{dx}{dt} - 2nx\frac{dx}{dt} \qquad \text{...(ii)}$$

At origin, $$\left.\frac{dy}{dt}\right|_{y=0} = m\left.\frac{dx}{dt}\right|_{x=0} \qquad \text{...(iii)}$$

Again differentiating (ii) w.r.t. time

$$\frac{d^2y}{dt^2} = m\frac{d^2x}{dt^2} - 2n\left(\frac{dx}{dt}\right)^2 - 2nx\frac{d^2x}{dt^2}$$

using equation (i),

$$-a = m \times 0 - 2n\left(\frac{dx}{dt}\right)^2 - 2nx \times 0$$

or $$\left.\frac{dx}{dt}\right|_{x=0} = \sqrt{\frac{a}{2n}} \qquad \text{...(iv)}$$

Using equation (iv) in equation (iii) we get,

$$\left.\frac{dy}{dt}\right|_{y=0} = m\sqrt{\frac{a}{2n}} \qquad \text{...(v)}$$

Equation (iv) and (v) represents the velocity along x and y-axis respectively which are perpendicular to each other, therefore resultant velocity at origin,

$$v = \sqrt{\left(\frac{dx}{dt}\right)_{x=0}^2 + \left(\frac{dy}{dt}\right)_{y=0}^2}$$

$$= \sqrt{\frac{a}{2n} + m^2\frac{a}{2n}} = \sqrt{\frac{a}{2n}(1+m^2)}$$

12. **(a):** In order to find out the trajectory of a moving particle we have to interpret a relationship between its x and y coordinates, which can be done by integrating given equations of velocities along x and y axes.

Velocity along x-axis, $v_x = \frac{dx}{dt} = 8t - 2$

$\Rightarrow \quad x = \frac{8t^2}{2} - 2t + c$

At, $t = 2, x = 14 \Rightarrow c = 2$

$\therefore \quad x = 4t^2 - 2t + 2$ ...(i)

Again, $v_y = \frac{dy}{dt} = 2 \Rightarrow y = 2t + c$

At $t = 2, y = 4, \Rightarrow c = 0$

$\therefore \quad y = 2t$ ...(ii)

Eliminating t from (i) and (ii),

$x = y^2 - y + 2$

This is the required equation of trajectory, which is a equation of parabola.

13. **(d) :** In order to find out the displacement of particle we integrate twice the given equation of acceleration.

Since $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$\therefore \quad x = \iint a\,dt\,dt$ ...(i)

Substituting value of a in equation (i),

$$x = \iint(-r\omega^2 \sin \omega t)dt\,dt = \int\frac{r\omega^2}{\omega}\cos \omega t\,dt$$

$$= \frac{r}{\omega}.\omega \sin \omega t = r\sin \omega t$$

$x = r \sin \omega t$

14. **(d) :** Let time taken is t.

Now $\quad s = ut + \frac{1}{2}at^2$

$15 = 2t - \frac{0.1}{2}t^2 \qquad \therefore \quad 150 = 20t - \frac{1}{2}t^2$

$t^2 - 30t - 10t + 300 = 0$

$t(t - 30) - 10(t - 30) = 0$

$t = 30 \quad$ or $\quad t = 10$

But $t = 30$ s is not admissible as velocity reverse the direction which is not possible for given problem.

15. **(b):** As acceleration is same for both bodies the relative distance will not change as both are moving with same velocity.

16. **(c):** Let balls meet after t sec. The distance travelled by the ball coming down is

$s_1 = \frac{1}{2}gt^2$

Distance travelled by the other ball

$s_2 = 40t - \frac{1}{2}gt^2$

$\because \quad s_1 + s_2 = 100$ m

$\therefore \quad 40t - \frac{1}{2}gt^2 + \frac{1}{2}gt^2 = 100$

$t = \frac{100}{40} = 2.5$ s

17. **(b):** Average speed = $\frac{\text{Total distance}}{\text{Total time}}$

Total time = 20 + 20 + 40 = 80 s.

Total distance $s = s_1 + s_2 + s_3$

$\because \quad s_1 = \frac{1}{2}at^2 = \frac{1}{2} \times (40/20)\ 20 \times 20 = 400$ m

$s_2 = 40 \times 20 = 800$ m

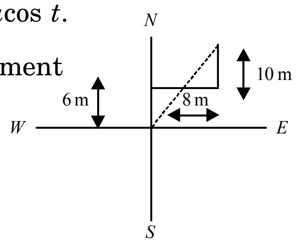
$s_3 = \frac{1}{2} \cdot \left(\frac{40}{40}\right) \times 40 \times 40 = 800$ m

$\therefore$ Average velocity $= \frac{2000}{80} = 25$ m/s

18. **(b):** $v = \frac{dx}{dt} = \frac{d}{dt}(a \cos t) = -a \sin t$

$\frac{dv}{dt} = \frac{d}{dt}(-a \sin t) = -a \cos t.$

19. **(d):** Resultant displacement



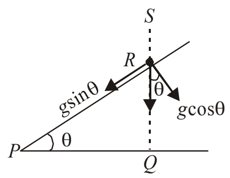
$R = \sqrt{x^2 + y^2 + z^2}$

$R = \sqrt{6^2 + 8^2 + 10^2}$

$= \sqrt{36 + 64 + 100}$

$= \sqrt{200} = \sqrt{2} \times 10\,\text{m}$

20. **(d) :** The distance covered by the particle in time t is given by



$PR = \frac{1}{2} g \sin\theta t^2$ ... (i)

Also from ΔPRQ,

$\frac{PQ}{PR} = \cos\theta$ ... (ii)

From equations (i) and (ii),

$\frac{PQ}{\cos\theta} = \frac{1}{2} g \sin\theta t^2$ or, $t = 2\sqrt{\frac{PQ}{g \sin 2\theta}}$

As θ increases, sin2θ first increases and then decreases. Thus time firstly decreases and then increases.

21. **(b) :** Given that displacement of the body,

$y \propto t^3$ or $y = kt^3$

$\therefore \quad v = \frac{dy}{dt} = 3kt^2$ or $a = \frac{dv}{dt} = 6kt \propto t$

$\therefore$ The magnitude of the acceleration of the body increases with time.

22. **(d) :** The average speed of an object is greater or equal to the magnitude of average velocity over a given time interval.

23. **(a) :** Let us take down as +*ve*, here,

$u = -2$ ms^{-1}, $a = +\ 10$ ms^{-2}

$t = 2$ s

As $v = u + at \Rightarrow v = (-2) + (10)(2)$

$v = +\ 18$ ms^{-1}.

24. **(b) :** $u = 100$ ms^{-1}, $a = 10$ms^{-2},

$v = 2u = 200$ms^{-1}

$t = \left(\frac{v-u}{a}\right) = \left(\frac{200-100}{10}\right) = 10$ s

25. **(b) :**

	y	$\frac{dy}{dt}$	$\frac{d^2y}{dt^2}$
(a)	$y = at$	a	0
(b)	$y = at + bt^2$	$a + 2bt$	$2b$
(c)	$y = at + bt^2 + ct^3$	$a + 2bt + 3ct^2$	$2b + 6ct$
(d)	$y = at^{-1} + bt$	$-at^{-2} + b$	$2at^{-3}$

$a = \frac{d^2y}{dt^2}$ is finite and constant in case (b).

26. **(d) :** To go down, the stone takes a time t_1,

Using, $s = ut + at^2$; $h = (0 \cdot t_1) + \frac{1}{2}gt_1^2$

$\Rightarrow t_1 = \sqrt{\frac{2h}{g}}$

The sound takes a time $t_2 = \frac{h}{c}$ to reach back to the ear. Total time taken $= (t_1 + t_2)$

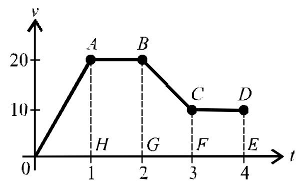
$= \sqrt{\frac{2h}{g}} + \frac{h}{c}$

27. **(b) :** The slope of *v*-*t* graph is acceleration, as $a = \frac{dv}{dt}$. The *v*-*t* graph has a constant negative slope in the first half, it means acceleration is constant and negative in the first half.
In the second half the graph has a constant positive slope in the *v*-*t* graph. Hence the acceleration is constant and positive in the second half.

28. **(c) :** The area below the *v*-*t* graph shows displacement. The shaded area is below the portion of the graph which has a positive slope. The positive slope of *v*-*t* graph shows accelerated motion. Here shaded area depicts distance moved during accelerated motion.

29. **(d) :** When an object falls down its acceleration is downwards and constantly *g*. Its velocity is downward. On bouncing its velocity is upward but acceleration is still downward and constantly *g*. *v*-*t* graph should have same slope in both phases of motion, one half must be positive velocity and the other half must be negative velocity.

30. **(c) :** Area under the graph represents distance moved in 4 s.

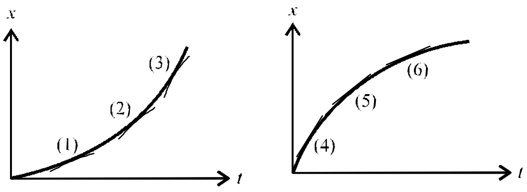


= Area OAH + Area $ABGH$ + Area $BCFG$ + Area $CDEF$

$= \frac{1}{2}(1)(20) + (20)\cdot(1) + \frac{1}{2}(20+10)(1) + (10)\cdot(1)$

= 10 + 20 + 15 + 10 = 55 m.

31. (c) :



(i) (ii)

In Fig. (i) the slopes at 1, 2, 3 go on increasing. It means velocity goes on increasing. So this depicts uniform acceleration. In Fig. (ii) the slopes at 4, 5, 6 go on decreasing. It means velocity goes on decreasing. So this depicts uniform retardation.

32. (a) : Area under the triangle of the $v - t$ graph indicates to displacement of the rocket. Height reached $= \frac{1}{2} \times (132\text{s})(1200\text{ms}^{-1}) = 1200 \times 66\text{m}$

33. (c) : The ball during its flight is under uniform acceleration of g, downwards. If we take up as positive, then acceleration is negative. So the slope of v-t graph must be negative. Further when the ball goes up, its velocity goes on decreasing and becomes zero. Therefor its direction changes or velocity becomes negative.

34. (a) : During the first 4 s, the particle is uniformly accelerating as seen by the straight line graph of positive slope. Here u = 10 ms^{-1}, v = 15 ms^{-1}, t = 4 s.

$\Rightarrow a = \left(\frac{v-u}{t}\right) = \frac{15-10}{4} = \frac{5}{4}\text{ms}^{-2}$

Using $s = ut + \frac{1}{2}at^2$

Distance travelled in 1 s = s_1

$= (10)(1) + \frac{1}{2}\left(\frac{5}{4}\right)1^2$

Distance travelled in 4 s = s_4

$= (10)(4) + \frac{1}{2}\left(\frac{5}{4}\right)4^2$

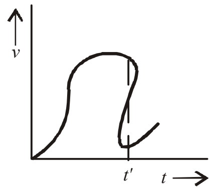
Distance travelled between 1 s and 4 s

$= s_4 - s_1$

$= (10)[4-1] + \frac{1}{2}\cdot\left(\frac{5}{4}\right)\cdot(4^2 - 1^2)$

$= (10)\cdot(3) + \frac{5}{8}(15) = 30 + \frac{75}{8} = \frac{315}{8}$ m ≈ 39 m.

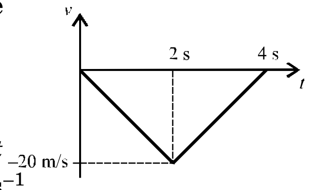
35. (c) : The figure (i) cannot be realized in practice. At the peak points the acceleration or $\frac{dv}{dt}$ is not defined.



In practice some value of acceleration will be there. Further the vertical lines in the figure (i) indicate that the acceleration is infinite, which again is not realized.

At the instant t', the particles's velocity is shown to have three different values. It can have only one value. Hence this case is not practical. The cases shown in fig. (ii) and (iv) are practical.

36. (b) : The v-t graph corresponding to the a-t graph would be



For u = 0,

$a = -10\text{ms}^{-2}$

and t = 2 s, $v = u + at$

$= 0 - (10\times2) = -20\text{ms}^{-1}$

As speed is $|v|$, the maximum speed corresponds to the maximum magnitude of velocity = 20 m/s, which happens after 2 s.

37. (d) : The maximum acceleration corresponds to the greatest slope in the v-t graph. This is seen between the 30$^{\text{th}}$ and 40$^{\text{th}}$ second where

$a = \frac{\Delta v}{\Delta t}$

$a_{\text{max}} = \frac{(60-20)\text{ ms}^{-1}}{10\text{ s}} = 4\text{ ms}^{-2}$.

38. (b) : $v_{PQ} = v_P - v_Q = 0 \Rightarrow v_P = v_Q$

same velocity ⇒ same slope in displacement-time graph.

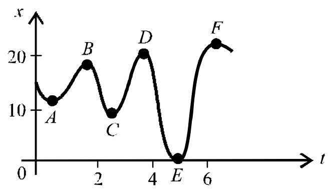
39. (c) : Average velocity in 20 s

$= \frac{\text{Total displacement at }20^{\text{th}}\text{ s}}{20\text{ s}}$

$= \frac{20\text{ m}}{20\text{ s}} = 1$ m/s.

40. (b) : $a = \frac{dv}{dt}$ = slope in the v-t graph. As is evident, II has uniform and positive slope so it indicates uniform and positive acceleration.

41. (a) :



(a) At A, B, C, D, E and F, $\frac{dx}{dt} = 0$ or $v = 0$. The body has come to rest six times.

(b) At the sixth second, $\frac{dx}{dt} = 0$, this cannot be the maximum speed.

