CHAPTER 6

Gravitation

Newton's Law of Gravitation

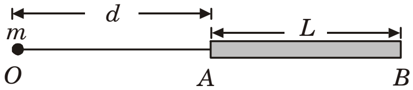
The interaction between the masses of two bodies is known as the gravitation.

- Any two particles in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them and it acts along the line joining the masses, i.e., if two particles of masses m_1 and m_2 be separated by a distance r, the force of attraction F is given by,

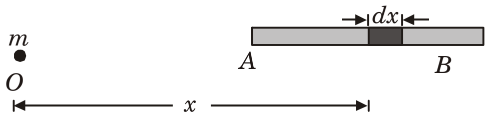
$$F = G\frac{m_1 m_2}{r^2}$$

- G is the universal gravitational constant. It is basically a conversion factor to adjust the number and units so that they come out to the correct value. $G = 6.7 \times 10^{-11}$ Nm²/kg²
  The dimension of G is given by $[M^{-1}L^{3}T^{-2}]$

Illustration - 1 : Find the gravitational force in micronewton of attraction on the point mass m placed at O by a thin rod of mass M and length L as shown in figure.
[Take m = 10 kg, $G = 6.6 \times 10^{-11}$ Nm²/kg², M = 1000 kg, L = 1 m, d = 10 cm



Soln.:



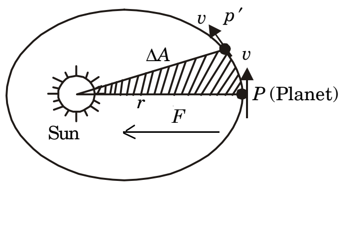
First we need to find the force due to an element of length dx. The mass of the element is $dm = \left(\frac{M}{L}\right)dx$

so, $dF = G\frac{Mm}{L}\frac{dx}{x^2}$

$\therefore$ The net gravitational force is $F = \frac{GMm}{L}\int_{d}^{d+L}\frac{dx}{x^2} = \frac{GMm}{L}\left[\frac{1}{d} - \frac{1}{L+d}\right] = \frac{GMm}{d(L+d)} = 6\ \mu\text{N}$

Notice that when $d >> L$, we find $F = \frac{GMm}{d^2}$, the result for two point masses.

Kepler's Law of Planetary Motion



- **First law.** The orbit of an object moving around another in space is elliptical with the stationary objects located at one of the focal points of the ellipse.
  In other words the earth travels around the sun in an ellipse, and the sun is at a focal point of that ellipse.
- **Second law.** The position vector from the sun to the planet sweeps out equal area in equal time, i.e., areal velocity (the area covered by the position vector of planet per unit time) of a planet around the sun always remains constant.

$$\frac{\Delta A}{\Delta t} = \text{Constant}$$

- **Third law.** The square of the time period of a planet around the sun is proportional to the cube of the semi-major axis of the ellipse or mean distance of the planet from the sun. i.e.

$$T^2 \propto a^3$$

Illustration - 2 : A saturn year is 29.5 times the earth year. How far is saturn from the sun (M) if the earth is 1.5×10^8 km away from the sun?

Soln.: It is given that $T_s = 29.5\ T_e$; $R_e = 1.5 \times 10^{11}$ m

Now according to Kepler's third law $\frac{T_s^2}{T_e^2} = \frac{R_s^3}{R_e^3}$

$$R_s = R_e\left(\frac{T_s}{T_e}\right)^{2/3} = 1.5\times10^{11}\left(\frac{29.5\ T_e}{T_e}\right)^{2/3} = 1.43\times10^{12}\text{ m} = 1.43\times10^{9}\text{ km}$$

Gravity and Acceleration due to Gravity

- Gravity is the force of attraction exerted by the earth towards its centre on a body lying on or near the surface of earth. Gravity is basically a special case of gravitation and is also known as earth's gravitation.
- When a body is falling under the action of gravity the body would possess an acceleration which is called acceleration due to gravity, usually denoted by the letter 'g'.

For a body of mass 'm'

Force due to gravity, $F = mg$ ...(1)

From Newton's law of gravitation,

$$F = G\frac{M_e m}{R_e^2} \quad ...(2)$$

M_e = Mass of the earth

R_e = Radius of the earth

From (1) and (2), $mg = G\frac{M_e m}{R_e^2}$

$$g = \frac{GM_e}{R_e^2}$$

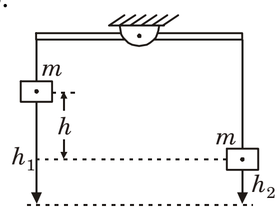
Variation of acceleration due to gravity

- **Variation with altitude:** Let us consider an object of mass m at a height h above the surface of the earth. If g' be the acceleration due to gravity at this place, then,

$$g' = \left(1 - \frac{2h}{R_e}\right)g$$

Thus, the decrease in the acceleration due to gravity $= g - g' = \frac{2h}{R_e}g$

Illustration - 3 : Two equal masses m and m are hung from a balance whose scale pans differ in vertical height by h. Calculate the error in weighing, if any in terms of density of earth ρ.



Soln.: As with height g varies as $g' = \frac{g}{[1 + h/R]^2} = g\left[1 - \frac{2h}{R}\right]$

and in accordance with figure, $h_1 > h_2$,

so W_1 will be lesser than W_2. $\therefore\ W_2 - W_1 = mg'_2 - mg'_1 = 2mg\left[\frac{h_1}{R} - \frac{h_2}{R}\right]$

or $W_2 - W_1 = 2m\frac{GM}{R^2}\frac{h}{R}$ $\left[\text{as } g = \frac{GM}{R^2} \text{ and } (h_1 - h_2) = h\right]$

or $W_2 - W_1 = \frac{2mhG}{R^3}\times\left(\frac{4}{3}\pi R^3\rho\right) = \frac{8}{3}\pi\rho Gmh$ $\left[\text{as } M = \frac{4}{3}\pi R^3\rho\right]$

- **Variation with depth:** For an object of mass m at a depth h below the surface of the earth,

$$g' = \left(1 - \frac{h}{R_e}\right)g$$

Decrease in acceleration due to gravity at this place $= g - g' = \frac{h}{R_e} g$

- **Variation of 'g' due to shape of the earth:** The earth is not a perfect sphere. The diameter in the equator plane is about 21 km more than the diameter along the poles. Due to this, the acceleration due to gravity is more at the poles and less at the equator.
- **Variation of 'g' due to rotation of the earth:** The earth rotates about its own axis from west to east. All the bodies on the surface of the earth, except those lying on the poles, execute circular motion about the axis of the earth. If λ be the latitude at any place, then the value of acceleration due to gravity at the place is given by,

$$g' = g - \omega^2 R_e \cos^2\lambda$$

where ω is the angular velocity of motion of the earth around its polar axis.

At the equator, $\lambda = 0$

$g' = g - \omega^2 R_e$

At the poles, $\lambda = 90°$

$g' = g$

- **Variation of g due to some other features:** The surface of the earth is uneven. The presence of mountains, plateaus and valleys would cause a variation in g.

Further, the earth's density is not uniform, the inner core is heavier than the mantle. The density of the crust of earth varies from region to region over earth's surface. Thus g varies from region to region.

Illustration - 4 : A body is suspended on a spring balance in ship sailing along the equator with a speed v. Show that scale reading will be very close to $W_0(1 \pm 2\omega v/g)$ where ω is the angular speed of the earth and W_0 is the scale reading when the ship is at rest. Explain also the significance of plus or minus sign.

Soln.: We know that at equator due to rotation of earth $g' = (g - R\omega^2)$, so if m is the mass of body

$$W_0 = m(g - R\omega^2) = m\left[g - \frac{V^2}{R}\right] \quad [\text{as } V = R\omega] \qquad .....(i)$$

Now when the ship is moving along the equator with a speed v, the speed of ship relative to the centre of earth will be $V \pm v$. The plus sign holds if the ship is moving in the direction of motion of earth, *i.e.*, west to east and minus if the ship is moving in opposite direction. So the weight recorded by the spring balance in the moving ship will be

$$W = m\left[g - \frac{(V \pm v)^2}{R}\right] \qquad .....(ii)$$

Dividing equation (ii) by (i), $\frac{W}{W_0} = \left[1 - \frac{(V \pm v)^2}{gR}\right]\left[1 - \frac{V^2}{gR}\right]^{-1}$

or $\frac{W}{W_0} = \left[1 - \frac{(V \pm v)^2}{gR}\right]\left[1 + \frac{V^2}{gR}\right]$ $\quad$ [as $(1 - x)^{-1} = 1 + x$ if $x << 1$]

or $\frac{W}{W_0} = \left[1 - \frac{(V \pm v)^2}{gR} + \frac{V^2}{gR} + ...\right] \simeq \left[1 \mp \frac{2Vv}{gR}\right]$

or $W \simeq W_0\left[1 \mp 2\frac{\omega v}{g}\right]$ [as $V = R\omega$]

Here negative sign holds for the ship moving in the direction of motion of earth, i.e., from west to east, *i.e.,* the body in the moving ship will weigh less if the ship is moving from west to east, in the direction of motion of earth and more if the ship is moving from east to west, opposite to the motion of earth.

Gravitational Field and Gravitational Potential

- The space surrounding a material body in which another body experiences force by virtue of its mass is called gravitational field.
- The intensity of gravitational field at any point is the force on the unit mass placed at that point i.e.

$$\vec{E} = \frac{\vec{F}}{m} = G\frac{M_e}{R_e^{\,3}}\hat{R}_e$$

Magnitude of $E = g$

- The gravitational potential at a point in the gravitational field of a body is defined as the work done in bringing a unit mass from infinity to that point.
- The gravitational force on a unit mass placed at a distance r from the object of mass $M = \frac{GM}{r^2}$
- Work done by this force in moving the unit mass through a distance dr $= \frac{GM}{r^2}\text{d}r.$
- Thus, the gravitational potential at a point r from the body is given by,

$$V = \int_{\infty}^{r}\frac{GM}{r^2}\text{d}r$$

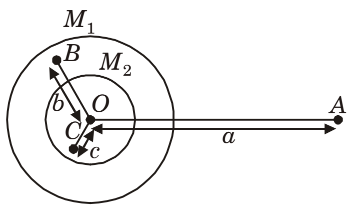
$$V = -\frac{GM}{r}$$

- **Gravitational potential energy.** The gravitational potential energy of a body at a point is defined as the work done in bringing the body from infinity to that point.

$$U = -\frac{GMm}{r}$$

Illustration - 5 : Two concentric shells of masses M_1 and M_2 are situated as shown in figure. Find the force on a particle of mass m when the particle is located at (a) $r = a$ (b) $r = b$ (c) $r \le c$. The distance r is measured from the centre of the shell.

[$M_1 = 500$ kg, $M_2 = 1500$ kg, $a = \sqrt{6.6}$ m, $b = \sqrt{3.3}$, $G = 6.6 \times 10^{-11}$ Nm²/kg²]



Take 10^{-8} N as unit.

Soln.: We know that attraction at an external point due to spherical shell of mass M is $\frac{GM}{r^2}$ while at an internal point is zero. So

(a) For $r = a$, the point is external for both the shell ; so $E_A = \frac{G(M_1 + M_2)}{a^2}$

$\therefore\ F_A = mE_A = \frac{G[M_1 + M_2]m}{a^2} = 2m$ units

(b) For $r = b$, the point is external to the shell of mass M_2 and internal to the shell of mass M_1 ; so

$E_B = \frac{GM_2}{b^2} + 0$

$\therefore\ F_B = mE_B = \frac{GM_2 m}{b^2} = 3m$ units

(c) For $r = c$, the point is internal to both the shells, so $E_C = 0 + 0 = 0$ $\quad\therefore\ F_C = mE_C = 0$

Motion of the Satellites-Orbital Velocity

- The celestial bodies which revolve around the planets in close and stable orbits are called satellites.
- Satellites revolve round the earth in definite orbits and are kept in these orbits by the gravitational attraction of the earth. If a satellite of mass m circles round the earth at a height of h above the surface of the earth, then its velocity v in the orbit is given by the following equation,

$$\frac{mv^2}{R_e + h} = G\frac{M_e m}{(R_e + h)^2}, \qquad \text{or,} \quad v = \sqrt{\frac{GM_e}{(R_e + h)}}$$

- If g is the acceleration due to gravity on the earth, then,

$$g = \frac{GM_e}{R_e^2} \Rightarrow GM_e = gR_e^2 \qquad \therefore v = \sqrt{\frac{gR_e^2}{(R_e + h)}}$$

$$v = R_e\sqrt{\frac{g}{R_e + h}}$$

- The time period of revolution of the satellite about the earth is given by,

$$T = \frac{2\pi r}{v} = \frac{2\pi(R_e + h)}{R_e\sqrt{\frac{g}{R_e + h}}}$$

$$T = \frac{2\pi}{R_e}\sqrt{\frac{(R_e + h)^3}{g}}$$

- For a satellite revolving very near to earth's surface, $R_e >> h$, then

$$v = \sqrt{gR_e} = 8 \text{ km/sec}$$

and $T = 2\pi\sqrt{\frac{R_e}{g}}$ = 84 minutes

Illustration - 6 : A small satellite revolves round a planet of mean density 10 gm/cm^3, the radius of the orbit being slightly greater than the radius of the planet. Calculate the time of revolution of the satellite. Take $G = 6.6 \times 10^{-8}$ C.G.S. unit.

Soln.:
- Mean density of planet, $\rho = 10$ gm/cm^3
- Radius of satellite $\simeq$ Radius of planet

To calculate the time of revolution of the satellite.

Let M is the mass of the planet and r is the radius of the orbit of the satellite which is nearly equal to the radius of the planet.