(c) Between A and B, C and D, E and F, slope is positive. Between B and C, D and E and beyond F, slope is negative hence velocity is negative.

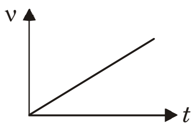
(d) The total displacement is positive. So average velocity will be positive.

42. **(d) :** The slope of the line in displacement-time graph is the velocity.

Slope = tan θ

Here, $\frac{v_1}{v_2} = \frac{\tan 30°}{\tan 45°} = \frac{1}{\sqrt{3}}$.

43. **(b) :**



(a) Fig (a) shows body at rest

(b) Fig (b) shows uniform speed, due to constant slope in s-t graph.

(c) In a-t graph, uniform speed would mean zero acceleration throughout.

(d) In v-t graph slope is –ve the motion, would be retarded so there is no possiblility of uniform speed.

44. **(a) :** The slope of curve in x-t graph shows the velocity of the particle. As is evident, in portion OA slope goes on decreasing, hence velocity is decreasing or acceleration is negative.
In section AB, slope is constant. Hence velocity is constant or acceleration is 0. In the portion BC, the slope increases or velocity increases. So acceleration is positive.
In the section CD, slope is constant, velocity is constant, so acceleration is 0.

45. **(a) :** $s_{12} = u_{12} \cdot t + \frac{1}{2}a_{12}t^2$

Now, $u_{12} = u_1 - u_2 = (+u) - (-u) = 2u$

$a_{12} = a_1 - a_2 = g - g = 0 \Rightarrow S_{12} = (2u)t$

It must be a line of constant slope passing through the origin.

46. **(b) :** Here $u = 0$

For $t = 2$ s, $s = x$

Using $s = ut + \frac{1}{2}at^2$

$x = \frac{1}{2}a \times 2^2 = 2a$

Next 2 s, the particle travels y.

$\Rightarrow t = (2 + 2)$ s = 4 s, $s = x + y$

$\Rightarrow (x + y) = \frac{1}{2}a(4)^2 = 8a$

$\Rightarrow (x + y) - x = 8a - 2a = 6a$

or $y = 6a = 3(2a) \Rightarrow y = 3x$.

47. **(a) :** Let the height of each storey be y.

$\Rightarrow$ 9th storey's height = $9y$

The ball is dropped $\Rightarrow u = 0$

$s = ut + \frac{1}{2}at^2$

$9y = (0)(3 \text{ s}) + \frac{1}{2}(10 \text{ ms}^{-2})(3 \text{ s})^2$

$9y = \frac{10 \times 9}{2}$

$y = 5$ m

In 1 s, the ball travels,

$s = \frac{1}{2}(10 \text{ ms}^{-2})(1 \text{ s})^2 = 5$ m

Thus in 1 s, the ball travels 1 storey.

48. **(c) :** Take down as positive. Then, $u = -40$ ms^{-1},

$s = 100$ m, $a = g = +10$ ms^{-2}

Using $s = ut + \frac{1}{2}at^2 \Rightarrow 100 = -40t + \left(\frac{1}{2}\right)10t^2$

$5t^2 - 40t - 100 = 0$; or $t^2 - 8t - 20 = 0$

$t^2 - 10t + 2t - 20 = 0$;

$t(t - 10) + 2(t - 10) = 0$

$(t - 10)(t + 2) = 0 \Rightarrow t = 10$ or $t = -2$

t can't be –ve, hence $t = 10$ s.

49. **(a) :** Take west as positive, then

$v_{BT} = v_B - v_T = 10 - (-5) = 15\frac{\text{m}}{\text{s}}$

$s_{BT} = v_{BT} \cdot t = \left(15\frac{\text{m}}{\text{s}}\right) \cdot (8 \text{ s}) = 120$ m.

50. **(b) :** $u = 0$, using $v^2 = u^2 + 2gh$; $v = \sqrt{2gh}$

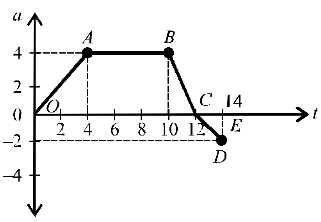
$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{h_1}{h_2}}$.

51. **(a) :** $h = -v_0t + \frac{1}{2}gt^2$

or $\frac{1}{2}gt^2 - v_0t - h = 0 \Rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 + 2gh}}{g}$

t is minimum if v_0 is minimum.

52. **(b) :** $v = \int a dt$ or the area under the a-t graph curve will give us velocity.



v = Area $OABC$ – Area CDE

$= \frac{1}{2}(AB + OC)$

$\times$ altitude $- \frac{1}{2}(CE) \cdot (ED)$

$= \frac{1}{2}(6 + 12) \times 4 - \frac{1}{2}$ (2) (2)

$= \frac{18 \times 4}{2} - 2 = 36 - 2 = 34$ m/s.

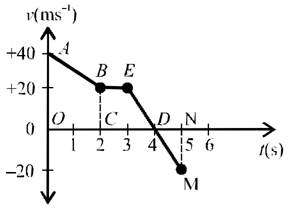
53. **(a) :** Average velocity in 10 s

$= \frac{\text{Distance travelled in 10 s}}{10\,\text{s}} = \frac{\frac{1}{2} \times AD \times OD}{10\,\text{s}}$

$= \frac{1}{2} \times \frac{60 \text{ m s}^{-1} \times 10\,\text{s}}{10\,\text{s}} = 30 \text{ ms}^{-1}$.

54. **(c) :** Velocity is negative, where slope is negative in the x-t graph, namely at point E.

55. **(d) :** Distance travelled in 5 s = Area $OABC$ + Area $BCDE$ + Area DNM



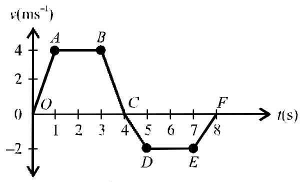
$= \frac{1}{2}(OA + BC) \times OC + \frac{1}{2}(BE + CD) \times BC$

$+ \frac{1}{2} \times DN \times NM$

$= \frac{1}{2}(40 + 20) \times 2 + \frac{1}{2}(1 + 2) \times 20 + \frac{1}{2}(1)\cdot(20)$

$= 60 + 30 + 10 = 100$ m.

56. **(a) :**



Displacement in 8 s

= + Area $OABC$ − Area $CDEF$

$= \frac{1}{2}(AB + OC) \times$ altitude

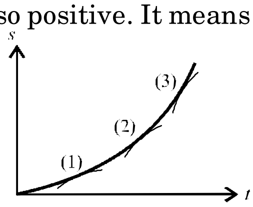
$- \frac{1}{2}(CF + DE) \times$ altitude

$= \frac{1}{2}(2 + 4)\cdot(4) - \frac{1}{2}(4 + 2) \times 2 = 12 - 6 = 6$ m.

57. **(b) :** As the body falls, if its displacement is positive, acceleration is also positive. It means the velocity or slope in x-t graph goes on increasing with time.

Slope $= m$

$m_3 > m_2 > m_1$



58. **(c) :** The slope of the v-t graph is acceleration

$a = \frac{\Delta v}{\Delta t} = \left(\frac{15 - 0}{4 - 1}\right) = \frac{15}{3}\text{ms}^{-2} = 5 \text{ ms}^{-2}$.

For the 1st second, the speed is 0.

$\Rightarrow$ The body is at rest.

59. **(a) :** $x = 6 - 4t + 6t^2$

$v = \frac{dx}{dt} = -4 + 12t$

So v-t graph is a straight line and at $t = 0$, $v = -4$ ms^{-1}.

60. **(a) :** The slope of the s-t graph is velocity. At $x = 0$, a certain slope exists, it means body starts with an initial velocity. Later the slope goes on decreasing to zero. It means velocity decreases until it becomes zero.

61. **(b) :** $v^2 = u^2 + 2as$, as $u = 0$, $v^2 = 2as$

The graph is a parabola facing the s-axis.

62. **(a) :** Let us take, down as negative. The acceleration is always downwards except when the body hits the horizontal surface. During the time of collision an upward force acts which gives the ball a momentary positive acceleration. Hence fig. (II) corresponds to the acceleration.

Initially as ball falls and accelerates down with g, the velocity goes on increasing in the negative direction. Then an elastic collision reverses the direction of velocity with the same magnitude. Again it accelerates downward with g. Hence fig. (I) depicts this behaviour.

63. **(d) :** As both the curves have a constant slope or $\frac{dx}{dt}$, their speeds are constant.

$(\text{Slope})_A > (\text{Slope})_B \Rightarrow v_A > v_B$.

64. **(b) :** The table shows a constantly accelerating body.

$a = \frac{\Delta v}{\Delta t} = 4\frac{\text{m}}{\text{s}^2}$

Constant acceleration means a constant force along the path is acting. This is possible with fig. (b).

65. **(b) :** In the first two seconds, the body moves with uniform speed, as indicated by the constant slope.

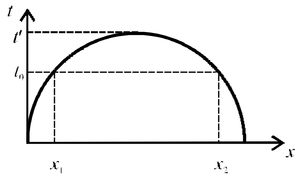
$v_1 = \frac{s}{t} = \frac{20 \text{ m}}{2\,\text{s}} = 10$ m/s

In the next 4 s, the velocity is again constant but in the opposite direction as indicated by the constant negative slope.

$v_2 = \frac{-20\text{ m}}{4\text{ s}} = -5\frac{\text{m}}{\text{s}}$

$\left|\frac{v_1}{v_2}\right| = \left(\frac{10}{5}\right) = 2 \Rightarrow \frac{v_1}{v_2} = \frac{2}{1}.$

66. (d) : At time t_0, the body is present at x_1 and x_2. This is an impossible situation.



At t', $\frac{dt}{dx} = 0$

or $\frac{dx}{dt} = \infty \Rightarrow$ velocity of the body is infinity. This is again impossible.

67. (c) : Area under the speed-time graph represents distance.

$s = \int v dt$.

68. (a) : Displacement $= +(4\times2)-(2\times2)+(2\times2) = 8$ m.
Distance $= |4 \times 2| + |2 \times 2| + |2 \times 2| = 16$m.

69. (b) : The v-t graph has three lines of equal slope but positive, negative and positive signs.

Hence it shows the accelerations $\frac{dv}{dt}$ are of constant magnitude but changing direction at the end of every two second.

70. (d) : $u = 0$

$\frac{h}{2} = \frac{1}{2}g(2)^2$

$\Rightarrow \; h = 4g$

Now, time taken to fall through the height of the tower is t.

$h = \frac{1}{2}gt^2 \Rightarrow 4g = \frac{1}{2}gt^2$

or $t^2 = 8$; $t = \sqrt{8} = 2.828$ s.

71. (c) : $s_1 = \frac{1}{2}g(n^2)$

$s_2 = \frac{1}{2}g(n - N)^2$

$s_1 - s_2 = 1 = \frac{1}{2}g[n^2 - (n - N)^2]$

$\Rightarrow \frac{2}{g} = (n + n - N)N$

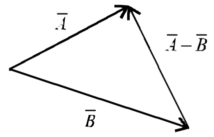
$\frac{2}{g} = (2n - N)\cdot N$

$2n - N = \frac{2}{gN}$

$2n = \frac{2}{gN} + N$

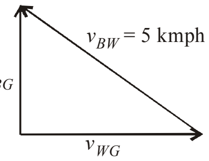
$\Rightarrow \; n = \left(\frac{1}{gN} + \frac{N}{2}\right).$

72. (b) : $\vec{A} - \vec{B}$ is in the same plane as $\vec{A}$ and $\vec{B}$, if $\vec{A}$ and $\vec{B}$ are not parallel.



$(\vec{A} \times \vec{B})$ is normal to the plane containing $\vec{A}$ and $\vec{B}$. Hence it is normal to $(\vec{A} - \vec{B})$ as well.

73. (b) : $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$
B-Boat, G-ground, W-water



$v_{BG} = \frac{1\text{ km}}{15\text{ minutes}} = 4\text{ km hr}^{-1}$

$\vec{v}_{BW} = \vec{v}_{BG} - \vec{v}_{WG}$

$v_{WG} = \sqrt{5^2 - 4^2} = 3\text{ km hr}^{-1}$.

74. (d) : $(\vec{A} + \vec{B}) = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

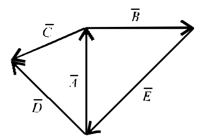
since $A = B = |\vec{A} + \vec{B}|$

$A = \sqrt{A^2 + A^2 + 2A^2\cos\theta}$

$A^2 = 2A^2 + 2A^2 \cos\theta$

$\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$.

75. (d) : $\vec{A} + \vec{B} + \vec{E} = 0$ as they form a triangle when taken in order.



$\Rightarrow \vec{E} = -(\vec{A} + \vec{B})$.

76. (b) : $|\vec{a} + \vec{b}| = \sqrt{a^2 + b^2 + 2ab\cos\theta}$

$|\vec{a} + \vec{b}|_{max} = a + b$, when $\theta = 0°$

$|\vec{a} + \vec{b}|_{min} = a - b$ when $\theta = 180°$

Here, $\frac{a+b}{a-b} = \frac{3}{1} \Rightarrow a + b = 3a - 3b$

so, $4b = 2a$

or $a = 2b \Rightarrow |\vec{a}| = 2|\vec{b}|$.

77. (a) : $R^2 = (3P)^2 + (2P)^2 + 2(3P)(2P)\cos\theta$

$\Rightarrow R^2 = 13P^2 + 12P^2\cos\theta$ ...(1)

When the first force is doubled, the resultant is doubled,

so, $(2R)^2 = (6P)^2 + (2P)^2 + 2(6P)(2P)\cos\theta$

$\Rightarrow 4R^2 = 36P^2 + 4P^2 + 24P^2\cos\theta$

$\Rightarrow R^2 = 10P^2 + 6P^2\cos\theta$...(2)

Equating (1) & (2), we get

$13P^2 + 12P^2\cos^2\theta = 10P^2 + 6P^2\cos\theta$

$3P^2 = -6P^2\cos\theta$

so, $\cos\theta = -\frac{1}{2} \Rightarrow \theta = 120°$.

78. (a) : The linear velocity of a rotating body is given by $\vec{v} = \vec{\omega} \times \vec{r}$

As $\vec{\omega} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{r} = 4\hat{j} - 3\hat{k}$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 0 & 4 & -3 \end{vmatrix}$$

Opening by the Ist column,

$= \hat{i}[(-2)(-3) - (4)(2)] - 1(-3\hat{j} - 4\hat{k})$

$\vec{v} = -2\hat{i} + 3\hat{j} + 4\hat{k}$

$|\vec{v}| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$.

79. (a) : $W = \vec{F} \cdot \vec{s}$

$\vec{F} = 2\hat{i} + 2\hat{j}$ N and $\vec{s} = 2\hat{i} + 2\hat{k}$ m

$W = 2(2) + 2(0) + 0(2) = 4$ J

$P = \frac{W}{t} = \frac{4\text{ J}}{16\text{ s}} = 0.25$ Js^{-1}.

80. (d) : $\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} - 2\hat{j} + \hat{k}$

Area of parallelogram = $|\vec{A} \times \vec{B}|$

Now, $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 3 & -2 & 1 \end{vmatrix} = 4\hat{i} + 8\hat{j} + 4\hat{k}$

$|\vec{A} \times \vec{B}| = \sqrt{4^2 + 8^2 + 4^2} = \sqrt{96} = 4\sqrt{6}$ units.

81. (d) : $\vec{A} + \vec{B} = (3\hat{i} + 2\hat{i}) + (-4\hat{j} + 4\hat{j}) + (-2\hat{k} - 5\hat{k})$

$= 5\hat{i} + 0\hat{j} - 7\hat{k}$

$(\vec{A} + \vec{B}) \cdot \hat{j} = 0$

$\Rightarrow (\vec{A} + \vec{B})$ is perpendicular to the y-axis.

82. (d) : $R = \frac{u^2 \sin 2\theta}{g}$

If u is doubled, R the range will be quadrupled.

83. (c) : For vertical motion, the initial speed for both the stones is same, g is same, hence the time when they reach the ground will be same.

$t = \sqrt{\frac{2h}{g}}$.

84. (a) : $R = \frac{u^2 \sin 2\theta}{g}$

If R_1 be the range of stone thrown at an angle of $(45 - \theta)$, then

$R_1 = \frac{u^2 \sin 2(45 - \theta)}{g}$

$= \frac{u^2 \sin(90 - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

Similarly, if R_2 be the range of stone thrown at an angle of $(45 + \theta)$, then

$R_2 = \frac{u^2 \sin 2(45 + \theta)}{g}$

$= \frac{u^2 \sin(90 + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$

$\Rightarrow R_1 = R_2$ or $R_1 : R_2 = 1 : 1$.

85. (b) : $R = \frac{u^2 \sin 2\theta}{g}$ or $R \propto \sin 2\theta$

$R_{30°} \propto \sin 60° = \frac{\sqrt{3}}{2}$

$R_{45°} \propto \sin 90° = 1$

$R_{55°} \propto \sin 110° = \sin 70° < 1$

$R_{65°} \propto \sin 130° = \sin 50° < 1$

Hence the body thrown at 45°, will have the largest range.

86. (d) : Equations of kinematics donot depend on mass.

87. (d) : $H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow u_{30°}^2 \sin^2 30° = u_{60°}^2 \sin^2 60°$

$\Rightarrow \left(\frac{u_{30°}}{u_{60°}}\right) = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{1}$.

88. (a) : For motor-cycle to land safely across the ditch, it should clear a range = 11′ + 5′ = 16′. Here 16′ is the width of the ditch plus the length of the bike.

$R = \frac{u^2 \sin 2\theta}{g} \Rightarrow 16 \text{ ft} = u^2 \cdot \frac{\sin 2(15°)}{32\text{ ft}}$

or $u^2 = \frac{16\text{ ft} \times 32\text{ ft}}{\sin 30° \text{ ft}} = \frac{16\text{ ft} \times 32\text{ ft}}{1/2}$; $u = 32$ ft/s.

89. (a) : Time of fall: $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 490}{9.8}} = 10$ s

Horizontal speed = $\left(216 \frac{\text{km}}{\text{hr}}\right) \times \frac{5}{18} = 12 \times 5$ms^{-1}

= 60 ms^{-1}

Horizontal distance = 60 ms^{-1} × 10 s = 600 m.

90. (c) : $x = 2t + 4t^2$

$v_x = \frac{dx}{dt} = 2 + 8t$; $a_x = \frac{dv_x}{dt} = 8$ ms^{-2}; $y = 5t$

$v_y = \frac{dy}{dt} = 5$; $a_y = \frac{dv_y}{dt} = 0$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 0^2}$

at $t = 5s$, $a = 8$ms^{-2}.

91. (c) : $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Here $\theta = 60°$, $x = 50\text{m}$, $u = 25\text{ms}^{-1}$, $g = 10\text{ms}^{-2}$

$$\Rightarrow y = (50)\tan 60° - \frac{10(50)^2}{2(25^2)\cos^2 60°}$$

$$= 50\sqrt{3} - \frac{20}{1/4}$$

$$= 50(1.7) - 80 = 85 - 80 = 5 \text{ m}.$$

92. (d) : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 980}{9.8}} = 10\sqrt{2}$ s.

93. (b) : Trajectory on earth:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Trajectory on planet

$$y = x\tan\theta - \frac{g'x^2}{2u'^2\cos^2\theta}$$

As the trajectories are identical, it means the equations are identical.

$$\Rightarrow \frac{g}{u^2} = \frac{g'}{u'^2}$$

$$g' = g\left(\frac{u'}{u}\right)^2 = \left(9.8\frac{\text{m}}{\text{s}^2}\right)\cdot\left(\frac{3}{5}\right)^2 = 3.5\frac{\text{m}}{\text{s}^2}.$$

94. (d) : Time of flight : $t = \frac{2u\sin\theta}{g}$

$$\Rightarrow 5 \text{ s} = \frac{2(u\sin\theta)}{10\text{ ms}^{-2}}$$

or $u\sin\theta = 25 \text{ ms}^{-1}$

Now, Range: $R = \frac{2(u\cos\theta)(u\sin\theta)}{g}$

$$200 = \frac{2(u\cos\theta)(25)}{10}$$

$$\Rightarrow u\cos\theta = \frac{200 \times 10}{25 \times 2} = 40 \text{ ms}^{-1}$$

The horizontal component of the velocity of the arrow is '$u\cos\theta$' = 40 ms^{-1}.

95. (b) : Maximum Range : $R_{\max} = \frac{v^2\sin 2(45°)}{g}$

$$\Rightarrow = \frac{v^2}{g}$$

Maximum Area $= \pi\left(\frac{R_{\max}^2}{2}\right)^2 = \pi\left(\frac{v^2}{g}\right)^2 = \frac{\pi v^4}{g^2}$.

96. (d) : $R = \frac{u^2\cdot\sin 2\theta}{g}$

As $R_A = R_B$,

hence $\frac{v^2\sin 2\theta}{g} = \frac{\left(\frac{v}{2}\right)^2\cdot\sin 2(15°)}{g}$

$$\Rightarrow \sin 2\theta = \frac{1}{4}\cdot\left(\frac{1}{2}\right) = \frac{1}{8}$$

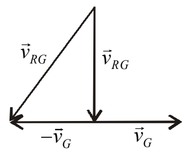
$$\theta = \frac{1}{2}\cdot\sin^{-1}\left(\frac{1}{8}\right)$$

97. (b) : $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2 \Rightarrow (\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2$

$\vec{a}^2 + \vec{b}^2 + 2\vec{a}\cdot\vec{b} = \vec{a}^2 + \vec{b}^2 - 2\vec{a}\cdot\vec{b} \Rightarrow \vec{a}\cdot\vec{b} = 0$

$\Rightarrow \vec{a}$ is perpendicular to $\vec{b}$.

98. (c) :



$\vec{v}_{RG} = \vec{v}_R - \vec{v}_G$, R-rain, G-girl

$\vec{v}_{RG} = \sqrt{v_R^2 + v_G^2} = \sqrt{4^2+3^2} = 5$ km/h.

99. (c) : If we assume horizontal and vertical as X and Y axes, then

$\overrightarrow{OA} = r\hat{j}$, $\overrightarrow{OC} = r\hat{i}$

$\overrightarrow{OB} = r\cos 45°\,\hat{i} + r\sin 45°\,\hat{j}$

$$\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} = \left(r + \frac{r}{\sqrt{2}}\right)\hat{i} + \left(r + \frac{r}{\sqrt{2}}\right)\hat{j}$$

$$|\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}| = \sqrt{\left(r + \frac{r}{\sqrt{2}}\right)^2 + \left(r + \frac{r}{\sqrt{2}}\right)^2}$$

$$= \sqrt{r^2\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2 \times 2} = r\left(1+\sqrt{2}\right).$$

100. (b) : $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$

$$= (2\hat{i} - 3\hat{j} - 2\hat{k}) + (5\hat{i} + 8\hat{j} + 6\hat{k}) + (-4\hat{i} - 5\hat{j} + 5\hat{k}) + (-3\hat{i} + 4\hat{j} - 7\hat{k}) = 0\hat{i} + 4\hat{j} + 2\hat{k}.$$

The X-component of force is missing, so the particle initially at rest will accelerate on the YZ plane.

101. (d) : $\cos\theta = \frac{(\hat{i}+\hat{j})\cdot(\hat{i}-\hat{j})}{|\hat{i}+\hat{j}||\hat{i}-\hat{j}|} = \frac{1-1}{\sqrt{2}\cdot\sqrt{2}} = 0$

$\Rightarrow \theta = 90°$.

102. (c) : $\vec{a} = \hat{i} + 4\hat{j}$; $\vec{b} = 7\hat{i} + 3\hat{j}$

Area of parallelogram $= |\vec{a}\times\vec{b}| = |3\hat{k} + (-28)\hat{k}|$

$= |-25\hat{k}| = 25$ units.

103. (b) : $\vec{d}_1 = \vec{a} + \vec{b}$ and $\vec{d}_2 = \vec{a} - \vec{b}$

$$(\vec{d}_1 \times \vec{d}_2) = (\vec{a}+\vec{b})\times(\vec{a}-\vec{b})$$

$$= (\vec{a}\times\vec{a}) - (\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a}) - (\vec{b}\times\vec{b})$$

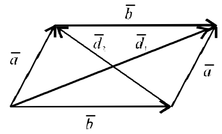
$$= -(\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a}) = 2(\vec{b}\times\vec{a})$$

$$\frac{1}{2}|(\vec{d}_1\times\vec{d}_2)| = |(\vec{b}\times\vec{a})|$$

= Area of parallelogram

As $\vec{d}_1 = 3\hat{i} + \hat{j} + 2\hat{k}$; and $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$

As $\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & -3 & 4 \end{vmatrix}$



$= \hat{i}(4+6) - \hat{j}(12-2) + \hat{k}(-9-1) = 10\hat{i} - 10\hat{j} - 10\hat{k}$

Area of parallelogram $= \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$

$= \frac{1}{2}|10\hat{i} - 10\hat{j} - 10\hat{k}| = \frac{1}{2}\sqrt{10^2 + 10^2 + 10^2}$

$= \frac{10}{2}\sqrt{3} = 5\sqrt{3} = 8.66$ units.

104. (a) : $\vec{r} = \vec{r}_{head} - \vec{r}_{tail} = (2\hat{i} + \hat{j} + \hat{k}) - (2\hat{i} - 3\hat{j} + \hat{k})$
$= 4\hat{j}$

$\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$

Angular momentum $(\vec{L}) = \vec{r} \times \vec{p}$

$= 4\hat{j} \times (2\hat{i} + 3\hat{j} + \hat{k}) = -8\hat{k} + 4\hat{i} = 4\hat{i} - 8\hat{k}$

105. (a) : $\vec{A} = -3\hat{i} + 2\hat{j} - 4\hat{k}$; $\vec{B} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & -4 \\ -1 & 2 & 1 \end{vmatrix}$

$= \hat{i}(2+8) - \hat{j}(-3-4) + \hat{k}(-6+2) = 10\hat{i} + 7\hat{j} - 4\hat{k}$

Area of triangle $= \frac{1}{2}|\vec{A} \times \vec{B}| = \frac{1}{2}\sqrt{10^2 + 7^2 + 4^2}$

$= \frac{\sqrt{165}}{2}$ units.

106. (d) : As stated.

107. (d) : If vertical is Y-axis and horizontal is X-axis, then $u_y = 0$, $a_y = g$, $s_y = h$

$\Rightarrow s_y = u_y t + \frac{1}{2}a_y t^2$.

Here, $t = \sqrt{\frac{2h}{g}}$

For the two bullets, u_x is different but that does not influence time of fall t.

108. (d) : When projectile falls back to the ground, its displacement in the vertical direction is zero.

109. (d) : $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 490 \text{ m}}{9.8 \text{ m/s}^2}} = 10$ s.

110. (d) : $v_{highest} = u \cos\theta = (10 \text{ m s}^{-1}) \cos 60°$

$= 10 \times \frac{1}{2} \text{ ms}^{-1} = 5 \text{ m s}^{-1}$.

111. (a) : At the highest point of the trajectory, the velocity becomes horizontal. Thus, it is normal to the acceleration due to gravity which is directed downwards.