The orbital velocity of the satellite round the planet, $v = \sqrt{\frac{GM}{r}}$ ...(1)

Mass of the planet, $M = \frac{4}{3}\pi r^3 \rho$

Putting the value of M in equation (1), we get,

$$v = \sqrt{\frac{4}{3}\pi r^3 \rho . \frac{G}{r}} = r\sqrt{\frac{4}{3}\pi\rho G}$$

Time of revolution, $T = \frac{2\pi r}{v} = \frac{2\pi}{\sqrt{\frac{4}{3}\pi\rho G}} = \sqrt{\frac{3\pi}{\rho G}} = \sqrt{\frac{3 \times 3.14}{10 \times 6.6 \times 10^{-8}}}$ = 3780 seconds = 1 hr 3 min

Geostationary Satellite and Parking Orbit

If the time period of the satellite is exactly equal to the period of revolution of the earth, then it will appear to be stationary at the same place on the earth. This is known as *parking orbit*. And such a satellite is called a geo-static or geo-stationary or geo-synchronous satellite and is used as a communication satellite.

Energy of an orbiting satellite

Kinetic energy of the satellite $= \frac{1}{2}mv^2$

$$= \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 = \frac{GMm}{2r}$$

Gravitational potential energy of the satellite $= -\frac{GMm}{r}$

$\therefore$ Total energy of the satellite $= \frac{GMm}{2r} - \frac{GMm}{r}$

$$E = -\frac{GMm}{2r}$$

Escape Velocity

- The escape velocity on earth is defined as the minimum velocity with which a body has to be projected vertically upward from the surface of the earth, so that it just goes beyond the gravitational attraction of the earth and never returns on it.
- It can be proved using the concept of gravitational potential energy that the escape velocity on earth's surface is given by,

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$$

- We know that the orbital velocity of a satellite close to the earth surface is given by, $v_0 = \sqrt{gR_e}$

$\therefore$ $v_e = \sqrt{2}v_0$

$v_e = 11.2$ km/sec

Illustration - 7 : An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.

(a) Determine the height of the satellite above the earth's surface.

(b) If the satellite is stopped suddenly in its orbit and allowed to fall freely on to the earth, find the speed with which it hits the surface of the earth.

Take, radius of the earth = 6400 km and $g = 9.8$ m/s^2.

Soln.: If R be the radius of the earth and h be the height of the satellite above the earth's surface, the orbital velocity v_0 and the escape velocity v_e of the satellite are given by,

$$v_0 = \sqrt{\frac{gR^2}{R+h}} \quad \text{and} \quad v_e = \sqrt{2gR}$$

(a): It is given that, $v_0 = \frac{1}{2}v_e$

i.e. $\sqrt{\frac{gR^2}{R+h}} = \frac{1}{2}\sqrt{2gR}$ or $\frac{gR^2}{R+h} = \frac{1}{2}gR$

$\therefore$ $h = R = 6400$ km

(b): Potential energy of the satellite at a distance $R + h$ (= $2R$) from the centre of the earth $= -\frac{GMm}{2R}$

Where M and m are the masses of the earth and the satellite respectively.

Potential energy of the satellite at a distance R from the centre of the earth i.e., on the surface of the earth

$= -\frac{GMm}{R}$

$\therefore$ Change in potential energy of the satellite = $-\dfrac{GMm}{R}+\dfrac{GMm}{2R} = -\dfrac{GMm}{2R}$

This decrease in potential energy will be converted into kinetic energy and thus,

$$\frac{1}{2}mv^2 = \frac{GMm}{2R} \text{ or, } v^2 = \frac{GM}{R} = gR \qquad \left[\because g = \frac{Gm}{R^2}\right]$$

$\therefore$ $v = \sqrt{gR} = \sqrt{9.8\times 6400\times 10^3} = 7.92 \times 10^3$ m/s = 7.92 km/s

Thus the satellite would hit the surface of the earth with a speed of 7.92 km/s.

Weightlessness	Weightlessness is a situation in which the effective weight of the body becomes zero. Example when body is in weightlessness state. (i) When the body is taken at the centre of the earth. (ii) When the body is lying in a freely falling lift. (iii) When the body is inside a space craft or satellite which is orbiting around the earth. • Inertial mass of a body is related to its inertia in linear motion and is defined by Newton's second law of motion. $m_i = F/a$ • Gravitational mass of a body is related to gravitational pull on the body and is defined by Newton's law of gravitation. $m_G = \dfrac{F}{(GM/R^2)}$ • Gravitational mass of a body is affected by the presence of other bodies near it where as the inertial mass of a body remains unaffected by the presence of other bodies near it. • The gravitational mass is measured by spring balance where as inertial mass is measured by inertial balance.

COMPETITION WINDOW

UNIVERSAL LAW OF GRAVITATION

- Gravitational force is attractive force between two masses M_1 and M_2 separated by a distance r. It is given by $F = -G\,(M_1M_2/r^2)$, where G is universal gravitational constant, $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}. The negative sign shows that force is attractive.
- Dimensional formula of G is $[M^{-1}L^3T^{-2}]$.
- The force of gravitation is the central force. It acts along the line joining the particles. It is a conservative force. The work done by it is independent of the path followed. This force is attractive in nature.
- It is the weakest force in nature. It is 10^{38} times smaller than nuclear force and 10^{36} times smaller than electric force.
- Gravitation is independent of the presence of other bodies around it.
- The gravitational pull of the earth is called gravity.
- The gravitation forces between two bodies are action-reaction pairs. The law of gravitation is universal as it is applicable to all bodies, irrespective of their size, shape or position. This force does not depend upon the state of the bodies, nature of the intervening medium, temperature and other physical condition of the bodies.
- In motion of the planets and satellites, force of gravitation provides the necessary centripetal force due to which earth revolves around the sun and moon around the earth.

ACCELERATION DUE TO GRAVITY

- The value of acceleration due to gravity increases as we go from equator to the pole.
- The value of the acceleration due to gravity on the moon is about one sixth of that on the earth and on the sun is about 27 times that on the earth.
- Among the planets, the g is minimum on the mercury.
- Acceleration due to gravity on the surface of the earth is given by, $g = GM/R^2$, where M is the mass of the earth and R is the radius of the earth.
- Acceleration due to gravity at a height h above the surface of the earth is given by

  $$g_h = GM/(R+h)^2 \cong g\,(1 - 2h/R).$$

  The approximation is true when $h << R$.
- Value of g at depth d from earth's surface

  (a) $g' = g\left[1 - \frac{d}{R}\right]$ (b) $g' = \frac{GM}{R^3}(R-d)$.

  Again, the approximation is true for $d << R$.
- The value of g at a height h from the surface and $h << R$ is $g' = g\left(1 - \frac{2h}{R}\right)$.
- The value of g at latitude λ is

  (a) $g' = g - \omega^2 R_e \cos^2\lambda$

  (b) At the equator $\lambda = 0$. $\therefore$ $g' = g - \omega^2 R_e$

  (c) At the poles $\lambda = \pi/2$. $\therefore$ $g' = g$.
- The decrease in g with latitude is caused by the rotation of the earth about its own axis. A part of g

is used to provide the centripetal acceleration for rotation about the axis.

- The rotation of the earth about the sun has no effect on the value of g.
- Decrease in g in going from poles to the equator is about $0 \cdot 35\%$.
- The weight of the body varies in the same manner as g does. ($W = mg$).

GRAVITATIONAL FIELD AND POTENTIAL

- The gravitational force of attraction acting on a body of unit mass at any point in the gravitational field is defined as the intensity of gravitational field (E_g) at that point. $E_g = \frac{F}{m} = \frac{GM}{r^2}$.
- The gravitational potential energy of a mass m at a point above the surface of the earth at a height h is given by $\frac{-GMm}{R+h}$. The negative sign implies that as R increases, the gravitational potential energy decreases and becomes zero at infinity.
- The body is moved from the surface of earth to a point at a height h above the surface of earth then change in potential energy will be mgh.
- Gravitational potential at a point above the surface of the earth at a height h is $-GM/(R + h)$. Its unit is joule/kilogram.
- Gravitational mass, M_g is defined by Newton's law of gravitation.

$$M_g = \frac{F_g}{g} = \frac{W}{g} = \frac{\text{Weight of body}}{\text{Acceleration due to gravity}}, \qquad \frac{(M_1)_g}{(M_2)_g} = \frac{F_{g_1} g_2}{g_1 F_{g_2}}$$

SATELLITE

- Let ω_0 be the angular speed of the satellite and v_0 be orbital speed of the satellite, then $v_0 = (R + h)\omega_0$, where R = radius of the earth and h = height of the satellite above the surface of the earth.
Let g be the acceleration due to gravity on the surface of the earth, T be the time period of the satellite and M be the mass of the earth. Then different quantities connected with satellite at height h are as follows:

(a) $\omega_0 = \left[\frac{GM}{(R+h)^3}\right]^{1/2} = R\left[\frac{g}{(R+h)^3}\right]^{1/2}$.

(b) $T = \frac{2\pi}{\omega_0}$ and frequency of revolution, $\upsilon = \frac{\omega_0}{2\pi}$.

(c) $v_0 = \left[\frac{GM}{R+h}\right]^{1/2} = R\left[\frac{g}{R+h}\right]^{1/2}$

Very near the surface of the earth, we get the values by putting $h = 0$. That is:

(i) $\omega_0 = \left[\frac{GM}{R^3}\right]^{1/2} = \left[\frac{g}{R}\right]^{1/2}$ (ii) $v_0 = \left[\frac{GM}{R}\right]^{1/2} = [g\ R]^{1/2}$

(iii) $T \cong 2\pi\left[\frac{R}{g}\right]^{1/2}$ = 5078 sec = 1 hour 24.6 minute.

- Altitude or height of satellite above the earth's surface, $h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$.
- Angular momentum, $L = mv(R+h) = [m^2GM(R+h)]^{1/2}$.
- Above the surface of the earth, the acceleration due to gravity varies inversely as the square of the distance from the centre of the earth. $g' = \frac{gR^2}{(R+h)^2}$.
- The gravitational potential energy of a satellite of mass m is $U = \frac{-GMm}{r}$, where r is the radius of the orbit.
It is negative.

- Kinetic energy of the satellite is $K = \frac{1}{2}mv_0^2 = \frac{GMm}{2r}$.
- Total energy of the satellite $E = U + K = -\frac{GMm}{2r}$. –ve sign indicates that work is done to bring or to bound the satellite close to the earth.
- Total energy of a satellite at a height equal to the radius of the earth is given by

$$-\frac{GMm}{2(R+R)} = -\frac{GMm}{4R} = \frac{1}{4}mgR .$$

where $g = GM/R^2$ is the acceleration due to gravity on the surface of the earth.
When the total energy of the satellite becomes zero or greater than zero, the satellite escapes from the gravitational pull of the earth.
- If the radius of planet decreases by $n\%$, keeping the mass unchanged, the acceleration due to gravity on its surface increases by $2n\%$ *i.e.* $\frac{\Delta g}{g} = -\frac{2\Delta R}{R}$
- If the mass of the planet increases by $m\%$ keeping the radius constant, the acceleration due to gravity on its surface increases by $m\%$ $\left[\frac{\Delta g}{g} = \frac{\Delta M}{M}\right]$ where R = constant.
- If the density of planet decreases by $p\%$ keeping the radius constant, the acceleration due to gravity decreases by $p\%$.
- If the radius of the planet decreases by $q\%$ keeping the density constant, the acceleration due to gravity decreases by $q\%$.
- For the planets orbiting around the sun, the angular speed, the linear speed and kinetic energy change with time but the angular momentum remains constant.
- The minimum velocity with which a body must be projected in the atmosphere so as to enable it to just overcome the gravitational attraction of the earth is called escape velocity. *i.e.* $v_e = \sqrt{2gR}$, where R = radius of earth.
- The relation between orbital velocity of satellite and escape velocity is $v_e = \sqrt{2}\,v_o$. *i.e.* if the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape.
- There is no atmosphere on the moon because the escape velocity on the moon is low.
- If the orbital radius of the earth around the sun be one fourth of the present value, then the duration of the year will be one eighth of the present value.
- Weightlessness is a situation in which the effective weight of the body becomes zero. Circumstances when this condition arises.
  (i) When the body is taken at the centre of the earth.
  (ii) When a body is lying in a freely falling lift, ($a = g$).
  (iii) When the body is inside a space craft or satellite which is orbiting around the earth.
- **Kepler's first law (law of orbit)** - Every planet revolves around the sun in an elliptical orbit. The sun is situated at one focus of the ellipse.
- **Kepler's second law (law of area)** - The radius vector drawn from the sun to a planet sweeps out equal areas in equal intervals of time. *i.e.* the areal velocity of planet around the sun is constant.
- **Kepler's third law (law of period)** - The square of the time period of revolution of a planet around the sun is directly proportional to the cube of semi-major axis of its elliptical orbit, *i.e* $T^2 \propto a^3$, where a = semi-major axis of the elliptical orbit of the planet around the sun.
- **Shape of orbit of a satellite** - The shape of orbit of a satellite depends upon its velocity of projection v from earth.
  (a) If $v < v_o$, the satellite falls back to earth following a spiral path.
  (b) If $v = v_o$, the satellite revolves in circular path/orbit around earth.

(c) If $v > v_o$, but less than escape velocity v_e, *i.e.* $v_o < v < v_e$, the satellite shall revolve around earth in elliptical orbit.
(d) If $v = v_e$, the satellite will escape away following a parabolic path.
(e) If $v > v_e$, the satellite will escape away following a hyperbolic path.

- **Geo-stationary satellite/Parking orbit**
  (a) Time period = 24 hour
  It is synchronous with earth.
  (b) The angular velocity of satellite must be in the same direction as that of the earth. It thus revolves around earth from west to east. Its relative angular velocity with respect to earth is zero.
  (c) The orbit of satellite should be circular.
  (d) The orbit should be in equatorial plane of earth. It contains centre of earth as well as equator.
  (e) It should be at 36000 km from the surface of earth. It is thus (36000 + 6400) km or 42400 km from the centre of earth.
  Radius of parking orbit = 42400 km.
  (f) A satellite revolving in stable orbit does not require any energy from an external source. The work done by the satellite in completing its orbit is zero.
  (g) Acceleration due to gravity is used up in providing centripetal acceleration to the satellite. Hence effective g inside the satellite is zero.
  (h) Its orbital velocity = 3.1 km/sec
  Its angular velocity = $\frac{2\pi}{24}$ radian/hour.