112. (a) : $\vec{u}_1 = -4\hat{i}$ ms^{-1}, $\vec{u}_2 = +1\hat{i}$ ms^{-1}

$\vec{v}_1 = \vec{u}_1 + \vec{a}t = (-4\hat{i}) - (10\hat{j})t$

$\vec{v}_2 = \vec{u}_2 + \vec{a}t = (+1\hat{i}) - (10\hat{j})t$

When $\vec{v}_1$ becomes perpendicular to $\vec{v}_2$

$\vec{v}_1 \cdot \vec{v}_2 = 0 \Rightarrow (-4\hat{i} - 10t\hat{j}) \cdot (1\hat{i} - 10t\hat{j}) = 0$

$\Rightarrow -4 + 10^2 t^2 = 0$

or $t^2 = \frac{4}{10^2}$; $t = \frac{2}{10}$ s $= \frac{1}{5}$ s

Separation = Relative velocity along x-axis $\times \frac{1}{5}$s

$= 5 \times \frac{1}{5}$ m $= 1$ m.

113. (a) : $u = 30$ ms^{-1}, $\theta = 30°$

$v_x = u_x = u\cos\theta$

$= 30 \cos 30° = 15\sqrt{3}$

$v_y = u_y - gt = u\sin\theta - gt$

For $t = 1.5$ s, $v_y = 30 \cdot \sin 30° - 10(1.5)$

$\Rightarrow v_y = \frac{30}{2} - 15 = 0$

After, $t = 1.5$ s;

$\vec{v} = v_x\hat{i} + v_y\hat{j} = 15\sqrt{3}\hat{i}$

The velocity's angle with the horizontal is 0°, as $\tan\theta = \left(\frac{v_y}{v_x}\right) = \frac{0}{15\sqrt{3}} = 0$

$\Rightarrow \theta = \tan^{-1} 0 = 0°$.

114. (b) : $R = \frac{u^2 \sin 2\theta}{g}$

$R_{max} = \frac{u^2}{g}$ when $\theta = 45°$; $\Rightarrow 20 \text{ m} = \frac{u^2}{9.8}$

$u = (20 \times 9.8)^{1/2} = 14$ ms^{-1}.

115. (b) : $(\text{KE})_{initial} = \frac{1}{2}mu^2 = 800$ J

For maximum range, $\theta = 45°$

$(\text{KE})_{highest} = \frac{1}{2}m(u\cos\theta)^2 = \left(\frac{1}{2}mu^2\right)\cos^2 45°$

$= (800 \text{ J})\left(\frac{1}{2}\right) = 400$ J.

116. (b) : $t = \frac{u\sin(90-\beta)}{g} = \frac{u\cos\beta}{g}$

$H = \frac{u^2\sin^2(90-\beta)}{2g} = \frac{(u\cos\beta)^2}{2g}$

$\Rightarrow u\cos\beta = \sqrt{2gH} \Rightarrow t = \frac{\sqrt{2gH}}{g} = \sqrt{\frac{2H}{g}}$.

117. (b) : $v_x = 98\ \text{ms}^{-1}$

$v_y = u_y + a_y t$

$= 0 + (9.8\ \text{ms}^{-2})\ (10\ \text{s}) = 98\ \text{ms}^{-1}$

$\tan\theta = \left(\frac{v_y}{v_x}\right) = \left(\frac{98\ \text{ms}^{-1}}{98\ \text{ms}^{-1}}\right) = 1 \Rightarrow \theta = 45°.$

118. (b) : $\Delta p_{\text{vertical}} = (mv\sin\theta) - (-mv\sin\theta)$

$= 2\ mv\ \sin\theta = 2\ mv\ \sin 45° = \sqrt{2}mv.$

119. (b) : $R = \frac{u^2\sin 2\theta}{g}$

$\frac{R_{45°}}{R_{15°}} = \frac{\sin 2(45°)}{\sin 2(15°)}$

$R_{45°} = (R_{15°})\frac{\sin 90°}{\sin 30°} = R_{15°}\left(\frac{1}{1/2}\right) = 2R_{15°}$

$= 2 \times (1.5\ \text{km}) = 3\ \text{km}.$

120. (a) : $H = \frac{u^2\sin^2\theta}{2g}$; $t = \frac{2u\sin\theta}{g}$

Now, $u\sin\theta = \left(\frac{gt}{2}\right) \Rightarrow H = \left(\frac{gt}{2}\right)^2 \cdot \frac{1}{2g}$

$H = \frac{g^2t^2}{8g} = \frac{gt^2}{8} = \frac{(10\ \text{ms}^{-2})(5\ \text{s})^2}{8}$

$= 31.25\ \text{m} = 31\ \text{m}.$

121. (d) : $v_x = u_x = 20\ \text{ms}^{-1}$

$v_y = u_y + a_y\ t = 0 + gt$

$= 0 + (10\ \text{ms}^{-2})\ (5\ \text{s}) = 50\ \text{ms}^{-1}$

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 50^2} \approx 54\ \text{ms}^{-1}.$

122. (b) : Relative velocity of the bird = 25 – (–5)

= 30 m/s

Distance = 210 m

$\therefore$ Time $= \frac{\text{Distance}}{\text{velocity}} = \frac{210}{30} = 7\,\text{sec}$.

123. (b) : The height h is covered in time interval t = 1s to t = 2s or t = 5s to t = 6s.

Let u be the initial velocity given to the ball when projected vertically upwards. Then

$h = s_2 - s_1 = s_5 - s_6$ ...(i)

$s_2 - s_1 = \left(u\times 2 - \frac{1}{2}\times 7.5\times 2^2\right) - \left(u\times 1 - \frac{1}{2}\times 7.5\times 1^2\right)$

$= u - \frac{3}{2}\times 7.5$ ...(ii)

$s_5 - s_6 = \left(u\times 5 - \frac{1}{2}\times 7.5\times 5^2\right) - \left(u\times 6 - \frac{1}{2}\times 7.5\times 6^2\right)$

$= -u + \frac{11}{2}\times 7.5$ ...(iii)

$\therefore\ u - \frac{3}{2}\times 7.5 = -u + \frac{11}{2}\times 7.5$

or $2u = \frac{14}{2}\times 7.5 = 7\times 7.5$

or $u = 7\times\frac{7.5}{2} = 26.25\ \text{m/s}$

$\therefore\ h = 26.25 - \frac{3}{2}\times 7.5 = 26.25 - 11.25 = 15.0\ \text{m}.$

124. (d) : Average speed $= \frac{\text{Total distance covered}}{\text{Total time taken}}$

For onwards journey to market,

$t_1 = \frac{\text{Distance}}{\text{speed}} = \frac{2.5}{5} = 0.5\ \text{h} = \frac{1}{2}\text{h} = 30\,\text{min}$

For backwards journey from market,

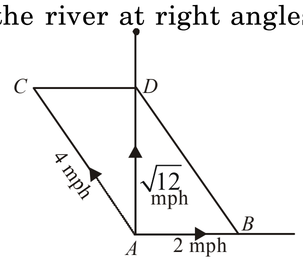
Time = (40 – 30) = 10 min

distance = velocity × time $= \frac{7.5\times 10}{60} = 1.25\ \text{km}$

Average speed $= \frac{2.5 + 1.25}{40/60} = \frac{3.75\times 60}{40}$ km/h

$= \frac{375\times 6}{4\times 100} = \frac{45}{8}$ km/h

125. (a) : Minimum time is taken when the boat crosses the river at right angles to its flow.



The resultant velocity of boat and river should be at right angles to flow of river.

AD represents resultant velocity (R)

By right angled triangle,

$(AC)^2 = (CD)^2 + (AD)^2$

$(4)^2 = (2)^2 + (AD)^2$

or $R = AD = \sqrt{12}$ mph

Width along AD = 4 miles

$\therefore$ Time $= \frac{\text{Distance}}{\text{Velocity}} = \frac{4}{\sqrt{12}} = \frac{4}{2\sqrt{3}} = \frac{2}{1.732}$

= 1.154 h

$\therefore$ Time = 1 hour 9 min.

126. (c) : Average velocity $v_a = \frac{\text{Total distance}}{\text{Total time}}$

For first half of journey,

Time $t_1 = \frac{\text{Distance}\ (x/2)}{v_0}$ or $t_1 = \frac{x}{2v_0}$ ...(i)

Remaining distance = $x/2$

Mean velocity for this distance $= \frac{v_1 + v_2}{2}$

$\therefore$ Time for this journey $= \frac{x/2}{(v_1+v_2)/2} = \frac{x}{v_1+v_2}$

$\therefore\ t_2 = \frac{x}{(v_1+v_2)}$ ...(ii)

For the whole journey,

Distance = x

Time = $t_1 + t_2 = \frac{x}{2v_0} + \frac{x}{(v_1+v_2)} = \frac{x(v_1+v_2+2v_0)}{2v_0(v_1+v_2)}$

$\therefore\ v_a = \frac{x\times 2v_0(v_1+v_2)}{x(v_1+v_2+2v_0)} = \frac{2v_0(v_1+v_2)}{(v_1+v_2+2v_0)}$

127. (b) : Effective acceleration in ascending lift $= (g + f)$

$\therefore \quad s = ut + \frac{1}{2}at^2$

$9.5 = 0 + \frac{1}{2}(g+f)t^2$ or $9.5 = \frac{1}{2}(32+6)t^2$

or $t^2 = \frac{9.5 \times 2}{38} = \frac{1}{2}$ or $t = \frac{1}{\sqrt{2}}$ sec.

128. (c) : The velocity upstream is (4 – 2) km/hr and downstream is (4 + 2) km/hr.

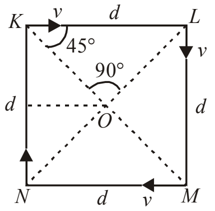
$\therefore$ Total time taken $= \frac{2\text{ km}}{2\text{ km/hr}} + \frac{2\text{ km}}{6\text{ km/hr}}$

= 80 minutes

129. (b)

130. (a) : The four persons, K, L, M and N will meet at O *i.e.* the centre of the diagonal of the square $KLMN$. The person K will travel a distance KO, with velocity along $KO = v\cos 45° = v/\sqrt{2}$.

Here $KO = d\cos 45° = d/\sqrt{2}$



$\therefore \quad$ Time of meeting, $t = \frac{\text{distance}}{\text{velocity}} = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v}$.

131. (b) : Let u be the initial velocity of the ball while going upwards. The final velocity of the ball at height x is, $v = 0$. So, $u = \sqrt{2gx}$.

Time of flight, $T = \frac{2u}{g} = \frac{2}{g}\sqrt{2gx} = 2\sqrt{\frac{2x}{g}}$

During time T, $(n - 1)$ balls will be in air and one ball will be in hand. So time for one ball in hand $= \frac{T}{n-1} = \frac{2\sqrt{2x/g}}{(n-1)} = \frac{2}{n-1}\sqrt{\frac{2x}{g}}$.

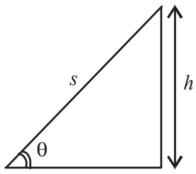
132. (c) : Let a be the uniform acceleration.
Then $v = at$ or $a = v/t$.
Velocity at $(t - 3)$ seconds, $v' = a(t - 3)$.
Displacement in last 3 seconds *i.e.* from $(t - 3)$ s to t s is

$= v' \times 3 + \frac{1}{2}a(3)^2 = a(t-3) \times 3 + \frac{1}{2}a \times 9$

$= \frac{v}{t}(3t - 9) + \frac{9}{2}\frac{v}{t} = 3v - \frac{9v}{2t} = 3v\left(1 - \frac{3}{2t}\right)$.

133. (a) : $h = \frac{1}{2}g\left(\frac{t}{2}\right)^2$ ... (i)

$s = \frac{1}{2}g\sin\theta t^2$ ... (ii)



$\frac{h}{s} = \frac{\frac{1}{2}g\left(\frac{t^2}{4}\right)}{\frac{1}{2}g\sin\theta(t^2)} \Rightarrow \sin^2\theta = \frac{1}{4}$

or $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

134. (c): The maximum height reached by a projectile is given by $H = \frac{u^2\sin^2\theta}{2g}$

i.e. H depends on initial velocity u, acceleration, g and angle of projection (θ) but does not depends on mass of projectile.

135. (b): When a body is moving in circular motion a centripetal acceleration acts along the radius and is directed towards the centre of the circular path and a tangential acceleration acts along the tangent.

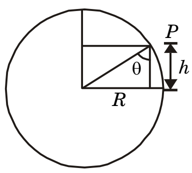
Total acceleration in circular motion $= \sqrt{a_C^2 + a_T^2}$

$= \sqrt{\left(\frac{v^2}{R}\right)^2 + a_T^2} = \sqrt{\left(\frac{30 \times 30}{500}\right)^2 + 2^2} = \sqrt{\frac{181}{25}}$

$= 2.7$ m/s^2

136. (d): Due to air, the final horizontal component of velocity of projectile becomes less than the initial horizontal component of velocity whereas the final vertical component velocity is more than the initial vertical velocity. Due to which the angle at which the projectile strikes the ground shows an increase.

137. (a): The block will lose contact where centripetal acceleration becomes equal to the component of acceleration due to gravity along the radius. Suppose this happens at a point P whose height w.r.t. ground is h. Hence the block has fallen through a distance $(R - h)$. So velocity at the point



$P \Rightarrow v = [2g(R - h)]^{1/2}$

Thus, $\frac{v^2}{R} = g\cos\theta$ or $\frac{2g(R-h)}{R} = g \times \frac{h}{R}$

or $h = \frac{2R}{3}$

138. (b): When a liquid is rotated in a test tube a centrifugal force acts on the liquid which depends on the mass of the liquid. Mercury being heavier will experience more centrifugal force than water, hence mercury will be forced to the outer part of rotational motion.