- **For a satellite orbiting near earth's surface**
  (a) Time period = 84 minute approximately
  (b) Orbital velocity = 8 km/sec
  (c) Angular speed $\omega = \frac{2\pi}{84}\frac{\text{radian}}{\text{min}} = 0.00125\ \frac{\text{radian}}{\text{sec}}$.

- **Inertial mass and gravitational mass**
  (a) Inertial mass = $\frac{\text{force}}{\text{acceleration}}$
  (b) Gravitational mass = $\frac{\text{weight of body}}{\text{acceleration due to gravity}}$
  (c) They are equal to each other in magnitude.
  (d) Gravitational mass of a body is affected by the presence of other bodies near it. Inertial mass of a body remains unaffected by the presence of other bodies near it.

Miscellaneous Xamples

1. For what value of angular velocity of the earth, a body becomes weightless at the equator?

Soln.: Considering the rotation of the earth about its axis, the apparent weight of a body is given by

$$W = mg - m\,\omega^2 R \cos^2 \lambda,$$

where m is the mass of the body, ω is the angular velocity of the earth, R is the radius of the earth, λ is the latitude of that place, g is acceleration due to gravity.

At the equator, $\lambda = 0°$, $\therefore$ $\cos\lambda = 1$

So, $W = mg - m\omega^2 R$

Now, the body will be weightless if $W = mg - m\omega^2 R = 0$

$$\text{or} \quad \omega = \sqrt{\frac{g}{R}}.$$

So, a body becomes weightless at the equator when the angular velocity of the earth becomes equal to $\sqrt{\frac{g}{R}}$.

---

2. Two artificial satellites are revolving round the earth at the same altitude. The mass of one is twice the other. Which of the satellite is moving faster?

Soln.: The orbital velocity v of a satellite revolving round the earth at a height h above the surface of the earth is given by

$$v = \sqrt{\frac{MG}{R + h}}$$

where M is the mass of the earth, R is the radius of the earth and G is the Gravitational constant.

It is clear from the above expression that orbital velocity is independent of the mass of the satellite and hence both the satellite will move with the same speed.

---

3. A body is taken from the centre of the earth to the moon. What will be the changes in the weight of the body?

Soln.: The weight of the body at the centre of the earth will be zero as acceleration due to gravity, g is zero there. The weight of the body will increase in moving from the centre of the earth due to increase in the value of g. At the surface of the earth the weight will be maximum. Again it will decrease (due to decrease in g) as the body is moved away from the surface of the earth. The weight will be again zero at some place where the gravitational forces of attraction on the body by the earth and the moon will be equal. Beyond this upto the moon the weight increases due to the gravitational force of the moon.

---

4. A satellite of mass m is moving in a circular orbit of radius r. Calculate its angular momentum with respect to the centre of the orbit in terms of the mass of the earth.

Soln.: The angular momentum of the satellite with respect to the centre of orbit is given by

$$\vec{L} = \vec{r} \times m\vec{v}$$

where $\vec{r}$ is the radius vector of the satellite with respect to the centre of the orbit and $\vec{v}$ is its velocity.

The angle between $\vec{r}$ and $\vec{v}$ is 90° in case of circular orbit.

$\therefore$ $L = mvr \sin 90° = mvr$ ...(i)

Now, the gravitational force of attraction between the earth and the satellite $= \frac{GMm}{r^2}$, where M is the mass of the earth.

This force provides the necessary centripetal force to the satellite for its circular motion.

$$\therefore \quad \frac{GMm}{r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad mv^2r = GMm$$

or $m^2v^2r^2 = GMm^2\, r$

or $mvr = (GMm^2r)^{1/2}$ ...(ii)

From (i) and (ii) we get $L = (GMm^2r)^{1/2}$

---

5. Why is the gravitational constant called universal? "The value of gravitational constant is 6.67×10^{-8} C.G.S. unit." What is meant by this statement?

Soln.: The gravitational constant is called universal because its value does not depend on the size or shape of the attracting bodies, nature of medium between them, state of the bodies (*i.e.* whether solid, liquid or gaseous) or any physical factor like temperature.

"The value of gravitational constant is 6.67×10^{-8} C.G.S. unit" means that when two bodies, each of mass 1 g are kept separated by a distance of 1 cm, each body attracts the other with a force of 6.67×10^{-8} dyne.

---

6. Will 1 kg sugar be more at the poles or at the equator?

Soln.: We know that the value of g is larger at the poles than at the equator due to the shape and rotation of the earth about its own axis. So, if sugar is weighed in a physical balance (*i.e.* mass) then there will be no difference. If it is weighed by a spring balance (*i.e.*, weight) calibrated at the equator, then 1 kg of sugar will have a lesser amount at the poles.

---

7. What will be the value of acceleration due to gravity if (i) the earth stops rotating, (ii) the speed of the earth increases?

Soln.: The variation in the value of acceleration due to the rotation of the earth is given by $g_\lambda = g\left(1 - \frac{R\omega^2 \cos^2\lambda}{g}\right)$, where g is the acceleration due to gravity in the absence of the rotation of the earth, ω is the angular velocity of the earth, R is the radius of the earth and λ is the latitude of the place. So

(i) if the earth stops rotating, $\omega = 0$ and hence the value of g_λ increases.

(ii) if the speed of rotation increases, *i.e.*, ω is more, g_λ will decrease.

But at the poles ($\lambda = 90°$), the rotation of the earth has no effect on the value of g_λ and is always equal to g.

---

8. The acceleration due to gravity on a satellite is 1.96 m/s^2, while on the earth it is 9.80 m/s^2. If the jumping from a height of 5 m is safe at the earth, then jumping from what height will be safe at that satellite?

Soln.: The safety of jump depends upon the amount of kinetic energy ($\frac{1}{2}mv^2$) with which a man hits the ground. The amount of kinetic energy depends on the potential energy (mgh) at the height from which the man is jumping. So in both cases, the potential energy at the respective safe height should be same.

That is, $mg_S\, h_S = mg_e\, h_e$ or $h_S = \frac{g_e h_e}{g_s} = \frac{9.8 \times 5}{1.96} = 25$ m

Therefore, safe height on the satellite is 25 m.

---

9. What are the conditions under which a rocket fired from the earth, launches an artificial satellite of earth?

Soln.: The rocket fired from the earth, launches an artificial satellite successfully under the following conditions:

(i) the rocket must carry the satellite to a suitable height above the surface of the earth.

(ii) From the desired height, the satellite must be projected with a suitable velocity, called the orbital velocity.

(iii) In the orbit, the air resistance should be negligible so that its velocity does not decrease and it does not burn out by the heat produced due to air resistance.

10. Calculate the maximum temperature of a planet of radius 600 km and mean density $\rho = 5 \times 10^3$ kg/m^3 to retain oxygen in its atmosphere.

Soln.: Escape velocity from the surface of the planet is given by

$$v_e = \sqrt{\frac{2MG}{r}} = \sqrt{\frac{2 \times \frac{4}{3}\pi r^3 \rho G}{r}} = \sqrt{\frac{8}{3} G\pi r^2 \rho}$$

where r is the radius of the planet and ρ is the mean density of the planet.

Now, to retain oxygen in the atmosphere of the planet the *r.m.s.* velocity (v_{rms}) of oxygen molecule in the atmosphere of the planet should not exceed the escape velocity of the planet.

We have, $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

where R is the universal gas constant, T is the temperature of the planet and M_0 is the molecular weight of oxygen.

Now, by problem, $(v_{rms})_{max} = v_e$ or $\sqrt{\frac{3RT_{max}}{M_0}} = \sqrt{\frac{8}{3} G\pi r^2 \rho}$

$$\therefore \quad T_{max} = \frac{8}{3} G \pi r^2 \rho \times \frac{M_0}{3R} \qquad \text{...(i)}$$

Here, $G = 6.67 \times 10^{-11}$ S.I. unit, $r = 600 \times 10^3$ m

$\rho = 5 \times 10^3$ kg/m^3, $M_0 = 32$

$R = 8.3 \times 10^3$ J/K-kg mole

From eqn (i), we get

$$T_{max} = \frac{8}{3} \times \frac{(6.67 \times 10^{-11})(3.14)(600 \times 10^3)^2 (5 \times 10^3)(32)}{3 \times (8.3 \times 10^3)} = 1291 \text{ K} = 1018°\text{C}$$

Therefore, the maximum temperature of the planet will be 1018°C.

11. (a) If the force of gravity acts on all bodies in proportion to their masses, why could not a heavy body fall faster than light body?

(b) What do you mean by escape velocity?

Soln. (a) Force due to gravity = mg.

$$\therefore \quad \text{acceleration} = \frac{\text{Force}}{\text{mass}} = \frac{mg}{m} = g$$

So acceleration of the body (if we neglect air resistance) is independent of mass.

(b) The escape velocity of a body is the minimum velocity with which it is to be projected so that it just overcomes the gravitational pull of the earth (or any other planet).

It is given by $v = \sqrt{2gR}$ where g is acceleration due to gravity and R the radius of earth.

EXERCISE

Multiple Choice Questions

1. The largest and the shortest distance of the earth from the sun are r_1 and r_2. Its distance from the sun when it is at perpendicular to the major-axis of the orbit drawn from the sun is

(a) $\frac{r_1 + r_2}{4}$ (b) $\frac{r_1 + r_2}{r_1 - r_2}$

(c) $\frac{2r_1r_2}{r_1 + r_2}$ (d) $\frac{r_1 + r_2}{3}$

2. Two spheres of masses m and M are situated in air and the gravitational force between them is F. The space around the masses is now filled with a liquid of specific gravity 3. The gravitational force will now be

(a) $3F$ (b) F

(c) $F/3$ (d) $F/9$

3. The acceleration due to gravity on the planet A is 9 times the acceleration due to gravity on planet B. A man jumps to a height of 2 m on the surface of A. What is the height of jump by the same person on the planet B ?

(a) (2/9) m (b) 18 m

(c) 6 m (d) (2/3) m

4. A body of mass m is placed on earth surface which is taken from earth surface to a height of $h = 3R$, then change in gravitational potential energy is

(a) $\frac{mgR}{4}$ (b) $\frac{2}{3}mgR$

(c) $\frac{3}{4}mgR$ (d) $\frac{mgR}{2}$.

5. With what velocity should a particle be projected so that its height becomes equal to radius of earth?

(a) $\left(\frac{GM}{R}\right)^{1/2}$ (b) $\left(\frac{8GM}{R}\right)^{1/2}$

(c) $\left(\frac{2GM}{R}\right)^{1/2}$ (d) $\left(\frac{4GM}{R}\right)^{1/2}$.

6. For a planet having mass equal to mass of the earth but radius is one fourth of radius of the earth. Then escape velocity for this planet will be

(a) 11.2 km/sec (b) 22.4 km/sec

(c) 5.6 km/sec (d) 44.8 km/sec.

7. Gravitational force is required for

(a) stirring of liquid

(b) convection

(c) conduction

(d) radiation.

8. A body of weight 72 N moves from the surface of earth at a height half of the radius of earth, then gravitational force exerted on it will be

(a) 36 N (b) 32 N

(c) 144 N (d) 50 N.

9. The escape velocity of a sphere of mass m is given by (G = Universal gravitational constant; M = Mass of the earth and R_e = Radius of the earth)

(a) $\sqrt{\frac{2GMm}{R_e}}$ (b) $\sqrt{\frac{2GM}{R_e}}$

(c) $\sqrt{\frac{GM}{R_e}}$ (d) $\sqrt{\frac{2GM + R_e}{R_e}}$

10. The escape velocity of a body on the surface of the earth is 11.2 km/s. If the earth's mass increases to twice its present value and radius of the earth becomes half, the escape velocity becomes

(a) 22.4 km/s (b) 44.8 km/s

(c) 5.6 km/s (d) 11.2 km/s.

11. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?

(a) 4 (b) 5

(c) 2 (d) 3.

12. What will be the formula of mass of the earth in terms of g, R and G ?

(a) $G\frac{R}{g}$ (b) $g\frac{R^2}{G}$

(c) $g^2\frac{R}{G}$ (d) $G\frac{g}{R}$.

13. A ball is dropped from a spacecraft revolving around the earth at a height of 120 km. What will happen to the ball?

(a) it will fall down to the earth gradually

(b) it will go very far in the space

(c) it will continue to move with the same speed along the original orbit of spacecraft

(d) it will move with the same speed, tangentially to the spacecraft.

14. The acceleration due to gravity g and mean density of the earth ρ are related by which of the following relations? (where G is the gravitational constant and R is the radius of the earth.)

(a) $\rho = \frac{3g}{4\pi GR}$ (b) $\rho = \frac{3g}{4\pi GR^3}$

(c) $\rho = \frac{4\pi gR^2}{3G}$ (d) $\rho = \frac{4\pi gR^3}{3G}$.

15. Two particles of equal mass go around a circle of radius R under the action of their mutual gravitational attraction. The speed v of each particle is

(a) $\frac{1}{2}\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{4Gm}{R}}$

(c) $\frac{1}{2R}\sqrt{\frac{1}{Gm}}$ (d) $\sqrt{\frac{Gm}{R}}$.

16. The earth (mass = 6×10^{24} kg) revolves around the sun with an angular velocity of 2×10^{-7} rad/s in a circular orbit of radius 1.5×10^8 km. The force exerted by the sun on the earth, in newton, is

(a) 36×10^{21} (b) 27×10^{39}

(c) zero (d) 18×10^{25}.

17. The radius of earth is about 6400 km and that of mars is 3200 km. The mass of the earth is about 10 times the mass of mars. An object weighs 200 N on the surface of earth. Its weight on the surface of mars will be

(a) 20 N (b) 8 N

(c) 80 N (d) 40 N.

18. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. The ratio of time periods of the planets is

(a) $\sqrt{10}$ (b) $10\sqrt{10}$

(c) 10 (d) $1/\sqrt{10}$

19. If the gravitational force between two objects were proportional to $1/R$ (and not as $1/R^2$), where R is the distance between them, then a particle in a circular path (under such a force) would have its orbital speed v, proportional to

(a) R (b) R^0 (independent of R)

(c) $1/R^2$ (d) $1/R$.

20. A satellite in force free space sweeps stationary interplanetary dust at a rate of $dM/dt = \alpha v$, where M is mass and v is the speed of satellite and α is a constant. The acceleration of satellite is

(a) $\frac{-\alpha v^2}{2M}$ (b) $-\alpha v^2$

(c) $\frac{-2\alpha v^2}{M}$ (d) $\frac{-\alpha v^2}{M}$

21. A planet is moving in an elliptical orbit around the sun. If T, V, E and L stand respectively for its kinetic energy, gravitational potential energy, total energy and magnitude of angular momentum about the centre of force, which of the following is correct ?

(a) T is conserved

(b) V is always positive

(c) E is always negative

(d) L is conserved but direction of vector L changes continuously.

22. The radius of orbit of a planet is two times that of the earth. The time period of planet is

(a) 4.2 year (b) 2.8 year

(c) 5.6 year (d) 8.4 year

23. The mean radius of earth is R, its angular speed on its own axis is ω and the acceleration due to gravity at earth's surface is g. What will be the radius of the orbit of a geostationary satellite ?

(a) $(R^2g/\omega^2)^{1/3}$ (b) $(Rg/\omega^2)^{1/3}$

(c) $(R^2\omega^2/g)^{1/3}$ (d) $(R^2g/\omega)^{1/3}$

24. The satellite of mass m is orbiting around the earth in a circular orbit with a velocity v. What will be its total energy ?

(a) $(3/4)mv^2$ (b) $(1/2)mv^2$

(c) mv^2 (d) $-(1/2)mv^2$

25. Orbit of a planet around a star is

(a) An ellipse (b) A circle

(c) A parabola (d) A hyperbola

26. The time period of a simple pendulum on a freely revolving artificial satellite is

(a) infinite (b) 24 hour

(c) 27 day (d) zero

27. v_e and v_p denote the escape velocities from the earth and another planet having twice the radius and the same mean density as that of the earth. Then

(a) $v_e = v_p/2$ (b) $v_e = v_p$

(c) $v_e = 2v_p$ (d) $v_e = \frac{v_p}{4}$

28. A planet moves around the sun. At a given point P, it is closest from the sun at a distance d_1 and has a speed v_1. At another point Q, when it is farthest from the sun at a distance d_2, its speed will be

(a) $\frac{d_1^2}{d_2^2} \cdot v_1$ (b) $\frac{d_2}{d_1} \cdot v_1$

(c) $\frac{d_2^2}{d_1^2} \cdot v_1$ (d) $\frac{d_1 v_1}{d_2}$

29. The gravitational field due to a mass distribution is $E = \frac{K}{x^3}$ in the x-direction. (K is a constant) Taking the gravitational potential to be zero at infinity, its value at a distance x is

(a) $\frac{K}{x}$ (b) $\frac{K}{x^2}$

(c) $\frac{K}{2x^2}$ (d) $\frac{K}{3x^2}$

30. A body is projected with velocity kv_e in vertically upward direction from the ground into space. It is given that v_e is escape velocity and $k < 1$. If air resistance is considered to be negligible, then the maximum height from the centre of earth to which it can go will be (R = Radius of earth)

(a) $\frac{R}{1-k}$ (b) $R(1-k)$

(c) $\frac{R}{1-k^2}$ (d) $\frac{R}{1+k^2}$

31. A small satellite is revolving near earth's surface. Its orbital velocity will be nearly

(a) 11.2 km/s (b) 8 km/s
(c) 6 km/s (d) 4 km/s

32. The change in potential energy when a body of mass m is raised to a height nR from the earth surface is (R = Radius of earth)

(a) $\frac{n}{n-1}\cdot mgR$ (b) $mgR\cdot\frac{n}{n+1}$

(c) $\frac{n^2}{n^2+1}\cdot mgR$ (d) $nmgR$

33. Two identical satellites A and B are circulating round the earth at the height of R and $2R$ respectively (where R is the radius of earth). The ratio of kinetic energy of A to that of B is

(a) 2/3 (b) 3/2
(c) 3/5 (d) 5/3

34. The distance of a geo-stationary satellite from the centre of the earth is nearest to (where R = 6400 km)

(a) $5R$ (b) $7R$
(c) $10R$ (d) $18R$

35. At what distance from the centre of the earth, the value of acceleration due to gravity g will be half that on the surface (R = radius of earth)?

(a) $2R$ (b) R
(c) $1.414R$ (d) $0.414R$

36. The acceleration due to gravity near the surface of a planet of radius R and density d is proportional to

(a) d/R^2 (b) dR^2
(c) $d.R$ (d) d/R

37. The ratio of radii of planets A and B is K_1 and ratio of accelerations due to gravity on them is K_2. The ratio of escape velocities from them will be

(a) K_1K_2 (b) $\sqrt{K_1K_2}$

(c) $\sqrt{\frac{K_1}{K_2}}$ (d) $\sqrt{\frac{K_2}{K_1}}$

38. Two planets at mean distances d_1 and d_2 from the sun have their frequencies n_1 and n_2 respectively. Then

(a) $n_1^2d_1^2 = n_2^2d_2^2$ (b) $n_2^2d_2^3 = n_1^2d_1^3$
(c) $n_1d_1^2 = n_2d_2^2$ (d) $n_1^2d_1 = n_2^2d_2$

39. If the density of earth is doubled keeping its radius constant then acceleration due to gravity will be (g = 9.8 m/s^2)

(a) 19.6 m/s^2 (b) 9.8 m/s^2
(c) 4.9 m/s^2 (d) 2.45 m/s^2

40. The kinetic energy needed to project a body of mass m from the surface of earth (radius R) to infinity is

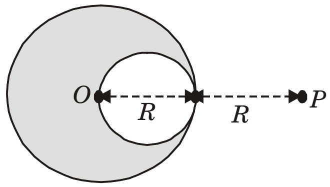
(a) $\frac{mgR}{2}$ (b) $2mgR$

(c) mgR (d) $\frac{mgR}{4}$

41. A spherical planet far out in space has a mass M_0 and diameter D_0. A particle of mass m falling freely near the surface of this planet will experience an acceleration due to gravity which is equal to

(a) GM_0/D^2 (b) $4mGM_0/D_0^2$
(c) $4GM_0/D_0^2$ (d) GmM_0/D_0^2

42. A solid sphere of uniform density and radius R applies gravitational force of attraction equal to F_1 on a particle placed at P, distance $2R$ from the centre O of the sphere. A spherical cavity of radius $R/2$ is now made in the sphere as shown in given figure. The sphere with cavity now applies a gravitational force F_2 on same particle placed at P. The ratio F_2/F_1 will be



(a) 1/2 (b) 7/9
(c) 3 (d) 7

43. Sun is about 330 times heavier and 100 times bigger in radius than earth. The ratio of mean density of the sun to that of earth is

(a) 3.3×10^{-6} (b) 3.3×10^{-4}
(c) 3.3×10^{-2} (d) 1.3

44. If the earth of radius R, while rotating with angular velocity ω become standstill, what will be the effect on the weight of a body of mass m at a latitude of 45°?

(a) remains unchanged
(b) decreases by $R\omega^2$
(c) increases by $R\omega^2$
(d) increases by $R\omega^2/2$

45. If V_e denotes escape velocity and V_0 denotes orbital velocity of a satellite revolving around a planet of radius R, then

(a) $V_e = \sqrt{2}\cdot V_0$ (b) $V_e = 2V_0$
(c) $2V_e = V_0$ (d) $\sqrt{2}\cdot V_e = V_0$

46. If R denotes the radius of orbit of a satellite of mass m revolving around a planet of mass M, the orbital velocity of the satellite is given by

(a) $v_0^2 = \frac{GM}{R}$ (b) $v_0^2 = \frac{GR}{M}$
(c) $v_0^2 = \frac{GmM}{R}$ (d) $v_0^2 = \frac{GM}{mR}$

47. If the change in the value of 'g' at a height h above the surface of the earth is the same as at a depth x below it, then (x and h being much smaller than the radius of earth)

(a) $x = 2h$ (b) $x = h$
(c) $x = h/2$ (d) $x = h^2$

48. Time period of revolution of a satellite around a planet of radius R is T. Period of revolution around another planet, whose radius is $3R$ but having same density is

(a) T (b) $3T$
(c) $9T$ (d) $3\sqrt{3}\cdot T$

49. Two planets have the same average density and their radii are R_1 and R_2. If acceleration due to gravity on these planets be g_1 and g_2 respectively, then

(a) $g_1/g_2 = R_1/R_2$ (b) $g_1/g_2 = R_2/R_1$
(c) $\frac{g_1}{g_2} = \frac{R_1^2}{R_2^2}$ (d) $\frac{g_1}{g_2} = \frac{R_1^3}{R_2^3}$

50. In some region, the gravitational field is zero. The gravitational potential in this region

(a) must be constant
(b) cannot be zero
(c) must be zero (d) must be variable

51. Ball pen functions on the principle of

(a) viscosity (b) surface tension
(c) capillary action (d) gravitational force

52. Two planets revolve around the sun. The periodic times and the mean radii of the orbits are T_1, T_2, and r_1 and r_2 respectively. The ratio $\frac{T_1}{T_2}$ is equal to

(a) $\left(\frac{r_1}{r_2}\right)^{1/2}$ (b) $\frac{r_1}{r_2}$
(c) $\left(\frac{r_1}{r_2}\right)^2$ (d) $\left(\frac{r_1}{r_2}\right)^{3/2}$

53. The mass and diameter of a planet have twice the value of the corresponding parameters of earth. Acceleration due to gravity on the surface of the planet is

(a) 9.8 m/sec^2 (b) 19.6 m/sec^2
(c) 980 m/sec^2 (d) 4.9 m/sec^2

54. Imagine a light plane revolving around a very massive star in a circular orbit of radius R with a period of revolution T. If the gravitational force of attraction between planet and star is proportional to $R^{-5/2}$ then T^2 is proportional to

(a) R^3 (b) $R^{3/2}$
(c) $R^{5/2}$ (d) $R^{7/2}$

55. If the earth stops rotating, the value of g at the equator will

(a) remain same (b) remain half
(c) decrease to quarter
(d) increase

56. The weight of a body at the centre of earth is

(a) same as on surface of earth
(b) half of that on surface
(c) infinite (d) zero

57. There is no atmosphere on moon because

(a) it is closer to earth
(b) it revolves round the earth
(c) it gets light from the sun
(d) the escape velocity of gas molecule is smaller than their root mean square velocity there.

58. The ratio of the K.E. required to be given to the satellite to escape earth's gravitational field to the K.E. required for maintaining the satellite in circular orbit just above earth's surface is

(a) 2 : 3 (b) 3 : 2
(c) 3 : 5 (d) 2 : 1

59. If earth is at one-fourth of its present distance from the sun, the duration of the year will be

(a) one-half the present year.
(b) one-fourth the present year.
(c) one-sixth the present year.
(d) one-eighth the present year.

60. A man inside an artificial satellite feels weightlessness because the force of attraction due to earth is

(a) zero at that place
(b) equal to centripetal force
(c) is balanced by the force of attraction due to moon
(d) non-effective due to particular design of satellite

61. The orbital velocity of an artificial satellite in a circular orbit just above the earth's surface is v_0. For a satellite orbiting at an altitude of half of the earth's radius, the orbital velocity is

(a) $\frac{3}{2}v_0$ (b) $\sqrt{\frac{2}{3}}\cdot v_0$

(c) $\sqrt{\frac{3}{2}}\cdot v_0$ (d) $\frac{2}{3}v_0$

62. The time period of a geostationary satellite is
(a) 12 hour (b) 18 hour
(c) 24 hour (d) One year

63. If the earth suddenly shrinks to half of its present radius, the acceleration due to gravity will be
(a) g (b) $2g$
(c) $4g$ (d) $8g$

64. An earth satellite S has an orbit radius which is 4 times that of a communication satellite C. The period of revolution of S is
(a) 2 days (b) 4 days
(c) 8 days (d) 16 days

65. The period of revolution of planet A around the sun is 8 times that of B. The distance of A from the sun is how many times greater than that of B from the sun?
(a) 4 (b) 5
(c) 6 (d) 8

66. Which physical quantity is constant for a satellite in orbit?
(a) Angular momentum
(b) Angular acceleration
(c) Angular velocity
(d) Kinetic energy

67. For a satellite escape velocity is 11 km/s. If the satellite is launched at an angle of 60° with the vertical, then escape velocity will be
(a) 11 km/s (b) $11\sqrt{3}$ km/s
(c) $\frac{11}{\sqrt{3}}$ km/s (d) $\frac{11}{2}$ km/s

68. If the angular speed of the earth is doubled, the value of acceleration due to gravity (g) at the north pole
(a) remains same (b) doubles
(c) reduced to half (d) becomes zero

69. An artificial satellite revolving in a circular orbit around the earth has a total energy E_0, being the sum of P.E. and K.E. Its potential energy is
(a) $2E_0$ (b) $E_0\sqrt{2}$
(c) $\frac{E_0}{2}$ (d) $2\sqrt{2}\ E_0$

70. A mass M is split into two parts, m and $(M-m)$. which are then separated by a certain distance. The gravitational force between these two parts will be maximum if the ratio $\frac{m}{M}$ is equal to
(a) 1/4 (b) 1/3
(c) 1/2 (d) 1/1

71. The intensity of gravitational field at the centre of a spherical shell of radius R and mass M will be
(a) GM/R^2 (b) GM/R
(c) zero (d) infinity

72. The earth revolves round the sun in one year. If the distance between them becomes double, the new period of revolution will be
(a) 8 year (b) 4 year
(c) $2\sqrt{2}$ year (d) 2 year

73. If the radius of earth shrinks by 1.5% (mass remaining same), then the value of acceleration due to gravity changes by
(a) 1% (b) 2%
(c) 3% (d) 4%

74. Two identical spheres, each of radius R, are placed in contact with each other. The force of gravitation between the two spheres will be proportional to
(a) R^3 (b) R^4
(c) R^5 (d) R^6

75. A body of mass m is taken from earth's surface to the height h equal to radius of earth. The increase in potential energy will be
(a) $mgR/4$ (b) $mgR/2$
(c) mgR (d) $2mgR$

76. A missile is launched with a velocity less than the escape velocity. The sum of its kinetic and potential energy is
(a) positive (b) negative
(c) zero (d) infinity

77. The escape velocity of a particle of mass m varies as
(a) m (b) m^0
(c) m^{-1} (d) m^{-2}

78. In a satellite if the time of revolution is T, then kinetic energy is proportional to
(a) T^{-1} (b) $T^{-2/3}$
(c) T^{-2} (d) $T^{-1/3}$

79. The distance of the centres of moon and earth is D. The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero?
(a) $D/2$ (b) $9D/10$
(c) $4D/3$ (d) $2D/5$

80. The height of the point vertically above earth's surface at which acceleration due to gravity becomes 1% of its value at the surface is (Radius of earth = R)

(a) $8R$ (b) $9R$
(c) $10R$ (d) $20R$

81. If a hole is bored along the diameter of the earth and a stone is dropped into hole.
(a) The stone reaches the centre of earth and stops there.
(b) The stone reaches the other side of earth and stops there.
(c) The stone reaches the other side of earth and escapes into space.
(d) The stone executes simple harmonic motion about the centre of the earth.