139. (b): When a vehicle takes a turn on the road, it travels along a nearly circular arc. There must exist some force which will produce the necessary centripetal acceleration. In order to take a safe turn the roads are banked at the turn so that the outer part of the road is a bit raised up as compared to the inner part. If the surface of the road makes an angle θ with the horizontal throughout the turn, then

$$\tan\theta = \frac{v^2}{rg}.$$

Here v = 60 km/hour = $\frac{60\times1000}{60\times60} = \frac{100}{6}$ m/s

$$\therefore\quad \tan\theta = \frac{\frac{100}{6}\times\frac{100}{6}}{100\times9.8} = \frac{(50/3)^2}{100\times9.8}$$

$$\therefore\quad \theta = \tan^{-1}\frac{(50/3)^2}{100\times9.8}$$

140. (d): The projected body has projectile motion, which can be supposed to be made of two simple motion i.e., motion in horizontal and motion in vertical direction.

The point of projection O, is the origin, the horizontal line through O in the plane of the motion is the x-axis and the vertical line through O is the y-axis. PM & QN are two walls, just clear by projectile.

The equation of trajectory of the particle is,

$$y = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha} \quad ...(1)$$

Since the projectile just clears the two walls PM and QN each of height h, the y-coordinate of points P & Q must satisfying equation (1), i.e.,

$$h = x\tan\alpha - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha}$$

or $gx^2 - 2u^2x\sin\alpha\cos\alpha + 2h\,u^2\cos^2\alpha = 0$ ...(2)

Let x_1 and x_2 be the x-coordinates of points P and Q. Then x_1 and x_2 are the roots of quadratic equation (2).

Comparing equation (2) with $ax^2 + bx + c = 0$, we find,

$$x_1 + x_2 = -\frac{b}{a} = \frac{2u^2\sin\alpha\cos\alpha}{g} \quad ...(3)$$

$$\text{and}\quad x_1x_2 = \frac{c}{a} = \frac{2hu^2\cos^2\alpha}{g} \quad ...(4)$$

Let R be the range of the projectile, i.e. let $OB = R$. From the symmetry of the path about the axis of the parabola, we have,

$NB = OM = x_1$

$\therefore\quad R = OB = ON + NB = x_2 + x_1$

From (3), $R = \frac{2u^2\sin\alpha\cos\alpha}{g}$ ...(5)

Distance between the walls = $2h = x_2 - x_1$

Squaring, $4h^2 = (x_2 - x_1)^2 = (x_2 + x_1)^2 - 4x_1x_2$

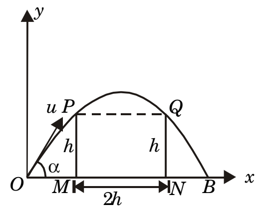
$$4h^2 = R^2 - \frac{8hu^2\cos^2\alpha}{g} \quad ...(6)$$

Solving equation (5) & (6), we get,

$R^2 - 4hR\cot\alpha - 4h^2 = 0$

$$\Rightarrow\quad R = \frac{4h\cot\alpha \pm \sqrt{16h^2\cot^2\alpha + 16h^2}}{2}$$

Taking only +ve value, we get,



$R = 2h\cot\alpha + 2h\,\text{cosec}\,\alpha$

$$= 2h\left(\frac{\cos\alpha}{\sin\alpha} + \frac{1}{\sin\alpha}\right)$$

$$= 2h\frac{(1+\cos\alpha)}{\sin\alpha}$$

$R = 2h\cot\alpha/2$

141. (c): In order to hit the target located at the same level with the cannon, the shell fired from cannon has projectile motion.

Let total time of motion = t

For projectile motion of shell, $t = \frac{2u\sin\alpha}{g}$

α = angle of projection

or $\sin\alpha = \frac{9.8\times t}{2\times240} = \frac{9.8t}{480}$ ...(1)

and horizontal range, $R = u\cos\alpha\times t$

or $\cos\alpha = \frac{R}{ut} = \frac{5100}{240t} = \frac{85}{4t}$ .....(2)

From equation (1) and (2),

$$\frac{(9.8)^2t^2}{(480)^2} + \frac{(85)^2}{(4t)^2} = 1$$

On simplifying, $t^4 - 2400t^2 + 1083750 = 0$

Solving for t^2 we get,

$$t^2 = \frac{2400\pm\sqrt{1425000}}{2} = \frac{2400\pm1194}{2}$$

Thus, t = 42.39 sec = 0.71 min

And t = 24.55 sec = 0.41 min

Thus the shell hit the target after 0.41 min or 0.71 min later depending on angle of projection.

142. (c): Radius of horizontal track = 10 m

Speed of car = 10 m/sec

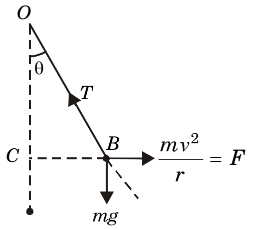
Length of rod = 1.00 m

Let θ be the angle which the rod makes with the vertical & m be the mass of plumb. T be the tension in the rod.

Resolving mg and F into two rectangular components we have,

Force parallel to rod,

$mg\cos\theta + \frac{mv^2}{r}\sin\theta = T$



Force perpendicular to rod,

$$mg\sin\theta - \frac{mv^2}{r}\cos\theta$$

The rod will be balanced if no perpendicular force act on it,

$\therefore \quad mg\sin\theta - \frac{mv^2}{r}\cos\theta = 0$

or $\quad mg\sin\theta = \frac{mv^2}{r}\cos\theta$

or $\quad \tan\theta = \frac{v^2}{rg} = \frac{10\times10}{10\times10} = 1$

$\therefore \quad \theta = 45°$

143. (d) : The horizontal velocity of the projectile remains constant throughout the journey.

Since the body is projected horizontally, the initial velocity will be same as the horizontal velocity at any point.

Since $x = 2t$, $\frac{dx}{dt} = 2$

$\therefore$ Horizontal velocity = 2 m/s

$\therefore$ Initial velocity = 2 m/s.

144. (a): $h_1 = \frac{u^2\sin^2\theta}{2g}$,

$h_2 = \frac{u^2\sin^2(90°-\theta)}{2g}$, $R = \frac{u^2\sin 2\theta}{g}$

Range R is same for angles θ and $(90° - \theta)$

$\therefore \quad h_1 h_2 = \frac{u^2\sin^2\theta}{2g}\times\frac{u^2\sin^2(90-\theta)}{2g}$

$= \frac{u^4(\sin^2\theta)\times\sin^2(90-\theta)}{4g^2}$

$[\because \sin(90-\theta) = \cos\theta]$

$= \frac{u^4(\sin^2\theta)\times\cos^2\theta}{4g^2} = \frac{u^4(2\sin\theta\cos\theta)^2}{4g^2\times4}$

$= \frac{u^4(\sin 2\theta)^2}{16g^2}$ $\quad [\because \sin 2\theta = 2\sin\theta\cos\theta]$

$= \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$

or $\quad R^2 = 16h_1h_2$ or $\quad R = 4\sqrt{h_1 h_2}$

145. (c) : The bullet performs a horizontal journey of 100 m with constant velocity of 1500 m/s.

The bullet also performs a vertical journey of h with zero initial velocity and downward acceleration g.

$\therefore$ For horizontal journey, time $(t) = \frac{\text{Distance}}{\text{Velocity}}$

$\therefore \quad t = \frac{100}{1500} = \frac{1}{15}$ sec ...(i)

The bullet performs vertical journey for this time.

For vertical journey, $h = ut + \frac{1}{2}gt^2$

$h = 0 + \frac{1}{2}\times10\times\left(\frac{1}{15}\right)^2$

or $\quad h = \frac{10}{2\times15\times15}$ m $= \frac{10\times100}{2\times15\times15}$ cm

or $\quad h = \frac{20}{9}$ cm = 2.2 cm

The gun should be aimed $\left(\frac{20}{9}\right)$ cm above the target.

146. (c) : For circular/angular motion, the formula for angular displacement θ and angular acceleration α is

$\theta = \omega t + \frac{1}{2}\alpha t^2$ where ω = initial angular velocity

or $\quad \theta = 0 + \frac{1}{2}\alpha t^2$

or $\quad \theta = \frac{1}{2}\times(2)(10)^2$ or $\quad \theta = 100$ radian

2π radian are covered in 1 revolution

$\therefore$ 1 radian is covered in $\frac{1}{2\pi}$ revolution

or 100 radian are covered in $\frac{100}{2\pi}$ revolution

$\therefore$ Number of revolutions $= \frac{50}{3\cdot14} \approx 16$

147. (c) : An aeroplane is flying horizontally. It releases a packet/bomb while it is vertically above a point A. The packet/bomb strikes the ground at B such that AB = 3 km. As soon as the packet/bomb is dropped from aeroplane, it moves with horizontal velocity u.

For vertical journey of the packet/bomb,

$s = ut + \frac{1}{2}gt^2$

$2000 = 0 + \frac{1}{2}\times10\times t^2$

or $\quad 5t^2 = 2000$ or $\quad t^2 = 400$ or $\quad t = 20$ sec

For horizontal journey,

velocity $= \frac{\text{distance}}{\text{time}} = \frac{3\times1000}{20}\frac{\text{m}}{\text{s}}$

$= \frac{150\times(60\times60)}{1000}\frac{\text{km}}{\text{hour}}$

u = 540 km/hr

148. (c) : For particle P, motion between AC will be an accelerated one while between CB a retarded one. But in any case horizontal component of its velocity will greater than or equal to v. On the other hand, in case of particle Q, it is always equal to v. The path travelled by P is greater than the path travelled by Q. Therefore $t_Q < t_P$.

149. (a) : $\omega = \frac{\text{Angle in radian}}{\text{time}}$

or $\omega = \frac{100 \times 2\pi \text{ (radian)}}{60 \quad \text{(sec)}}$

or $\omega = \frac{10\pi}{3} = \frac{10 \times 3\cdot14}{3}$ = 10.47 radian/sec

150. (c) : The projectile acquires a horizontal velocity when fired from cart moving horizontally by virtue of inertia of motion. The projectile thus performs horizontal journey with speed of the cart. Therefore the projectile should be launched in such a direction that its horizontal velocity remains same as that of the cart which is true for vertical upward direction.

Time for journey of cart = $\frac{\text{Distance}}{\text{Speed}} = \frac{80}{30}$ sec

$\therefore$ time of journey = $\frac{8}{3}$ sec ...(i)

The projectile is fired in upward vertical direction. It is at 90° to horizontal. It should rise for time t and then fall from highest point for time t, such that $2t = \frac{8}{3}$ sec

$\therefore \quad t = 8/6$ sec

$\therefore$ Time of upward journey of projectile = $\frac{8}{6}$ sec

$\therefore \; v = u - gt$ or $0 = u - \frac{10 \times 8}{6}$

or $u = \frac{40}{3}$ m/s in vertical upward direction from cart.

$\therefore \quad u = \frac{40}{3}$ m/s at 90°

151. (a) : The person will catch the ball if his speed and horizontal speed of the ball are same

$= v_0 \cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} = \cos 60° \; \therefore \; \theta = 60°.$

152. (d) : Range of projectile = $R = \frac{u^2 \sin 2\theta}{g}$

$\therefore \quad R = \left(\frac{50 \times 1000}{60 \times 60}\right)^2 \times \frac{\sin 60°}{10}$ or $R = \frac{125 \times 125 \times \sqrt{3}}{9 \times 9 \times 2 \times 10}$ m

or $R = 9.65\sqrt{3}$ m or $R = 9.65\sqrt{3} \times 10^{-3}$ km.

153. (b) : $v = 3\hat{i} + 8\hat{j}$

$\therefore \quad v_x = v\cos\theta = 3$ = horizontal velocity

$v_y = v\sin\theta = 8$ = vertical velocity

Time of flight of projectile = $2t$

where t = time during which v_y becomes zero

$\therefore$ Final velocity (v) = Initial velocity (u) $- gt$

or $0 = v_y - gt$ or $t = \frac{v_y}{g} = \frac{8}{10} = 0.8$ sec

$\therefore$ Time of flight = $(2t)$ = 1.6 sec.

154. (d) : For angular motion, $\theta = \omega t + \frac{1}{2}\alpha t^2$

$\theta = 0 + \frac{1}{2} \times \frac{\pi}{6} \times (60)^2$ rad = 300 rad.

2π radian = 1 rotation

$\Rightarrow$ 1 radian = $\frac{1}{2\pi}$ rotation

For θ radians, rotations made = $N = \frac{300 \times \pi}{2\pi}$

$N = 150$.

155. (a) : For a projectile motion, the trajectory is a parabola.

It is a first degree equation in y and second degree equation in x for a parabola. Hence the form of equation is

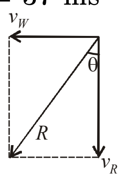
$y = ax + bx^2$.

Infact, $y = x\tan\theta - \left[\frac{g}{2u^2\cos^2\theta}\right]x^2$.