82. A satellite with kinetic energy E_k is revolving round the earth in a circular orbit. How much more kinetic energy should be given to it so that it may just escape into outer space?
(a) E_k (b) $2E_k$
(c) $\frac{E_k}{2}$ (d) $3E_k$

83. The angular speed of earth, so that the object on equator may appear weightless is (g = 10 m/s^2, radius of earth = 64000 km)
(a) 1.25×10^{-3} rad/s (b) 1.56×10^{-3} rad/s
(c) 1.25×10^{-1} rad/s (d) 1.56 rad/s

84. Periodic time of a satellite revolving above Earth's surface at a height equal to R, radius of Earth is (g = acceleration due to gravity at Earth's surface)
(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $4\sqrt{2}\,\pi\sqrt{\frac{R}{g}}$
(c) $2\pi\sqrt{\frac{R}{g}}$ (d) $8\pi\sqrt{\frac{R}{g}}$

85. If radius of earth's orbit is made 1/4, the duration of a year will become
(a) 4 times (b) 8 times
(c) $\frac{1}{4}$ times (d) $\frac{1}{8}$ times

86. A body of weight W newton is at the surface of the earth. Its weight at a height equal to half the radius of the earth will be
(a) $\frac{W}{2}$ (b) $\frac{2W}{3}$
(c) $\frac{4W}{9}$ (d) $\frac{8W}{27}$

87. A body revolved around the sun 27 times faster than the earth. What is the ratio of their radii?
(a) 1/3 (b) 9/1
(c) 1/27 (d) 1/4

88. The escape velocity of a body on an imaginary planet which is thrice the radius of the earth are double the mass of the earth is (v_e is the escape velocity from earth)
(a) $\sqrt{\frac{2}{3}}\cdot v_e$ (b) $\sqrt{\frac{3}{2}}\cdot v_e$
(c) $\frac{\sqrt{2}}{3}\cdot v_e$ (d) $\frac{2}{\sqrt{3}}\cdot v_e$

89. The acceleration due to gravity at a place is π^2m/s^2. Then the time period of a simple pendulum of length one metre is
(a) $\frac{2}{\pi}$ s (b) 2π s
(c) 2 s (d) π s

90. Radius of orbit of satellite of earth is R. Its kinetic energy is proportional to
(a) $\frac{1}{R}$ (b) $\frac{1}{\sqrt{R}}$
(c) R (d) $\frac{1}{R^{3/2}}$

91. A satellite is launched into a circular orbit of radius R around earth while a second satellite is launched into an orbit of radius 1.02 R. The percentage difference in the time periods of the two satellites is
(a) 0.7 (b) 1.0
(c) 1.5 (d) 3.0

92. Weight of a body of mass m decreases by 1% when it is raised to height h above the earth's surface. If the body is taken to a depth h in a mine, change in its weight is
(a) 2% decrease (b) 0.5% decrease
(c) 1% increase (d) 0.5% increase

93. A particle falls towards earth from infinity. Its velocity on reaching the earth would be
(a) infinity (b) $\sqrt{2gR}$
(c) $2\sqrt{Rg}$ (d) zero

94. The depth at which the effective value of acceleration due to gravity is $\frac{g}{4}$ is (R = Radius of earth)
(a) R (b) $\frac{3R}{4}$
(c) $\frac{R}{2}$ (d) $\frac{R}{4}$

95. The escape velocity from the earth is v_e. The escape velocity from a planet whose radius is 4 times and density is nine times that of the earth, is
(a) $36\,v_e$ (b) $12\,v_e$
(c) $6\,v_e$ (d) $20\,v_e$

Assertion and Reason | AIIMS

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Of the statements, mark the correct answer -

(a) *If both assertion and reason are true and reason is the correct explanation of assertion*
(b) *If both assertion and reason are true but reason is not the correct explanation of assertion*
(c) *If assertion is true but reason is false*
(d) *If both assertion and reason are false.*

1. **Assertion :** Smaller the orbit of the planet around the sun, shorter is the time it takes to complete one revolution.
   Reason : According to Kepler's third law of planetary motion, square of time period is proportional to cube of mean distance from sun.
2. **Assertion :** The value of acceleration due to gravity does not depend upon mass of the body.
   Reason : Acceleration due to gravity is a constant quantity.
3. **Assertion :** The difference in the value of acceleration due to gravity at pole and equator is proportional to square of angular velocity of earth.
   Reason : The value of acceleration due to gravity is minimum at the equator and maximum at the pole.
4. **Assertion :** There is no effect of rotation of earth on acceleration due to gravity at poles.
   Reason : Rotation of earth is about polar axis.
5. **Assertion :** Gravitational potential of earth at every place on it is negative.
   Reason : Every body on earth is bound by the attraction of earth.
6. **Assertion :** A planet moves faster, when it is closer to the sun in its orbit and vice versa.
   Reason : Orbit velocity in orbit of planet is constant.
7. **Assertion :** The time period of geostationary satellite is 24 hours.
   Reason : Geostationary satellite must have the same time period as the time taken by the earth to complete one revolution about its axis.
8. **Assertion :** When distance between two bodies is doubled and also mass of each body is doubled, gravitational force between them remains the same.
   Reason : According to Newton's law of gravitation, force is directly proportional to masses of bodies and inversely proportional to square of distance between them.
9. **Assertion :** A body becomes weightless at the centre of earth.
   Reason : As the distance from centre of earth decreases, acceleration due to gravity increases.
10. **Assertion :** Space rockets are usually launched in the equatorial line from west to east.
    Reason : The acceleration due to gravity is minimum at the equator.
11. **Assertion :** The binding energy of a satellite depends upon the mass of the satellite.
    Reason : Binding energy is the negative value of total energy of satellite.
12. **Assertion :** Every body in this universe attracts every other body with a force which is inversely proportional to the square of distance between them.
    Reason : We can not move even a finger without disturbing all the stars.
13. **Assertion :** The speed of satellite always remains constant in an orbit.
    Reason : The speed of a satellite doesn't depend in its path.
14. **Assertion :** Gravitational field intensity is zero both at centre and infinity.
    Reason : The dimensions of gravitational field intensity is $[LT^{-2}]$.
15. **Assertion :** The square of the period of revolution of a planet is proportional to the cube of its distance from the sun.
    Reason : Sun's gravitational field is inversely proportional to the square of its distance from the planet.

MCQs NEET / AIPMT AIIMS

Multiple Choice Questions

1. Infinite number of bodies, each of mass 2 kg are situated on x-axis at distances 1 m, 2 m, 4 m, 8 m, ..., respectively, from the origin. The resulting gravitational potential due to this system at the origin will be

(a) $-\frac{4}{3}G$ (b) $-4G$

(c) $-G$ (d) $-\frac{8}{3}G$ **(NEET 2013)**

2. A body of mass 'm' is taken from the earth's surface to the height equal to twice the radius (R) of the earth. The change in potential energy of body will be

(a) $3mgR$ (b) $\frac{1}{3}mgR$

(c) $mg2R$ (d) $\frac{2}{3}mgR$ **(NEET 2013)**

3. The radius of a planet is twice the radius of earth. Both have almost equal average mass-densities. V_P and V_E are escape velocities of the planet and the earth, respectively, then

(a) $V_P = 1.5\ V_E$ (b) $V_P = 2\ V_E$

(c) $V_E = 3\ V_P$ (d) $V_E = 1.5\ V_P$

(NEET Karnataka 2013)

4. A particle of mass 'm' is kept at rest at a height $3R$ from the surface of earth, where 'R' is radius of earth and 'M' is mass of earth. The minimum speed with which it should be projected, so that it does not return back, is

(g is acceleration due to gravity on the surface of earth)

(a) $\left(\frac{GM}{2R}\right)^{1/2}$ (b) $\left(\frac{gR}{4}\right)^{1/2}$

(c) $\left(\frac{2g}{R}\right)^{1/2}$ (d) $\left(\frac{GM}{R}\right)^{1/2}$

(NEET Karnataka 2013)

5. Gravitational potential of the body of mass m at a height h from surface of earth of radius R is (Take g = acceleration due to gravity at earth's surface)

(a) $-g(R + h)$ (b) $-g(R - h)$

(c) $g(R + h)$ (d) $g(R - h)$

(AIIMS 2013)

6. What will be the effect on the weight of a body placed on the surface of earth, if earth suddenly stops rotating?

(a) No effect

(b) Weight will increase

(c) Weight will decrease

(d) Weight will become zero

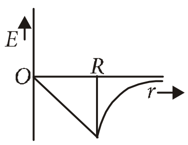
(AIIMS 2013)

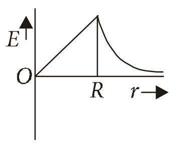
7. A black hole is an object whose gravitational field is so strong that even light cannot escape from it. To what approximate radius would earth (mass = 5.98×10^{24} kg) have to be compressed to be a black hole?

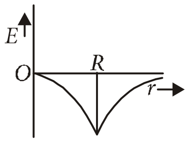
(a) 10^{-9} m (b) 10^{-6} m

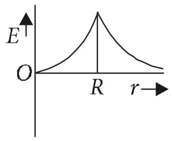
(c) 10^{-2} m (d) 100 m **(AIPMT 2014)**

8. Dependence of intensity of gravitational field (E) of earth with distance (r) from centre of earth is correctly represented by

(a) 

(b) 

(c) 

(d) 

(AIPMT 2014)

9. A satellite is in an orbit around the earth. If its kinetic energy is doubled, then

(a) it will maintain its path

(b) it will fall on the earth

(c) it will rotate with a great speed

(d) it will escape out of earth's gravitational field.

(AIIMS 2014)

10. Kepler's third law states that square of period of revolution (T) of a planet around the sun, is proportional to third power of average distance r between sun and planet *i.e.* $T^2 = Kr^3$ here K is constant.
If the masses of sun and planet are M and m respectively then as per Newton's law of gravitation force of attraction between them is $F = \frac{GMm}{r^2}$, here G is gravitational constant.
The relation between G and K is described as

(a) $K = G$ (b) $K = \frac{1}{G}$
(c) $GK = 4\pi^2$ (d) $GMK = 4\pi^2$

(AIPMT 2015, Cancelled)

11. Two spherical bodies of mass M and $5M$ and radii R and $2R$ are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body before collision is

(a) $7.5R$
(b) $1.5R$
(c) $2.5R$
(d) $4.5R$

(AIPMT 2015, Cancelled)

12. A remote-sensing satellite of earth revolves in a circular orbit at a height of 0.25×10^6 m above the surface of earth. If earth's radius is 6.38×10^6 m and $g = 9.8$ ms^{-2}, then the orbital speed of the satellite is

(a) 9.13 km s^{-1} (b) 6.67 km s^{-1}
(c) 7.76 km s^{-1} (d) 8.56 km s^{-1}

(AIPMT 2015)

13. A satellite S is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. Then,

(a) the linear momentum of S remains constant in magnitude.
(b) the acceleration of S is always directed towards the centre of the earth.
(c) the angular momentum of S about the centre of the earth changes in direction, but its magnitude remains constant.
(d) the total mechanical energy of S varies periodically with time.

(AIPMT 2015)

14. At what height from the surface of earth the gravitation potential and the value of g are -5.4×10^7 J kg^{-2} and 6.0 m s^{-2} respectively? Take the radius of earth as 6400 km.

(a) 1400 km (b) 2000 km
(c) 2600 km (d) 1600 km

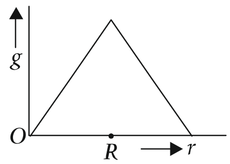
(NEET-I 2016)

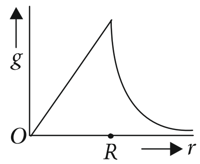
15. The ratio of escape velocity at earth (v_e) to the escape velocity at a planet (v_p) whose radius and mean density are twice as that of earth is

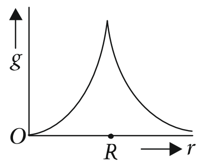
(a) 1 : 4 (b) $1:\sqrt{2}$
(c) 1 : 2 (d) $1:2\sqrt{2}$

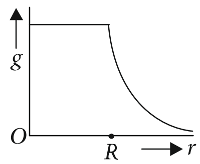
(NEET-I 2016)

16. Starting from the centre of the earth having radius R, the variation of g(acceleration due to gravity) is shown by

(a)


(b)


(c)


(d)


(NEET-II 2016)

17. A satellite of mass m is orbiting the earth (of radius R) at a height h from its surface. The total energy of the satellite in terms of g_0, the value of acceleration due to gravity at the earth's surface, is

(a) $\frac{mg_0R^2}{2(R+h)}$ (b) $-\frac{mg_0R^2}{2(R+h)}$
(c) $\frac{2mg_0R^2}{R+h}$ (d) $-\frac{2mg_0R^2}{R+h}$

(NEET-II 2016)

18. The additional kinetic energy to be provided to a satellite of mass m revolving around a planet of mass M to transfer it from a circular orbit of radius R_1 to another of radius R_2 ($R_2 > R_1$) is

(a) $GmM\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$ (b) $GmM\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

(c) $2GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (d) $\frac{1}{2}GmM\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

(AIIMS 2016)

19. The acceleration due to gravity at a height 1 km above the earth is the same as at a depth d below the surface of earth. Then

(a) $d = 1$ km (b) $d = \frac{3}{2}$ km

(c) $d = 2$ km (d) $d = \frac{1}{2}$ km

(NEET 2017)

20. Two astronauts are floating in gravitational free space after having lost contact with their spaceship. The two will

(a) move towards each other.

(b) move away from each other.

(c) will become stationary.

(d) keep floating at the same distance between them.

(NEET 2017)

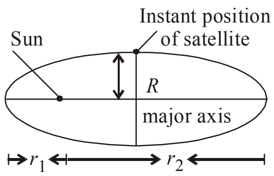
Gravitation

CHAPTER 6

Multiple Choice Questions

1. **(c) :** Applying the properties of ellipse, we have

$$\frac{2}{R} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2}$$



$$R = \frac{2r_1 r_2}{r_1 + r_2}$$

2. **(b) :** The gravitational force does not depend upon the medium in which objects are placed.

3. **(b) :** The velocity of the mass while reaching the surface of both the planets will be same.

i.e., $\sqrt{2g'h'} = \sqrt{2gh}$

$\sqrt{2 \times g \times h'} = \sqrt{2 \times 9g \times 2}$ $\quad 2h' = 36 \Rightarrow h' = 18$ m.

4. **(c) :** Gravitational potential energy on earth's surface = $-\frac{GMm}{R}$, where M and R are the mass and radius of the earth respectively, m is the mass of the body and G is the universal gravitational constant.

Gravitational potential energy at a height $h = 3R$

$$= -\frac{GMm}{R+h} = -\frac{GMm}{R+3R} = -\frac{GMm}{4R}$$

∴ Change in potential energy

$$= -\frac{GMm}{4R} - \left(-\frac{GMm}{R}\right) = -\frac{GMm}{4R} + \frac{GMm}{R} = \frac{3}{4}\frac{GMm}{R}.$$

Again, we have, $\frac{GMm}{R^2} = mg$

(where g is acceleration due to gravity on earth's surface).

∴ $\frac{GMm}{R} = mgR$.

∴ Change in potential energy = $\frac{3}{4}mgR$.

5. **(a) :** Use, $v^2 = \dfrac{2gh}{1+\dfrac{h}{R}}$ given $h = R$.

∴ $v = \sqrt{gR} = \sqrt{\frac{GM}{R}}$.

6. **(b) :** $v_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}}$

If R is 1/4th then $v_e = 2\, v_{e\text{–earth}}$
$= 2 \times 11.2 = 22.4$ km/sec.

7. **(b)**

8. **(b) :** $F_{\text{surface}} = G\frac{Mm}{R_e^2}$

$$F_{R_e/2} = G\frac{Mm}{(R_e + R_e/2)^2} = \frac{4}{9} \times F_{\text{surface}} = \frac{4}{9} \times 72 = 32 \text{ N}.$$

9. **(b) :** The gravitational potential energy of a body of mass m placed on earth's surface is given by

$U = -\frac{GM_e m}{R_e}$

Therefore in order to take a body from the earth's surface to infinity, the work required is $\frac{GM_e m}{R_e}$.

Hence it is evident that if we throw a body of mass m with such a velocity that its kinetic energy is $\frac{GM_e m}{R_e}$, then it will move outside the gravitational field of earth. Hence,

$\frac{1}{2}mv_e^2 = \frac{GM_e m}{R_e}$ or $v_e = \sqrt{\frac{2GM_e}{R_e}}$.

10. **(a) :** Escape velocity of a body (v_e) = 11.2 km/s; New mass of the earth $M_e = 2\,M_e$ and new radius of the earth (R_e) = 0.5 R_e.

Escape velocity (v_e) = $\sqrt{\frac{2GM_e}{R_e}} \propto \sqrt{\frac{M_e}{R_e}}$.

Therefore $\frac{v_e}{v'_e} = \sqrt{\frac{M_e}{R_e} \times \frac{0.5R_e}{2M_e}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

or, $v'_e = 2v_e = 22.4$ km/sec.

11. **(a) :** Period of revolution of planet A (T_A) = $8T_B$. According to Kepler's III law of planetary motion $T^2 \propto R^3$.

Therefore $\left(\frac{r_A}{r_B}\right)^3 = \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{8T_B}{T_B}\right)^2 = 64$

or $\frac{r_A}{r_B} = 4$ or $r_A = 4r_B$.

12. **(b) :** The gravitational force (F) = $\frac{GMm}{R^2}$ and $F = mg$. Equating both the values of gravitational force, $\frac{GMm}{R^2} = mg$ or $M = g\,\frac{R^2}{G}$, where M is the mass of the earth.

13. **(c) :** Since no external torque is applied therefore, according to law of conservation of angular momentum, the ball will continue to move with the same angular velocity along the original orbit of the spacecraft.

14. **(a) :** Acceleration due to gravity (g) = $G\times\frac{M}{R^2}$

$= G\frac{(4/3)\pi R^3\times\rho}{R^2} = G\times\frac{4}{3}\pi R\times\rho$ or $\rho = \frac{3g}{4\pi GR}$.

15. **(d) :** The two masses, separated by a distance $2R$ are going round their common centre of mass, the centre of the circle.

Attractive force = $-G\frac{mm}{4R^2}$. But the two masses are going round the centre of mass or the reduced mass $\mu=\frac{mm}{m+m}$ is going round a circle of radius = distance of separation

$\therefore$ Centrifugal force = $\frac{m}{2}\omega^2.2R = \frac{m}{2}v^2.\frac{1}{2R}$

$\frac{m}{2}\times\frac{v^2}{2R}=\frac{Gm^2}{4R^2} \Rightarrow v=\sqrt{\frac{Gm}{R}}$

16. **(a) :** Mass (m) = 6×10^{24} kg;
Angular velocity (ω) = 2×10^{-7} rad/s and radius (r) = 1.5×10^8 km = 1.5×10^{11} m.
Force exerted on the earth = $mR\omega^2$
$= (6\times10^{24})\times(1.5\times10^{11})\times(2\times10^{-7})^2 = 36\times10^{21}$ N.

17. **(c) :** Radius of earth (R_e) = 6400 km; Radius of mars (R_m) = 3200 km; Mass of earth (M_e) = 10 (M_m) and weight of an object on earth (W_e) = 200 N.

$\frac{W_m}{W_e}=\frac{mg_m}{mg_e}=\frac{M_m}{M_e}\times\left(\frac{R_e}{R_m}\right)^2 = \frac{1}{10}\times(2)^2 = \frac{4}{10}=\frac{2}{5}$

or $W_m = W_e\times\frac{2}{5} = 200\times0.4 = 80$ N.

18. **(b) :** Distance of two planets from sun (r_1) = 10^{13} m and (r_2) = 10^{12} m.
Relation between time period (T) and distance of the planet from the sun is $T^2\propto r^3$
or $T\propto r^{3/2}$.

Therefore $\frac{T_1}{T_2}=\left(\frac{r_1}{r_2}\right)^{3/2}=\left(\frac{10^{13}}{10^{12}}\right)^{3/2} = 10^{3/2} = 10\sqrt{10}$.

19. **(b) :** Centripetal force (F)= $\frac{mv^2}{R}$ and the gravitational force (F) = $\frac{GMm}{R^2}=\frac{GMm}{R}$

(where $R^2\to R$). Since $\frac{mv^2}{R}=\frac{GMm}{R}$, therefore $v=\sqrt{GM}$. Thus velocity v is independent of R.

20. **(d) :** Rate of change of mass $\frac{dM}{dt}=\alpha v$.
Retarding force = Rate of change of momentum
= Velocity × Rate of change in mass = $-v\times\frac{dM}{dt}$
$= -v\times\alpha v = -\alpha v^2$. (Minus sign of v due to deceleration)

Therefore deceleration = $-\frac{\alpha v^2}{M}$.

21. **(c) :** In a circular or elliptical orbital motion torque is always acting parallel to displacement or velocity. So, angular momentum is conserved. In attractive field, potential energy is negative. Kinetic energy changes as velocity increase when distance is less. But if the motion is in a plane, the direction of L does not change.

22. **(b) :** $T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2}$ by Kepler's law

$T_2 = 1\times2^{3/2} = 2\sqrt{2} = 2\times1.41 = 2.82$ year
$T_2 = 2.80$ year

23. **(a) :** $\frac{GMm}{r^2}=m\omega^2r \Rightarrow r^3=\frac{GM}{\omega^2}=\frac{gR^2}{\omega^2}$
$\therefore$ $r = \{gR^2/\omega^2\}^{1/3}$.

24. **(d) :** Total energy = –K.E
$\text{K.E}=\frac{|\text{P.E.}|}{2}$, $\text{K.E.}=\frac{1}{2}mv^2$

25. **(a):** Planets revolve around sun in elliptical orbit.

26. **(a):** In artificial satellite, effective value of g is zero. Hence time period of simple pendulum is infinite.

27. **(a):** Escape velocity $v_e = \sqrt{\frac{2GM}{R}}$

$= \sqrt{\frac{2G}{R}\times\left(\frac{4}{3}\pi R^3\right)\rho} = \sqrt{\frac{8\pi G\rho}{3}}\cdot R$

$\frac{v_e}{v_p}=\frac{R}{2R}=\frac{1}{2}$

28. **(d):** Angular momentum is conserved.

$mv_1d_1 = mv_2d_2 \quad \therefore\ v_2 = \frac{v_1d_1}{d_2}$

29. **(c):** Gravitational Potential

$$V = \int_x^\infty E\cdot dx = \int_x^\infty \frac{K}{x^3}dx = \frac{K}{2x^2}$$

30. **(c):** Kinetic energy = Potential energy

$$\frac{1}{2}m(kv_e)^2 = \frac{mgh}{1+\frac{h}{R}}$$

$$\frac{1}{2}mk^2v_e^2 = \frac{mgRh}{(R+h)}$$

$\frac{1}{2}mk^2.2Rg = \frac{mgRh}{(R+h)} \Rightarrow k^2 = \frac{h}{R+h}$

$\therefore\ h = Rk^2 + hk^2$

$h(1-k^2) = Rk^2 \Rightarrow h = \frac{Rk^2}{1-k^2}$

Height from centre = R + height 'h' from surface

$r = R + \frac{Rk^2}{1-k^2} = \frac{R}{1-k^2}$

31. **(b):** $v_0 = \sqrt{gR}$ = 8 km/sec, R = Radius of earth. Orbit is near surface of earth.

32. **(b):** Change in PE = $\Delta U = \frac{mgh}{1+\frac{h}{R}} = \frac{mg\cdot nR}{1+\frac{nR}{R}} = \frac{nmgR}{(n+1)}$

33. **(b):** K.E. $= \frac{1}{2}Mv_0^2 = \frac{1}{2}M\left(\frac{GM}{r}\right)$, r = radius of orbit

$\frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2R+R}{R+R} = \frac{3R}{2R} = \frac{3}{2}$

34. **(b):** For geostationary satellite, distance from the surface of the earth is about 36000 km $\approx 6R$. Its distance from the centre of earth $\approx 7R$.

35. **(d):** $g' = g\left(\frac{R}{R+h}\right)^2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{R}{R+h}$

$R + h = \sqrt{2}\cdot R \Rightarrow h = \sqrt{2}\cdot R - R = 1.414R - R = 0.414R$

36. **(c):** $g = \frac{GM}{R^2} = G\frac{\text{volume}\times\text{density}}{R^2}$

$= G\cdot\frac{4}{3}\frac{\pi R^3\cdot d}{R^2} = \frac{4\pi G}{3}\cdot d\cdot R$ or $g \propto d\cdot R$

37. **(b):** Escape velocity $v_e = \sqrt{2Rg}$,

Given $R_1/R_2 = K_1$, $\frac{g_1}{g_2} = K_2$

$\frac{v_1}{v_2} = \sqrt{\frac{2R_1g_1}{2R_2g_2}} = \sqrt{\frac{R_1}{R_2}\cdot\frac{g_1}{g_2}} = \sqrt{K_1K_2}$

38. **(b):** Apply Kepler's law, $T^2 \propto R^3$

or $\frac{T^2}{R^3}$ = constant

We know $T = \frac{1}{n}$, $R = d$ given

$\frac{T^2}{R^3} = \frac{1}{n^2}\cdot\frac{1}{d^3}$ = constant or n^2d^3 = constant

For the two planets, $n_1^2d_1^3 = n_2^2d_2^3$

39. **(a) :** $g = \frac{GM}{R^2} = G.\frac{4}{3}\frac{\pi R^3\times\rho}{R^2} = \frac{4\pi G\rho R}{3}$,

$\rho_2 = 2\rho_1$ given

$\frac{g_1}{g_2} = \frac{4\pi G}{3}\cdot\rho_1R_1\times\frac{3}{4\pi G\rho_2R_2} = \frac{\rho_1}{\rho_2}\cdot\frac{R_1}{R_2} = \frac{\rho_1}{2\rho_1}\cdot\frac{R}{R} = \frac{1}{2}$

$g_2 = 2g_1 = 2 \times 9.8 = 19.6$ m/s^2

40. **(c):** $\frac{1}{2}mv_e^2 = \frac{1}{2}m\cdot 2gR = mgR$

41. **(c) :** Acceleration due to planet,

$g = \frac{GM_0}{R^2} = \frac{G_0M}{\left(\frac{D_0}{2}\right)^2} = \frac{4GM_0}{D_0^2}$.