156. (c) : $R = \sqrt{V_R^2 + V_W^2} = \sqrt{35^2 + 12^2} = 37 \text{ ms}^{-1}$

$\tan\theta = \frac{v_W}{v_R} = \frac{12}{35}$

or $\theta = \tan^{-1}\left(\frac{12}{35}\right)$



157. (d) : Using parallelogram law of addition of vectors, here

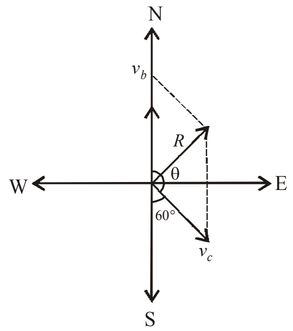
$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\alpha = \frac{A\sin\theta}{B + A\cos\theta}$

158. (a) : v_b = velocity of boat, v_c = velocity of current

Here $\theta = (180 - 60)° = 120°$

$\Rightarrow R = \sqrt{v_b^2 + v_c^2 + 2v_b v_c \cdot \cos\theta}$



$= \sqrt{25^2 + 10^2 + 2(25)(10)\cos 120°}$ = 22 kmph

159. (b) : $\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}\left(3t\,\hat{i} + 2t^2\hat{j} + 5\,\hat{k}\right)$

At t = 1s, $\bar{v} = 3\hat{i} + 4t\hat{j} = 3\hat{i} + 4\hat{j}$

For θ with x-axis,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53°.$$

160. (c) : The position of the particle is given by

$s = (u_x + u_y) + \frac{1}{2}(a_x + a_y)t^2$

Hence, $x = 5t + 1.5t^2$, $y = 1t^2$

Given $x(t)$ = 84 m, t = ?

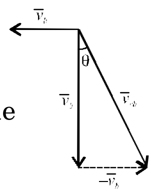
$5\,t + 1.5\,t^2 = 84 \Rightarrow t = 6$ s

At t = 6 s, $y = 6^2 = 36$ m.

161. (b) : $\bar{v}_b$ = Velocity of bicycle

$\bar{v}_r$ = velocity of rain

$\bar{v}_{rb}$ = velocity of rain w.r.t bicycle



$\tan\theta = \frac{v_b}{v_r} = \frac{12}{35}$

Therefore, the woman should hold her umbrella at an angle of $\theta = \tan^{-1}\left(\frac{12}{35}\right)$ with the vertical towards west.

162. (b) : $R = \frac{u^2 \sin 2\theta}{g}$

For θ = 45 ± α, 2θ = 90 ± 2α

$R = \frac{u^2 \sin(90 \pm \alpha)}{g}$

Therefore, ranges are equal for elevations which exceed or fall short of 45° by equal amounts.

163. (a) : $v_y^2 = u_y^2 + 2a_y.s_y$

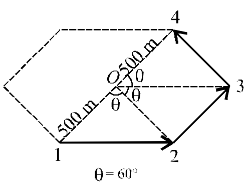
$v_y^2 = 0^2 + 2\,(+\,9.8)\,(490) \Rightarrow v_y = 98$ m/s

$v = \sqrt{v_x^2 + v_y^2} = \sqrt{15^2 + 98^2} = 99\ \text{ms}^{-1}$

164. (d) : $R = \frac{u^2 \cdot \sin 2\theta}{g} = \frac{(28)^2 \cdot \sin 2(30°)}{9.8} = 69$ m.

165. (b) : Impulse = Change in momentum = $m\Delta\bar{v}$
Impulse is a vector, formed by the product of scalar mass and vector velocity.

166. (c) :



The displacement from 1 to 4 is 1000 m.

167. (c)

168. (c) : To of wind with respect to the boat or $\bar{v}_{WB} = \bar{v}_W - \bar{v}_B$; 72 cos 45° = 51

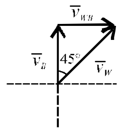
Thus, it is evident that $\bar{v}_{WB}$ is towards east.

169. (c) : $h_{\max} = \frac{u^2 \sin^2\theta}{2g} \Rightarrow 25 = \frac{40^2 \cdot \sin^2\theta}{2g}$

$\sin^2\theta = \frac{50g}{40^2} = \frac{50 \times 10}{40 \times 40} = \frac{5}{16}$

$\sin\theta = \frac{\sqrt{5}}{4} \Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta} = \frac{\sqrt{11}}{4}$

$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta\cos\theta}{g}$



$= \frac{2(40)^2\left(\frac{\sqrt{5}}{4}\right)\left(\frac{\sqrt{11}}{4}\right)}{10}$

$= \frac{2 \times 40^2 \times \sqrt{55}}{4 \times 4 \times 10} = 20\sqrt{55} = 148.3 = 150$ m.

170. (a) : $R_{\max} = \frac{u^2 \sin 2(45°)}{g} = 100 \Rightarrow \frac{u^2}{g} = 100$

$H = \frac{u^2}{2g} = \frac{100}{2} = 50$ m.

171. (d) : $\bar{r} = 3t\,\hat{i} - 2t^2\,\hat{j} + 4\,\hat{k}$

$\bar{v} = \frac{d\bar{r}}{dt} = 3\,\hat{i} - 4t\,\hat{j}$

$\bar{a} = \frac{d\bar{v}}{dt} = -\,4\,\hat{j}$.

172. (b) : $\bar{u} = 10\,\hat{j}$ m/s

$\bar{a} = (8\hat{i} + 2\hat{j})$

$\bar{s} = ut + \frac{1}{2}\bar{a}t^2 = (10\hat{j})t + \frac{1}{2}(8\hat{i} + 2\hat{j})t^2$

When x-coordinate = 16 m $\Rightarrow 4t^2 = 16$; $t^2 = 4$

or t = 2s; $\bar{v} = \frac{d\bar{s}}{dt} = 8t\,\hat{i} + (10 + 2t)\,\hat{j}$

When x = 16 m, t = 2 s;

$\bar{v} = 8(2)\,\hat{i} + (10 + 2(2))\,\hat{j} = 21\ \text{ms}^{-1}$.

173. (c) : $2\hat{i} + 3\hat{j} = \lambda(\hat{i} + \hat{j}) + u(\hat{i} - \hat{j})$

$2\hat{i} + 3\hat{j} = (\lambda + u)\hat{i} + (\lambda - u)\hat{j}$

$\Rightarrow \lambda + u = 2$ and $\lambda - u = 3$

$\Rightarrow \lambda = \frac{5}{2}$ and $u = \frac{-1}{2}$

Now, unit vector along $\hat{i} + \hat{j}$ is $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$ and unit vector along $\hat{i} - \hat{j} = \frac{\hat{i} - \hat{j}}{\sqrt{2}}$

Thus, $2\hat{i}+3\hat{j} = \frac{5}{2}(\hat{i}+\hat{j}) - \frac{1}{2}(\hat{i}-\hat{j})$

$= \left(\frac{5}{\sqrt{2}}\right)\cdot\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

As, the components of $2\hat{i}+3\hat{j}$ along $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$ directions are $\left(\frac{5}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$.

174. (b) : Only (b) is correct, as all others are valid only for uniformly accelerated motion.

175. (d) : Vectors for different orientations of the axes will have different components.

176. (a) : $\frac{u^2\cdot\sin 2\theta}{g} = 3$ km

Here $\theta = 30°$, so $\frac{u^2}{g} = \frac{3}{\sin 60°}$ km

But $R_{max} = \frac{u^2}{g} = \frac{3\times 2}{\sqrt{3}} = \frac{6}{\sqrt{3}}$ km $= 2\sqrt{3}$
$= 3.46$ km.

177. (a) : The shell will limb and hit the plane, if their horizontal velocities are equal.

Plane's horizontal velocity $= \frac{720 \text{ km}}{\text{h}} \times \frac{5}{18}$
$= 200$m/s

The shell's horizontal component of velocity is 600 sin θ m/s

$\Rightarrow 600\sin\theta = 200;\ \sin\theta = \frac{2}{6} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{3}\right)$.

178. (d) : For the plane, not to be hit, it should fly above the maximum height of the projectile in the previous question.

$H_{max} = \frac{u^2\cos^2\theta}{2g};\ u = 600$ m/s, $\sin\theta = \frac{1}{3}$

so, $\cos\theta = \sqrt{1-\sin^2\theta} = \frac{\sqrt{8}}{3}$

$\Rightarrow H_{max} = \frac{600^2}{2\times 10} \times \left(\frac{8}{9}\right)$

$H_{max} = 16000$ m $= 16$ km.

179. (b) : $\tan\theta = \frac{v_y}{v_x} = \frac{v_{oy}-gt}{v_{ox}}$,

$\Rightarrow \theta = \tan^{-1}\left(\frac{v_{oy}-gt}{v_{ox}}\right)$

180. (c) : $h_m = \frac{u^2\sin^2\theta}{2g};\ R = \frac{u^2\sin 2\theta}{g}$

$\Rightarrow \frac{h_m}{R} = \frac{\sin^2\theta}{2\sin 2\theta} = \frac{\sin^2\theta}{4\sin\theta\cos\theta};\ \frac{h_m}{R} = \frac{\tan\theta}{4}$

or $\tan\theta = \frac{4h_m}{R}$ or $\theta = \tan^{-1}\left(\frac{4h_m}{R}\right)$.

181. (c) : $v = \frac{dx}{dt} = \frac{d}{dt}(a+bt^2) = 2bt = 5t$ ms^{-1}

$v(t = 2\text{ s}) = 5(2)$ ms^{-1} $= 10$ ms^{-1}

Average velocity $= \frac{x(4)-x(2)}{4-2}$

$= \frac{(a+16b)-(a+4b)}{2} = 6b$

$= 6(2.5)\text{ms}^{-1} = 15\text{ms}^{-1}$

182. (d) : Taking point of launch as origin and up as + ve, using

$v^2 = u^2 + 2as$

$0^2 = (20\text{ ms}^{-1})^2 + 2(-10\text{ ms}^{-2})\cdot H_{max}$

$\Rightarrow H_{max} = \frac{20^2}{20} = 20$ m

Hence, from the ground the maximum height reached

$= 25$ m $+ 20$ m $= 45$ m

Further, for time to reach ground.

$s = -25$ m, $u = +20$ ms^{-1}, $a = -10$ ms^{-2}

Using, $s = ut + \frac{1}{2}at^2 \Rightarrow -25 = 20t - 5t^2$

$5t^2 - 20t - 25 = 0$ or $5t^2 - 25t + 5t - 25 = 0$

$5t(t-5) + 5(t-5) = 0 \Rightarrow (5t+5)\cdot(t-5) = 0$

$t = -1$ s or $t = 5$ s

Rejecting –ve value of time, we get, $t = 5$ s.

183. (c) : This is known as Galileo's law of odd numbers.

t	*y*	*y in terms of* y_0	*Distance travelled in successive intervals*	*Ratio*
0	0	0	0	0
t	$\frac{1}{2}gt^2$	y_0	y_0	1
$2t$	$4\left(\frac{1}{2}gt^2\right)$	$4y_0$	$3y_0$	3
$3t$	$9\left(\frac{1}{2}gt^2\right)$	$9y_0$	$5y_0$	5
$4t$	$16\left(\frac{1}{2}gt^2\right)$	$16y_0$	$7y_0$	7

184. (d) : Take north as positive,

$v_A = 54$ kmh^{-1} $= 15$ ms^{-1}

$v_B = -90$ kmh^{-1} $= -25$ ms^{-1}

v_{MA} = –18 kmh^{-1} = –5 ms^{-1}

$v_{MA} = v_M - v_A$ or $v_M = v_{MA} + v_A$

$v_{MB} = v_M - v_B = (v_{MA} + v_A - v_B)$

= – 5 ms^{-1} + 15 ms^{-1} – (–25 ms^{-1})

$\Rightarrow v_{MB}$ = 35 ms^{-1}

185. (c) : Average speed cannot be less than the magnitude of average velocity, since path length is greater than or equal to the magnitude of displacement.

186. (a) : The sign of acceleration depends on the choice of the positive direction of the axis. For example, if the vertically upward direction is chosen to be positive, direction of the axis, the acceleration due to gravity is negative. If a particle is falling under gravity, the acceleration, though negative, results in increase in speed.

187. (d) : Since the slope of graph B is greater than that of graph A, the speed of B is greater than that of A, B walks faster than A. B overtakes A, at the point of intersection of the two lines. All other statements are incorrect.

188. (c) : The drunkard advances (+ 5 – 3) m = 2 m in 8 s. Hence 8 m in 32 s. Then in the next 5 s, he would move 5 m more or reach 13 m or the pit. Hence he reaches the pit in 37 s.

189. (b) : v_{CJ} = –1500 kmh^{-1} = $v_C - v_J$

or $v_C = v_{CJ} + v_J$ = (–1500 kmh^{-1}) + 500 kmh^{-1}

= –1000 kmh^{-1}

190. (a): For car, u = 126kmh^{-1} = $126 \times \frac{5}{18}$ ms^{-1}

= 35 ms^{-1}

s = 200 m, using $v^2 = u^2 + 2as$

$0 = 35^2 + 2(a)(200)$

$\Rightarrow a = \frac{-35^2}{2(200)}$ = –3.06 ms^{-2}

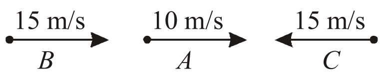
Using, $v = u + at$; $0 = 35 - (3.06)t$

$\Rightarrow t$ = 11.4 s

191. (d) : $s_{AB} = u_{AB} \cdot t + \frac{1}{2}a_{AB} \cdot t^2 = 0 \cdot t + \frac{1}{2}(1) \cdot (50)^2$

s_{AB} = 1250 m.

192. (b) :



v_A = 36 kmh^{-1} = $36 \times \frac{5}{18}$ ms^{-1} = 10 ms^{-1}

$v_B = v_C$ = 54 km h^{-1} = $\frac{54 \times 5}{18}$ ms^{-1} = 15 ms^{-1}

$s_{AB} = s_{AC}$ = 1000 m

$a_{AB} = a_A - a_B = 0 - a$

$s_{AB} = u_{AB}t + \frac{1}{2}a_{AB}t^2$

1000 m = $(10 - 15)\frac{\text{m}}{\text{s}} \cdot t + \frac{1}{2}(0 - a)t^2$

$\Rightarrow 1000 = -5t - \frac{1}{2}at^2 \Rightarrow a = -\frac{2(1000 + 5t)}{t^2}$

For car C,

$S_{AC} = u_{AC} \cdot t$

1000 m = $\left(\frac{25\text{ m}}{\text{s}}\right) \cdot t$

$t = \left(\frac{1000}{25}\right)$ = 40 s

$\Rightarrow a = -\frac{2(1000 + 5(40))}{40^2} = -\frac{2400}{1600}$ = – 1.5 m/s^2

193. (b) : The gap between successive buses is $v.T$, if v is their speed and T is their period. For a bus coming from behind the cycle, the $v_{BC} = v_B - v_C = (v - 20)$. The next bus crosses it in 18 min.

$\Rightarrow (v - 20) \cdot 18 = vT$

Similarly for a bus coming from the opposite side,

$(v + 20) \cdot 6 = v \cdot T$ ...(2)

Solving (1) and (2), we get

v = 40 kmh^{-1} and T = 9 min

194. (b) : Using, $s = ut + \frac{1}{2}at^2$

$s = 0$, u = 29.4 ms^{-1} and a = –9.8 ms^{-2}

$0 = 29.4\,t - \frac{9.8}{2}t^2$

$0 = t(29.4 - 4.9\,t)$

$\Rightarrow t = 0$ s or $t = \frac{29.4}{4.9}$ = 6s

0 s is the initial time, so the ball takes 6 s to return to the player's hand.

195. (a) : A ball thrown vertically up has zero speed at the highest point, but an acceleration vertically downwards.

196. (b) : v_{BP} = 150ms^{-1}, v_P = 30kmh^{-1}

= 8.3ms^{-1}

$\Rightarrow v_{BP} = v_B - v_P$

$\vec{v}_B = v_{BP} + v_P$ = (150 ms^{-1} + 8.3 ms^{-1})

= 158.3 ms^{-1}.

v_T = 192 kmh^{-1} = $192 \times \frac{5}{18}$ ms^{-1} = 53.3 ms^{-1}

$\Rightarrow v_{BT} = v_B - v_T$ = (158.3 – 53.3) ms^{-1}

= 105 ms^{-1}

197. (d) : Between $-0.5 < t < 0.5$, the graph has a negative slope.

$\Rightarrow \frac{dv}{dt} < 0$ or $v < 0$

At $t = -0.3$ s, the slope becomes more and more negative, so $\frac{dv}{dt} < 0$ or $a < 0$.

$x > 0$ is visible.

198. (b) : Average speed will be greatest among equal intervals of time when Δx is greatest and vice-versa.

199. (d) : As area under the curve in the third interval is largest, so distance travelled in the third interval is greatest.

200. (a): Using $s = ut + \frac{1}{2}at^2$

$s = 0$, $u = 49\ \text{ms}^{-1}$, $a = -9.8\ \text{ms}^{-2}$

$0 = 49t - \frac{9.8}{2}t^2$

$0 = t(49 - 4.9t) \Rightarrow t = 0$ or $t = 10$ s.

The ball comes back after 10 s. If we fix our frame of reference to the lift, the lift's initial speed will not change the answer got in the previous case. So, $t_1 = t_2 = 10$ s.

201. (b) : We can consider the moving belt as the frame of reference. Mother, father and child are stationary with reference to the belt. The child's speed w.r.t belt = 9 kmh^{-1}

$= \frac{9 \times 5}{18} = 2.5\ \text{ms}^{-1}$

$\Rightarrow$ Required time $= \frac{50\ \text{m}}{2.5\ \text{ms}^{-1}} = 20$ s

202. (b) : For the 2nd stone,

$x_2 = -15t + \frac{1}{2}(10)t^2$

$\Rightarrow x_2 = -15t + 5t^2$

Stone will hit the ground when

$x_2 = 200$ i.e. $200 = -15t + 5t^2$

$5t^2 - 15t - 200 = 0$

or $t^2 - 3t - 40 = 0 \Rightarrow (t - 8)(t + 5) = 0$

Rejecting the negative value of t, $\Rightarrow t = 8$ s.

So, $x_2 = -15t + 5t^2$ for $t \le 8$ s

$x_2 = 200$ for $t > 8$ s

For the first stone,

$x_1 = -30t + 5t^2$

For $0 \le t \le 8$ s; $x_2 - x_1 = 15t$ (straight part)

For $t > 8$ s

$x_2 - x_1 = 200 + 30t - 5t^2$ (curved part)

203. (d) : Area under the curve $= \frac{1}{2} \times 10 \times 12 = 60$ m.

204. (d) : Since acceleration is not constant in the given motion, all others are incorrect.

Assertion & Reason

1. (a) : If a ball is tossed up, after some time the ball reverses its direction of motion due to acceleration due to gravity.

2. (b) : For a particle going in a circle and completing a circle, average velocity is zero whereas average speed is not zero.

3. (c) : The uniform motion of a body means that the body is moving with constant speed, but if the direction of motion is changing (such as in uniform circular motion), its velocity changes and thus acceleration is produced in uniform motion.

4. (c) : $h = ut - \frac{1}{2}gt^2$ and $v^2 = u^2 - 2gh$.

The above equations are independent of mass.

5. (c) : If an object is stationary, then its position does not change with time. If the object is stationary at position $x(t) = x_0$ from the origin, the position - time graph for a stationary object is a straight line parallel to the time axis.

6. (d) : If a particle travels with speeds v_1, v_2, v_3 .... during time intervals t_1, t_2, t_3 .... respectively then total distance travelled $= v_1t_1 + v_2t_2 + v_3t_3 + ...$, total time taken $= t_1 + t_2 + t_3 + ...$ so, average speed,

$$v_{av} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + ...}{t_1 + t_2 + t_3 + ...}.$$

If $t_1 = t_2 = t_3 = ... = t$

then, $v_{av} = \frac{(v_1 + v_2 + ... + v_n)t}{nt} = \frac{v_1 + v_2 + ... + v_n}{n}$

7. (a) : An object is said to be in uniform motion if it undergoes equal displacement in equal intervals of time.

$$\therefore v_{av} = \frac{s_1 + s_2 + s_3 + ...}{t_1 + t_2 + t_3 + ...} = \frac{s + s + s + ...}{t + t + t + ...} = \frac{ns}{nt}$$

and $v_{\text{ins}} = \frac{s}{t}$.

Thus, in uniform motion average and instantaneous velocities have same value.

8. (b) : When a body is falling freely, only gravitational force acts on it in vertically downward direction. Due to this downward acceleration the velocity of a body increases and will be maximum when the body touches the ground. If downward accelerating force is balanced by the upward retarding force, the body falls with constant velocity. This constant velocity is called terminal velocity of body.

9. (c) : According to definition, displacement = velocity × time. Since displacement is a vector quantity so its value is equal to the signed sum of the area under velocity-time graph.

While considering the signed sum areas above the X-axis are considered positive and areas below the X-axis are considered negative.

10. **(b) :** At the highest point, the instantaneous velocity is acting horizontally and acceleration of projectile (= acceleration due to gravity) is acting vertically downward. Therefore, angle between velocity and acceleration at the highest point is 90°.

11. **(c) :** The maximum height to which a projectile rises above the point of projection is, $H = \frac{u^2 \sin^2\theta}{2g}$, which is independent of mass.

12. **(a) :** Horizontal range of projectile, $R = \frac{u^2 \sin 2\theta}{g}$

For maximum horizontal range, $\theta = 45°$

$\therefore \quad R_{max} = u^2/g$

Maximum height attained by projectile,

$$H = \frac{u^2 \sin^2\theta}{2g}$$

It is maximum when $\theta = 90°$.

$H_{max} = u^2/2g \quad \Rightarrow \quad R_{max} \neq H_{max}$.

13. **(a) :** The equation of the trajectory of a projectile is

$y = x\tan\theta - \frac{1}{2}\frac{g}{u^2\cos^2\theta}x^2$. Thus y component depends on x component.

14. **(c) :** Horizontal velocity provides the horizontal range. It does not effect time taken in the vertical direction.

15. **(a) :** As time of flight,

$$T = \frac{2u\sin\theta}{g} \quad \therefore \quad T' = \frac{2(nu)\sin\theta}{g} = nT$$

But range

$$R = \frac{u^2\sin 2\theta}{g} \quad \therefore \quad R' = \frac{n^2u^2\sin 2\theta}{g} = n^2R.$$

Hence range depends on initial velocity.

16. **(c) :** Range, $R = \frac{u^2\sin 2\theta}{g}$

when $\theta = 45°$, $R_{max} = \frac{u^2}{g}\sin 90° = \frac{u^2}{g}$

when $\theta = 135°$, $R_{max} = \frac{u^2}{g}\sin 270° = \frac{-u^2}{g}$

Negative sign shows opposite direction.

17. **(d) :** The horizontal range of projectile,

$$R = \frac{u^2\sin 2\theta}{g}$$

$$\therefore R_{max} = \frac{u^2}{g}$$

The maximum height attained by projectile

$$H = \frac{u^2\sin^2\theta}{2g}$$

If $H = R \quad \Rightarrow \frac{u^2\sin^2\theta}{2g} = \frac{u^2\sin 2\theta}{g}$,

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta \Rightarrow \tan\theta = 4$$

$\Rightarrow \theta = \tan^{-1}(4)$.

18. **(a) :** For maximum range, $\theta = 45°$. In that case maximum height

$$= \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{2g}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{u^2}{4g}$$

$= \frac{1}{4}\left(\frac{u^2}{g}\right)$ = 25% maximum range

Since angle for maximum range is always 45°, therefore the percentage also cannot vary. Reason is false.

19. **(b) :** The man should point his rifle at a point higher than the target since the bullet suffers a vertically downward deflection $\left(y = \frac{1}{2}gt^2\right)$ due to gravity.

20. **(a) :** Centripetal force is defined from formula

$$F = \frac{mv^2}{r}$$

$$\therefore \frac{F_1}{F_2} = \frac{v_1^2}{r_1} \times \frac{r_2}{v_2^2} = \left(\frac{v_1}{v_2}\right)^2 \frac{r_2}{r_1} = \frac{2}{4}$$

If $v_2 = 2v_1$, $r_2 = 2r_1 \quad \Rightarrow \quad F_2 = 2F_1$.

21. **(d) :** If μ_s is the coefficient of static friction between the tyres and the road, the magnitude of frictional force F cannot exceed $\mu_s mg$, so that $F \le \mu_s mg$

Thus for a safe turn : $\frac{mv^2}{r} \le \mu_s mg$

or, $\mu_s \ge \frac{v^2}{rg}$, or $v \le \sqrt{\mu_s rg}$

Therefore when the speed of car exceeds the value $\sqrt{\mu_s rg}$ then the car overturns. Since the inner wheels are moving in a circle of smaller radius, the maximum permissible velocity for them is less. Hence the inner wheels leave the ground first and the car will overturn on the outside.

22. **(a) :** During a turn $\tan\theta = \frac{v^2}{rg}$, where θ is angle of bending with vertical, when v is large and r is small, $\tan\theta$ increases. Therefore, as θ increases, so chances of skidding increase. Thus for a safe turn, θ should be small, for which v should be small and r should be large *i.e.* turning should be at a slow speed and along a track of larger radius.

23. **(b) :** When a ball is tossed up, at the highest point velocity is zero, but acceleration is g.

24. **(d) :** $a = -2t$

$$\frac{dv}{dt} = -2t$$

or $\int_4^v dv = -2\int_0^t t\,dt$

$v - 4 = -t^2$

or $v = 4 - t^2$

$\Rightarrow$ $v > 0$ for $t < 2$, $v = 0$

for $t = 0$, $v < 0$

for $t > 2$

The body reverses its direction of motion after 2 s. Till then distance travelled and displacement would be same on the straight line.

25. **(b) :** If the swimmer has a component against the river's velocity, then the drift can be lessened.

26. **(d) :**

$$R = \frac{u^2 \sin 2\theta}{2}$$

$$\frac{u^2 \sin(2(45-\theta))}{g} = \frac{u^2 \cos 2\theta}{g}$$

$$u^2 \sin\left(\frac{2(45+\theta)}{g}\right) = \frac{u^2 \cos 2\theta}{g}$$

27. **(b) :** Assertion is true, Reason is true but Reason is not a correct explanation for Assertion.

At the highest point, $y = \dfrac{u^2 \sin^2\theta}{2g}$

$x = \dfrac{R}{2} = \dfrac{u^2 \sin 2\theta}{2g}$

The radius vector locating highest point, tan $\alpha = \dfrac{y}{x}$

$$= \frac{\sin^2\theta}{\sin 2\theta} = \frac{\sin^2\theta}{2\sin\theta\cos\theta} = \frac{\tan\theta}{2}$$

$$= \frac{\tan(\tan^{-1}\theta)}{2} = 1 \text{ or } \alpha = \frac{\pi}{4}.$$

It velocity of projection be u, then position vector at any time t

$$r = (u\cos\theta.t)\hat{i} + \left(u\sin\theta t - \frac{1}{2}gt\right)^2 \hat{j}$$

Average velocity $= \dfrac{\overline{r}}{t} = u\cos\theta\hat{i} + \left(u\sin\theta - \dfrac{gt}{2}\right)\hat{j}$

$$(v_{AV}) = \sqrt{u^2\cos^2\theta + u^2\sin^2\theta - \frac{ug\sin\theta}{t} + \frac{g^2t^2}{4}}$$

$$= \sqrt{u^2\cos^2\theta + u^2\sin^2\theta - \frac{ug\sin\theta}{t} + \frac{g^2}{4t^2}}$$

$$= \sqrt{u^2 + \frac{g^2}{4t^2} - \frac{ug\sin\theta}{t}}$$

For $(v_{AV}) > u$

$\Rightarrow \dfrac{g^2}{4t^2} > \dfrac{ug\sin\theta}{t}$ or $t < \left(\dfrac{g}{4u\sin\theta}\right)$

QUESTIONS FROM PREVIOUS YEARS AIEEE/JEE MAIN

1. **(b) :** Ball A projected upwards with velocity u falls back to building top with velocity u downwards. It completes its journey to ground under gravity.