42. **(b) :** Gravitational force due to solid sphere,

$F_1 = \frac{GMm}{(2R)^2}$, where M and m are masses of the solid sphere and particle respectively and R is the radius of the sphere. The gravitational force on particle due to sphere with cavity = force due to solid sphere-force due to sphere creating cavity, assumed to be present above at that position.

i.e. $F_2 = \frac{GMm}{4R^2} - \frac{G(M/8)m}{(3R/2)^2} = \frac{7}{36}\frac{GMm}{R^2}$

So, $\frac{F_2}{F_1} = \frac{7GMm}{36R^2}\bigg/\left(\frac{GMm}{4R^2}\right) = \frac{7}{9}$.

43. **(b) :** As mass, $M = \frac{4}{3}\pi R^3\rho$ or $\rho = \frac{3M}{4\pi R^3}$

$\therefore\ \frac{\rho_s}{\rho_e} = \frac{M_s}{M_e}\times\frac{R_e^3}{R_s^3} = 330\times\left(\frac{1}{100}\right)^3 = 3.3\times10^{-4}$.

44. **(d) :** $g' = g - R\omega^2\cos^2\lambda$; when $\lambda = 45°$;

thus $g' = g - R\,\omega^2\left(\frac{1}{\sqrt{2}}\right)^2 = g - \frac{R\omega^2}{2}$

When earth stops rotating, $\omega = 0$, so $g' = g$

Increase in weight of body

$= g - \left(g - \frac{R\omega^2}{2}\right) = \frac{R\omega^2}{2}$.

45. **(a) :** $V_e = \sqrt{2Rg}$ and $V_0 = \sqrt{Rg}$

$\therefore\ V_e = \sqrt{2}\cdot V_0$

46. **(a) :** $\frac{mv^2}{R}$ = Centripetal force

$= \frac{GmM}{R^2}$ = Gravitational force

$v_0^2 = \frac{GM}{R}$

47. **(a) :** At height h, $g' = g\left(1-\frac{2h}{R}\right)$

At depth x, $g'' = g\left(1-\frac{x}{R}\right)$

Given $g' = g''$ $\therefore x = 2h$

48. **(a) :** Centripetal force = $mR\omega^2$

= Gravitational force = $\frac{GmM}{R^2}$

$$\omega^2 = \frac{GM}{R^3} = G\times\frac{\frac{4}{3}\pi R^3\cdot\rho}{R^3} = G\cdot\frac{4}{3}\pi\rho$$

$$\omega = \sqrt{\frac{4\pi G\rho}{3}} \text{ or } \frac{2\pi}{T} = \sqrt{\frac{4\pi G\rho}{3}}$$

T does not depend on R. Hence T remains as such.

49. **(a) :** $g = \frac{GM}{R^2} = G\cdot\frac{\frac{4}{3}\pi R^3\cdot\rho}{R^2} = \frac{4\pi\rho G}{3}\cdot R$

$$\frac{g_1}{g_2} = \frac{R_1}{R_2}$$

50. **(a) :** Gravitational intensity $E = -\frac{dV}{dx}$

= space rate of variation of potential

$\therefore \frac{dV}{dx} = 0$ or V = a constant

51. **(d) :** Ball pen functions on the principle of gravitational force.

52. **(d) :** $T^2 \propto r^3$ according to Kepler's law

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

53. **(d):** $g = \frac{GM}{R^2}$, $g' = \frac{G(2M)}{(2R)^2}$

$\therefore g' = \frac{1}{2}\cdot g = \frac{9.8}{2} = 4.9 \text{ m/s}^2$

54. **(d) :** Centripetal force = $mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2$

$$= \frac{4\pi^2 mR}{T^2}$$

Gravitational force = $KR^{-5/2}$ (Given)

$KR^{-5/2} = \frac{4\pi^2 mR}{T^2} \Rightarrow T^2K = 4\pi^2 m.R^{7/2}$

$\therefore T^2 \propto R^{7/2}$

55. **(d) :** Due to rotation, $g' = g\left(1-\frac{R\omega^2}{\lambda}\cdot\cos\phi\right)$

If $\omega = 0$, the negative part vanishes. g on equator therefore increases.

56. **(d) :** At the centre of earth $g = 0$

$\therefore$ weight = mg = zero.

57. **(d) :** RMS velocity is greater than escape velocity. Hence gas molecules escape away.

58. **(d) :** K.E. for escape = $\frac{1}{2}mv_e^2 = \frac{1}{2}m\,.\,(2Rg)$

$= mRg$

K.E. for orbital motion = $\frac{1}{2}mv_0^2 = \frac{1}{2}m\,(Rg)$

$= \frac{mRg}{2}$

Ratio of two K.E. = 2 : 1

59. **(d) :** By Kepler's law, $T^2 \propto r^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64} = \left(\frac{1}{8}\right)^2$$

$$\therefore \frac{T_1}{T_2} = \frac{1}{8}$$

60. **(b) :** The gravitational force or weight mg is used up in providing centripetal force to the satellite.

61. **(b) :** $v_0 = \sqrt{\frac{GM}{R}}$ $v_0' = \sqrt{\frac{GM}{3R/2}}$ $\therefore \frac{v_0'}{v_0} = \sqrt{\frac{2}{3}}$

$\therefore v_0' = \sqrt{\frac{2}{3}}\cdot v_0$

62. **(c) :** Time period of geostationary satellite is equal to 24 hours.

63. **(c) :** $g = \frac{GM}{R^2}$, $g' = \frac{GM}{(R/2)^2}$ $\therefore g' = 4\frac{GM}{R^2} = 4g$

64. **(c) :** Apply Kepler's law, $\left(\frac{T_s}{T_c}\right)^2 = (4)^3 = 64 = (8)^2 \therefore$

$T_S = 8T_C$

$\because$ Period $T_C = 1$ day $\Rightarrow$ $T_S = 8$ days.

65. **(a) :** Apply Kepler's law, $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 = 64$

$\left(\frac{R_1}{R_2}\right)^3 = 64 = (4)^3 \therefore \frac{R_1}{R_2} \Rightarrow \frac{4}{1}$, $R_1 = 4R_2$

66. **(a) :** Angular momentum is conserved in satellite revolution in an orbit.

67. **(a) :** Escape velocity does not depend upon angle of projection.

68. **(a) :** Angular speed of earth has no impact upon the acceleration due to gravity at poles.

69. **(a) :** Total energy = Kinetic energy + Potential energy

$$E_0 = \frac{1}{2}mv^2 - \frac{GmM}{R}$$

Centripetal force = Gravitational force

$$\frac{mv^2}{R} = \frac{GmM}{R^2} \quad \therefore \ v^2 = \frac{GM}{R}$$

$$E_0 = \frac{1}{2}m\frac{GM}{R} - \frac{GmM}{R} = -\frac{GmM}{2R}$$

$$\therefore \text{P.E.} = -\frac{GmM}{R} = 2E_0$$

70. **(c) :** $F = G\dfrac{m(M-m)}{R^2}$

For maximum force, $\dfrac{dF}{dm} = 0$

$$\frac{d}{dm}\left(\frac{GmM}{R^2} - \frac{Gm^2}{R^2}\right) = 0$$

or $M - 2m = 0 \Rightarrow \dfrac{m}{M} = \dfrac{1}{2}$

71. **(c) :** Intensity of gravitational field at the centre of a spherical shell is zero.

72. **(c) :** Apply Kepler's law $T^2 \propto R^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 \Rightarrow \left(\frac{1}{T_2}\right)^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$T_2 = \sqrt{8} = \sqrt{4\times 2} = 2\sqrt{2}$ year.

73. **(c) :** $g = \dfrac{GM}{R^2} \quad \therefore \quad \dfrac{\Delta g}{g} = -2\dfrac{\Delta R}{R} = -2 \times 1.5 = -3\%$

74. **(b) :** $F = \dfrac{GmM}{r^2} = \dfrac{G.\left(\frac{4}{3}\pi R^3\rho\right)^2}{(2R)^2}$

$\because \ m = M \quad r = 2R$

$$= \left(\frac{16}{9}\frac{G\pi^2\rho^2}{4}\right)\frac{R^6}{R^2}$$

$$F \propto \frac{R^6}{R^2} \propto R^4$$

75. **(b) :** $\Delta U = \dfrac{mgR}{1+\frac{h}{R}} = \dfrac{1}{2}mgR$ ($\because h = R$ Given)

76. **(b) :** At infinity total energy is zero. The nature of force is attractive. Hence the total energy is negative.

77. **(b) :** $v_e = \sqrt{\dfrac{2GM}{R}}$. It does not depend on mass of satellite m.

$v_e \propto m^0$

78. **(b) :** K.E. $= \dfrac{1}{2}Mv_0^2$, By Kepler's law, $T^2 \propto r^3$ or $T^2 = Kr^3$

$v_0 = \sqrt{\dfrac{GM}{r}}$ where r = radius of orbit

$$\therefore \text{K.E.} = \frac{1}{2}M\cdot\frac{GM}{r} = \frac{GM^2}{2}\cdot\frac{1}{r} = \frac{GM^2}{2}\left(\frac{K}{T^2}\right)^{\frac{1}{3}}$$

$\therefore$ KE $\propto T^{-2/3}$

79. **(b) :** Gravitational forces due to earth and moon are equal and opposite at the point x units away from earth if

$\dfrac{G\cdot M_e}{x^2} = \dfrac{G\cdot M_m}{(D-x)^2}$, M_m = Mass of moon

M_e = Mass of earth = 81 M_m

$$\frac{81M_m}{x^2} = \frac{M_m}{(D-x)^2} \Rightarrow 81 = \left(\frac{x}{D-x}\right)^2$$

$$\Rightarrow 9 = \frac{x}{D-x} \Rightarrow 9D - 9x = x \Rightarrow 10x = 9D$$

$$\therefore x = \frac{9}{10}D$$

80. **(b) :** $g_1' = \dfrac{GM}{(R+h)^2}, g = \dfrac{GM}{R^2}$

$\dfrac{g'_1}{g} = \dfrac{R^2}{(R+h)^2}$ Given $\dfrac{g'_1}{g} = \dfrac{1}{100}$

$\therefore \ \dfrac{1}{100} = \left(\dfrac{R}{R+h}\right)^2$ or $R + h = 10R$ or $h = 9R$

81. **(d) :** Acceleration due to gravity at the centre of earth = zero. This becomes the mean position of SHM of the particle released in the tunnel/hole.

82. **(a) :** Binding energy of a satellite = –K.E. of the satellite. If energy equal to E_k is provided to the satellite, it will escape into outer space.

83. **(a) :** Required condition is $mg - m\omega^2 r = 0$

Angular speed $\omega = \sqrt{\dfrac{g}{R}}$

$$= \sqrt{\frac{10}{6400\times10^3}} = \sqrt{\frac{1}{64\times10^4}}$$

$$\omega = \frac{1}{800} = 1.25 \times 10^{-3} \text{ rad/s}$$

84. **(b) :** $T = 2\pi\sqrt{\dfrac{(R+h)^3}{gR^2}} = 2\pi\sqrt{\dfrac{(2R)^3}{gR^2}}$

$$= 4\cdot\sqrt{2}\cdot\pi\sqrt{\frac{R}{g}}$$

85. **(d) :** Apply Kepler's law, T^2 = (constant) r^3.

Given $\dfrac{r_2}{r_1} = \dfrac{1}{4}$

$$\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} \Rightarrow T_2 = T_1\left(\frac{1}{4}\right)^{3/2} = T_1\left(\frac{1}{2}\right)^3 = \frac{T_1}{8}$$

86. **(c) :** $g' = g\left(\frac{R}{R+h}\right)^2 = \frac{4}{9}\cdot g \quad \therefore \frac{W'}{W} = \frac{4}{9}$

$\therefore \quad W' = \frac{4}{9}\cdot W$

87. **(b) :** Apply Kepler's law

$$\left(\frac{R_1}{R_2}\right)^3 = \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{2\pi}{\omega_1}\times\frac{\omega_2}{2\pi}\right)^2 = \left(\frac{\omega_2}{\omega_1}\right)^2$$

$$= \left(\frac{27\omega}{\omega}\right)^2 = (27)^2$$

$\frac{R_1}{R_2} = (27)^{2/3} = (3)^2 = 9 \qquad \frac{R_1}{R_2} = \frac{9}{1}$

88. **(a) :** $v_e = \sqrt{\frac{2GM}{R}}$, $v_e' = \sqrt{\frac{2G(2M)}{3R}} = \sqrt{\frac{2}{3}}\cdot v_e$

89. **(c):** $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{l}{\pi^2}} = \frac{2\pi}{\pi} = 2$ s

90. **(a) :** K.E. of satellite $= \frac{1}{2}\cdot\frac{GMm}{R}$

$\therefore$ K.E. $\propto \frac{1}{R}$

91. **(d) :** Apply Kepler's law, T^2r^{-3} = constant.
To study the small changes in T and r, take logs and differentiate.

$2\log T - 3\log r = \log$ (constant)

$2\cdot\frac{dT}{T} - 3\cdot\frac{dr}{r} =$ zero

$\frac{dT}{T} = \frac{3}{2}\cdot\frac{dr}{r} = -\frac{3}{2}\cdot\frac{(0.02)}{1} = 0.03$

$dT = T \times 0.03$

Percentage change in time period = 3

92. **(b) :** In a mine, $g = G\times\frac{4}{3}\frac{\pi R^3\cdot\rho}{R^2} = \frac{4\pi}{3}G\rho.R.$
Take logs.