$\therefore$ $v_A{}^2 = u^2 + 2gh$ ....(i)

Ball B starts with downwards velocity u and reaches ground after travelling a vertical distance h

$\therefore$ $v_B{}^2 = u^2 + 2gh$ ....(ii)

From (i) and (ii), $v_A = v_B$.

2. **(c) :** For first case,

$$u_1 = 50\frac{\text{km}}{\text{hour}} = \frac{50\times1000}{60\times60} = \frac{125}{9}\frac{\text{m}}{\text{sec}}$$

$\therefore$ Acceleration

$$a = -\frac{u_1^2}{2s_1} = -\left(\frac{125}{9}\right)^2 \times \frac{1}{2\times6} = -16 \text{ m/sec}^2$$

For second case,

$$u_2 = 100\frac{\text{km}}{\text{hour}} = \frac{100\times1000}{60\times60} = \frac{250}{9}\frac{\text{m}}{\text{sec}}$$

$$\therefore \quad s_2 = \frac{-u_2^2}{2a} = \frac{-1}{2}\left(\frac{250}{9}\right)^2 \times \left(-\frac{1}{16}\right) = 24 \text{ m}$$

or $s_2 = 24$ m.

3. **(d) :** Height of building = 10 m

The ball projected from the roof of building will be back to roof - height of 10 m after covering the maximum horizontal range.

Maximum horizontal range $(R) = \dfrac{u^2\sin 2\theta}{g}$

or $R = \dfrac{(10)^2 \times \sin 60°}{10} = 10 \times 0.866$ or $R = 8.66$ m.

4. **(c) :** Equation of motion : $s = ut + \dfrac{1}{2}gt^2$

$\therefore$ $h = 0 + \dfrac{1}{2}gT^2$ or $2h = gT^2$ ....(i)

After $T/3$ sec,

$$s = 0 + \frac{1}{2}\times g\left(\frac{T}{3}\right)^2 = \frac{gT^2}{18}$$

or $18\,s = gT^2$ .... (ii)

From (i) and (ii),

$18\,s = 2h$ or $s = \dfrac{h}{9}$ m from top.

$\therefore$ Height from ground $= h - \dfrac{h}{9} = \dfrac{8h}{9}$ m.

5. **(a) :** The person will catch the ball if his speed and horizontal speed of the ball are same

$$= v_0\cos\theta = \frac{v_0}{2} \Rightarrow \cos\theta = \frac{1}{2} = \cos 60°$$

$\therefore$ $\theta = 60°$.

6. (d) : Let a be the retardation for both the vehicles

For automobile, $v^2 = u^2 - 2\,as$

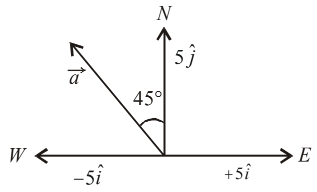
$\therefore \quad u_1^2 - 2as_1 = 0 \Rightarrow u_1^2 = 2as_1$

Similarly for car, $u_2^2 = 2as_2$

$\therefore \quad \left(\frac{u_2}{u_1}\right)^2 = \frac{s_2}{s_1} \Rightarrow \left(\frac{120}{60}\right)^2 = \frac{s_2}{20}$ or $s_2 = 80$ m.

7. **(b) :** Velocity in eastward direction $= 5\hat{i}$

velocity in northward direction $= 5\hat{j}$



$\therefore$ Acceleration $\vec{a} = \frac{5\hat{j} - 5\hat{i}}{10}$

or $\vec{a} = \frac{1}{2}\hat{j} - \frac{1}{2}\hat{i}$ or $|\vec{a}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2}$

or $|\vec{a}| = \frac{1}{\sqrt{2}}$ ms^{-2} towards north-west.

8. **(a) :** $t = ax^2 + bx$

Differentiate the equation with respect to t

$\therefore \quad 1 = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$

or $1 = 2axv + bv$ as $\frac{dx}{dt} = v$

or $v = \frac{1}{2ax + b}$

or $\frac{dv}{dt} = \frac{-2a(dx/dt)}{(2ax+b)^2} = -2av \times v^2$

or Acceleration $= -\,2av^3$.

9. **(*) :** For first part of journey, $s = s_1$,

$s_1 = \frac{1}{2} f t_1^2 = s$ .... (i)

$v = f\,t_1$ .... (ii)

For second part of journey,

$s_2 = vt$ or $s_2 = f\,t_1\,t$ ....(iii)

For the third part of journey,

$s_3 = \frac{1}{2}\left(\frac{f}{2}\right)(2t_1)^2$ or $s_3 = \frac{1}{2} \times \frac{4 f t_1^2}{2}$

or $s_3 = 2s_1 = 2s$ ....(iv)

$s_1 + s_2 + s_3 = 15s$

or $s + f\,t_1 t + 2s = 15s$

or $f\,t_1 t = 12s$ .... (v)

From (i) and (v), $\frac{s}{12s} = \frac{ft_1^2}{2 \times ft_1 t}$ or $t_1 = \frac{t}{6}$

or $s = \frac{1}{2}ft_1^2 = \frac{1}{2}f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$ or $s = \frac{ft^2}{72}$

* None of the given options provide this answer.

10. **(a) :** Initially, the parachutist falls under gravity

$\therefore \quad u^2 = 2ah = 2 \times 9.8 \times 50 = 980$ m^{2}s^{-2}

He reaches the ground with speed

$= 3$ m/s, $a = -2$ ms^{-2}

$\therefore \quad (3)^2 = u^2 - 2 \times 2 \times h_1$ or $9 = 980 - 4\,h_1$

or $h_1 = \frac{971}{4}$ or $h_1 = 242.75$ m

$\therefore$ Total height $= 50 + 242.75 = 292.75$
$= 293$ m.

11. **(b) :** $v = \alpha\sqrt{x}$

or $\frac{dx}{dt} = \alpha\sqrt{x}$ or $\frac{dx}{\sqrt{x}} = \alpha\,dt$ or $\int\frac{dx}{\sqrt{x}} = \alpha\int dt$

or $2x^{1/2} = \alpha t$ or $x = \left(\frac{\alpha}{2}\right)^2 t^2$

or displacement is proportional to t^2.

12. **(c) :** Given : velocity $v = v_0 + gt + ft^2$

$\therefore \quad v = \frac{dx}{dt}$ or $\int_0^x dx = \int_0^t v\,dt$

or $x = \int_0^t (v_0 + gt + ft^2)\,dt$

$x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3} + C$

where C is the constant of integration

Given : $x = 0, t = 0$. $\therefore$ $C = 0$

or $x = v_0 t + \frac{gt^2}{2} + \frac{ft^3}{3}$

At $t = 1$ sec $\therefore$ $x = v_0 + \frac{g}{2} + \frac{f}{3}$.

13. **(c) :** As $u = 0$, $v_1 = at$, $v_2 =$ constant for the other particle. Initially both are zero. Relative velocity of particle 1 w.r.t. 2 is velocity of 1 – velocity of 2. At first the velocity of first particle is less than that of 2. Then the distance travelled by particle 1 increases as $x_1 = (1/2)\,at_1^2$. For the second it is proportional to t. Therefore it is a parabola after crossing x-axis again. Curve (c) satisfies this.

14. **(b) :** $v = u + at$

$v = (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10$

$v = (3 + 4)\hat{i} + (4 + 3)\,\hat{j}$

$\Rightarrow \quad v = \sqrt{49 + 49} = \sqrt{98} = 7\sqrt{2}$ units

(This value is about 9.9 units close to 10 units. If (a) is given that is also not wrong).

15. **(d) :** The position vector of the particle from the origin at any time t is

$\vec{r} = v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\,\hat{j}$

$\therefore$ Velocity vector, $\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{v} = \frac{d}{dt}(v_0\cos\theta t\,\hat{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\,\hat{j})$

$= v_0\cos\theta\,\hat{i} + (v_0\sin\theta - gt)\,\hat{j}$

The angular momentum of the particle about the origin is

$$\vec{L} = \vec{r} \times m\vec{v}\ ;\ \vec{L} = m(\vec{r} \times \vec{v})$$

$$= m\Big[(v_0 \cos\theta t\,\hat{i} + (v_0 \sin\theta t - \frac{1}{2}gt^2)\,\hat{j}) \times (v_0 \cos\theta\,\hat{i} + (v_0 \sin\theta - gt)\,\hat{j})\Big]$$

$$= m\Big[(v_0^2 \cos\theta \sin\theta t - v_0 g t^2 \cos\theta)\hat{k} + (v_0^2 \sin\theta \cos\theta t - \frac{1}{2}gt^2 v_0 \cos\theta)(-\hat{k})\Big]$$

$$= m\Big[v_0^2 \sin\theta \cos\theta t\,\hat{k} - v_0 g t^2 \cos\theta\,\hat{k} - v_0^2 \sin\theta \cos\theta t\,\hat{k} + \frac{1}{2}v_0 g t^2 \cos\theta\,\hat{k}\Big]$$

$$= m\left[-\frac{1}{2}v_0 g t^2 \cos\theta\,\hat{k}\right] = -\frac{1}{2}mgv_0 t^2 \cos\theta\,\hat{k}$$

16. (a) : Here, $\vec{v} = K(y\hat{i} + x\hat{j})$

$\vec{v} = Ky\hat{i} + Kx\hat{j}$ ...(i)

$\because\ \vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$ ...(ii)

Equating equations (i) and (ii), we get

$\frac{dx}{dt} = Ky; \frac{dy}{dt} = Kx$

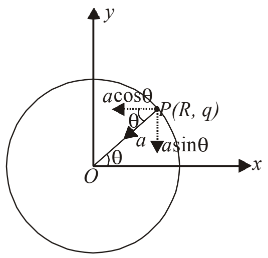
$\therefore\ \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{Kx}{Ky} = \frac{x}{y}$ ...(iii)

Integrating both sides of the above equation, we get

$\int y\,dy = \int x\,dx$

$y^2 = x^2$ + constant

17. (d) : For a particle in uniform circular motion,



Acceleration, $a = \frac{v^2}{R}$ towards the centre

From figure,

$\vec{a} = -a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

$= -\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$

18. (a) : $s = t^3 + 3$

$\therefore\ v = \frac{ds}{dt} = \frac{d}{dt}(t^3 + 3) = 3t^2$

Tangential acceleration, $a_t = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t$

At $t = 2$ s,

$v = 3(2)^2 = 12$ m/s, at $= 6(2) = 12$ m/s^2

Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}^2$$

Net acceleration,

$$a = \sqrt{(a_c)^2 + (a_t)^2} = \sqrt{(7.2)^2 + (12)^2} \approx 14 \text{ m/s}^2$$

19. (b) : $\frac{dv}{dt} = -2.5\sqrt{v}$ or $\frac{1}{\sqrt{v}}dv = -2.5\,dt$

On integrating, within limit ($v_1 = 6.25$ m s^{-1} to $v_2 = 0$)

$$\therefore \int_{v_1 = 6.25\ \text{ms}^{-1}}^{v_2 = 0} v^{-1/2}dv = -2.5\int_0^t dt$$

$$2 \times \left[v^{1/2}\right]_{6.25}^{0} = -(2.5)t \Rightarrow t = \frac{-2 \times (6.25)^{1/2}}{-2.5} = 2 \text{ s}$$

20. (c): Let u be the velocity of projection of the stone. The maximum height a boy can throw a stone is

$H_{max} = \frac{u^2}{2g} = 10$ m ...(i)

The maximum horizontal distance the boy can throw the same stone is

$R_{max} = \frac{u^2}{g} = 20$ m (Using (i))

21. (c) : Given : $u = \hat{i} + 2\hat{j}$

As $\vec{u} = u_x\hat{i} + u_y\hat{j}$ $\therefore$ $u_x = 1$ and $u_y = 2$

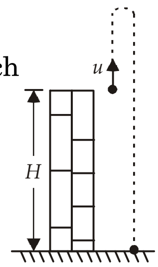
Also $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

$\therefore\ x = t$

and $y = 2t - \frac{1}{2} \times 10 \times t^2 = 2t - 5t^2$

Equation of trajectory is $y = 2x - 5x^2$.

22. (d) : Time taken by the particle to reach the top most point is, $t = \frac{u}{g}$ ...(i)



Time taken by the particle to reach the ground = nt

Using, $s = ut + \frac{1}{2}at^2$

$\Rightarrow\ -H = u(nt) - \frac{1}{2}g(nt)^2$

$\Rightarrow\ -H = u \times n\left(\frac{u}{g}\right) - \frac{1}{2}gn^2\left(\frac{u}{g}\right)^2$ [using (i)]

$\Rightarrow\ -2gH = 2nu^2 - n^2u^2$

$\Rightarrow\ 2gH = nu^2(n - 2)$

23. (a) : Using $h = ut + \frac{1}{2}at^2$

For stone 1, $y_1 = 10t - \frac{1}{2}gt^2$

For stone 2, $y_2 = 40t - \frac{1}{2}gt^2$

Relative position of the second stone with respect to the first, $\Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 - 10t + \frac{1}{2}gt^2$

$= 30\ t$

After 8 seconds, stone 1 reaches ground, *i.e.*, $y_1 = -240$ m

$\therefore\ \Delta y = y_2 - y_1 = 40t - \frac{1}{2}gt^2 + 240$

Therefore, it will be a parabolic curve till other stone reaches ground.

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