$\Rightarrow \quad \log g = \log\left(\frac{4\pi}{3}G\rho\right) + \log R$

By differentiation, $\frac{dg}{g} = \frac{dR}{R}$ ...(i)

On a mountain $g = \frac{GM}{R^2}$ .Take logs and differentiate

$\frac{dg}{g} = -2\frac{dR}{R} \quad \therefore \quad \frac{dR}{R} = \frac{1}{200}$

Put this in (i) $\frac{dg}{g}$ in mine $= -\frac{1}{200}$

$dg = -\frac{100}{200} = -\frac{1}{2} = -0.5\% = 0.5\%$ decrease.

93. **(b) :** Velocity of escape to infinity = velocity on reaching earth from infinity

$v_e = \sqrt{2gR}$

94. **(b) :** In a mine $g' = g\left(1-\frac{d}{R}\right)$

$\frac{g}{4} = g\left(1-\frac{d}{R}\right) \qquad \therefore \quad d = \frac{3R}{4}$

95. **(b) :** $v_e = \sqrt{\frac{2Gm}{r}} = \sqrt{2G\cdot\frac{4}{3}\pi\frac{r^3\cdot\rho}{r}} = \sqrt{\frac{8G\pi}{3}\cdot r^2\rho}$

$v'_e = \sqrt{\frac{8G\pi}{3}\cdot(4r)^2(9\rho)} = 12\sqrt{\frac{8G\pi}{3}r^2\rho} = 12\cdot v_e$

Assertion and Reason

1. **(a) :** According to Kepler's third law of motion, the square of the time period of a planet about the sun is proportional to the cube of the semi major axis of the ellipse or mean distance of the planet from the sun. *i.e.* $T^2 \propto a^3$, when a is smaller, shorter is the time period.
2. **(c) :** Acceleration due to gravity is given by $g = \frac{GM}{R^2}$
Thus it does not depend on mass of body on which it is acting. Also it is not a constant quantity and changes with change in value of both M and R (distance between two bodies). Even for earth it is a constant only near the earth's surface.
3. **(b) :** Acceleration due to gravity,
$g' = g - R_e\omega^2\cos^2\lambda$
At equator, $\lambda = 0° \;\therefore\; \cos 0° = 1$
$\therefore\; g_e = g - R_e\omega^2$
At poles, $\lambda = 90° \;\therefore\; \cos 90° = 0$
$\therefore\; g_p = g$
Thus, $g_p - g_e = g - g + R_e\omega^2 = R_e\omega^2$
Also, the value of g is maximum at poles and minimum at equator.
4. **(a) :** At poles, radius of horizontal circle is zero. $\therefore$Centripetal force $F = mr\omega^2 = 0$. Hence g at poles is not affected by rotation of earth.
5. **(a) :** The gravitational potential at a point in the gravitational field of earth is defined as the work done in bringing a unit mass from infinity to that point. It is attracted by the earth gravitational field, so work is done on the body, so the gravitational potential is negative.
6. **(c) :** According to Kepler's law of planetary motion, a planet revolves around the sun in such a way that its areal velocity is constant. *i.e.*, it move faster, when it is closer the sun and vice-versa.
7. **(a) :** As the geostationay satellite is established in an orbit in the plane of the equator at a

particular place, so it move in the same sense as the earth and hence its time period of revolution is equal to 24 hours, which is equal to time period of revolution of earth about its axis.

8. **(a) :** According to Newton's law of gravitation,

$F = \frac{Gm_1m_2}{r^2}$. When m_1, m_2 and r all are doubled,

$$F = \frac{G(2m_1)(2m_2)}{(2r)^2} = \frac{Gm_1m_2}{r^2},$$

i.e. F remains the same.

9. **(c) :** Variation of g with depth from surface of earth is given by $g' = g\left(1 - \frac{d}{R}\right)$.

At the centre of earth, $d = R$

$\therefore\ g' = g\left(1 - \frac{R}{R}\right) = 0$

$\therefore$ Apparent weight of body $= mg' = 0$

10. **(b) :** We know that earth revolves from west to east about its polar axis. Therefore, all the particles on the earth have velocity from west to east. This velocity is maximum in the equatorial line, as $v = R_e\omega$, where R_e is the radius of earth and ω is the angular velocity of revolution of earth about its polar axis. When a rocket is launched from west to east in equatorial plane, the maximum linear velocity is added to the launching velocity of the rocket, due to which launching becomes easier.

11. **(b) :** Binding energy is the minimum energy required to free a satellite from the gravitational attraction. It is the negative value of total energy of satellite. Let a satellite of mass m be revolving around earth of mass M_e and radius R_e.

Total energy of satellite = $P.E.$ + $K.E.$

$$= -\frac{GM_em}{R_e} + \frac{1}{2}mv^2$$

$$= -\frac{GM_em}{R_e} + \frac{m}{2}\frac{GM_e}{R_e} = -\frac{GMm}{2R_e}.$$

$\therefore$ Binding energy of satellite = – [total energy of satellite] which depend on mass of the satellite.

12. **(c) :** According to Newton's law of gravitation, every body in this universe attracts every other body with a force which is inversely proportional to the square of the distance between them. The distance of the finger from the stars is almost infinity for measuring gravitational fields.

13. **(d) :** If the orbital path of a satellite is circular, then its speed is constant and if the orbital path of a satellite is elliptical, then its speed in its orbit is not constant. In that case its areal velocity is constant.

14. **(b) :** Gravitational field intensity at a point distance r from centre of earth is $E = \frac{GM}{r^2}$.

When $r = \infty$, $E = 0$.

When point is inside the earth, then

$E = \frac{G}{r^2} \times \frac{4}{3}\pi r^3 \rho = \frac{4\pi G\rho r}{3}$ when $r = 0$, $E = 0$.

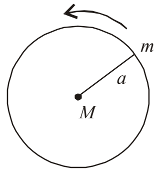
15. **(a) :** To make our calculations easy, let's take the semi-major axis of the ellipse be equal to the average distance of the Sun from the planet. By applying Newton's law,

$$\frac{GMm}{a^2} = m\omega^2 a$$

ω = angular velocity of the planet

$= \frac{2\pi}{T}$ (T = time period of the planet)

$\therefore\ \frac{GMm}{a^2} = m\frac{(2\pi)^2}{T^2}a$



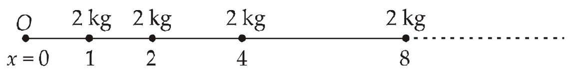
or, $T^2 = \left(\frac{4\pi^2}{GM}\right)a^3$

or, $T^2 \propto a^3$.

Which is also known as Kepler's third law or the law of period, according to which the square of period of any planet is proportional to the cube of the semi major axis of its orbit.

MCQs - NEET / AIPMT, AIIMS

1. **(b) :** The resulting gravitational potential at the origin O due to each of mass 2 kg located at positions as shown in figure is



$$V = -\frac{G\times2}{1} - \frac{G\times2}{2} - \frac{G\times2}{4} - \frac{G\times2}{8} - \ldots$$

$$= -2G\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\right] = -2G\left[\frac{1}{1-\frac{1}{2}}\right] = -2G\left[\frac{2}{1}\right] = -4G$$

2. **(d) :** Gravitational potential energy at any point at a distance r from the centre of the earth is

$$U = -\frac{GMm}{r}$$

where M and m be masses of the earth and the body respectively.

At the surface of the earth, $r = R$

$$\therefore\ U_i = -\frac{GMm}{R}$$

At a height h from the surface,

$r = R + h = R + 2R \quad (h = 2R \text{ (Given)})$

$= 3R$

$\therefore \quad U_f = -\frac{GMm}{3R}$

Change in potential energy,

$\Delta U = U_f - U_i$

$= -\frac{GMm}{3R} - \left(-\frac{GMm}{R}\right)$

$= \frac{GMm}{R}\left(1 - \frac{1}{3}\right) = \frac{2}{3}\frac{GMm}{R}$

$= \frac{2}{3}mgR \qquad \left(\because g = \frac{GM}{R^2}\right)$

3. **(b) :** Here, $R_P = 2R_E$, $\rho_E = \rho_P$

Escape velocity of the earth,

$$V_E = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2G}{R_E}\left(\frac{4}{3}\pi R_E^3 \rho_E\right)} = R_E\sqrt{\frac{8}{3}\pi G\rho_E} \quad \text{...(i)}$$

Escape velocity of the planet

$$V_P = \sqrt{\frac{2GM_P}{R_P}} = \sqrt{\frac{2G}{R_P}\left(\frac{4}{3}\pi R_P^3 \rho_P\right)} = R_P\sqrt{\frac{8}{3}\pi G\rho_P} \quad \text{...(ii)}$$

Divide (i) by (ii), we get

$$\frac{V_E}{V_P} = \frac{R_E}{R_P}\sqrt{\frac{\rho_E}{\rho_P}}$$

$$\frac{V_E}{V_P} = \frac{R_E}{2R_E}\sqrt{\frac{\rho_E}{\rho_E}} = \frac{1}{2}$$

or $V_P = 2V_E$

4. **(a) :** The minimum speed with which the particle should be projected from the surface of the earth so that it does not return back is known as escape speed and it is given by

$$v_e = \sqrt{\frac{2GM}{(R+h)}}$$

Here, $h = 3R$

$$\therefore \quad v_e = \sqrt{\frac{2GM}{(R+3R)}} = \sqrt{\frac{2GM}{4R}} = \sqrt{\frac{GM}{2R}}$$

$$= \sqrt{\frac{gR}{2}} \qquad \left(\because g = \frac{GM}{R^2}\right)$$

5. **(b) :** If a point mass m is placed at a height h from surface of earth, the potential energy is

$$U_h = -\frac{GMm}{(R+h)} = \frac{-gR^2 m}{R\left(1+\frac{h}{R}\right)} = \frac{-gR^2 m}{R}\left(1+\frac{h}{R}\right)^{-1}$$

$$\left(\because \quad g = \frac{GM}{R^2}\right)$$

$$U_h = \frac{-gR^2 m(R-h)}{R^2} = -gm(R-h)$$

$$\therefore \quad V = \frac{U_h}{m} = \frac{-gm(R-h)}{m} = -g(R-h)$$

6. **(b) :** The value of g at latitude λ is

$g' = g - R\omega^2\cos^2\lambda$.

If earth stops rotating, $\omega = 0$; $g' = g$.

It means the weight of body will increase.

7. **(c) :** Light cannot escape from a black hole,

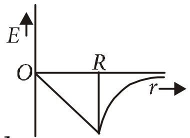
$v_e = c$

$$\sqrt{\frac{2GM}{R}} = c \quad \text{or} \quad R = \frac{2GM}{c^2}$$

$$R = \frac{2 \times 6.67 \times 10^{-11}\ \text{N m}^2\text{kg}^{-2} \times 5.98 \times 10^{24}\text{kg}}{(3 \times 10^8\ \text{m s}^{-1})^2}$$

$= 8.86 \times 10^{-3}$ m $\approx 10^{-2}$ m

8. **(a) :** For a point inside the earth *i.e.* $r < R$



$$E = -\frac{GM}{R^3}r$$

where M and R be mass and radius of the earth respectively.

At the centre, $r = 0$

$\therefore \quad E = 0$

For a point outside the earth *i.e.* $r > R$,

$$E = -\frac{GM}{r^2}$$

On the surface of the earth *i.e.* $r = R$,

$$E = -\frac{GM}{R^2}$$

The variation of E with distance r from the centre is as shown in the adjacent figure.

9. **(d) :** Kinetic energy, $K = \frac{1}{2}mv_0^2$

or $v_0 = \sqrt{\frac{2K}{m}}$

When kinetic energy is doubled, then

$$v_0' = \sqrt{\frac{2 \times 2K}{m}} = \sqrt{2}\sqrt{\frac{2K}{m}} = \sqrt{2}v_0$$

But $\sqrt{2}v_0 = v_e$ (escape velocity)

Hence satellite will escape out of earth's gravitational field.

10. **(d) :** Gravitational force of attraction between sun and planet provides centripetal force for the orbit of planet.

$$\therefore \quad \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v^2 = \frac{GM}{r} \quad \text{...(i)}$$

Time period of the planet is given by

$$T = \frac{2\pi r}{v}, \quad T^2 = \frac{4\pi^2 r^2}{v^2}$$

$$T^2 = \frac{4\pi^2 r^2}{\left(\frac{GM}{r}\right)} \quad \text{[Using equation (i)]}$$

$$T^2 = \frac{4\pi^2 r^3}{GM} \quad \text{...(ii)}$$

According to question,

$$T^2 = Kr^3 \quad \text{...(iii)}$$

Comparing equations (ii) and (iii), we get

$$K = \frac{4\pi^2}{GM} \quad \therefore \quad GMK = 4\pi^2$$

11. (a)

12. (c): The orbital speed of the satellite is

$$v_o = R\sqrt{\frac{g}{(R+h)}}$$

where R is the earth's radius, g is the acceleration due to gravity on earth's surface and h is the height above the surface of earth.

Here, $R = 6.38 \times 10^6$m, $g = 9.8$ m s^{-2} and

$h = 0.25 \times 10^6$ m

$$\therefore v_o = (6.38 \times 10^6 \text{ m})\sqrt{\frac{(9.8 \text{ m s}^{-2})}{(6.38 \times 10^6 \text{ m} + 0.25 \times 10^6 \text{ m})}}$$

$$= 7.76 \times 10^3 \text{ m s}^{-1} = 7.76 \text{ km s}^{-1}$$

$(\because 1 \text{ km} = 10^3 \text{ m})$

13. (b): The gravitational force on the satellite S acts towards the centre of the earth, so the acceleration of the satellite S is always directed towards the centre of the earth.

14. (c): Gravitation potential at a height h from the surface of earth, $V_h = -5.4 \times 10^7$ J kg^{-2}

At the same point acceleration due to gravity,

$g_h = 6$ m s^{-2}

$R = 6400$ km $= 6.4 \times 10^6$ m

We know, $V_h = -\frac{GM}{(R+h)}$, $g_h = \frac{GM}{(R+h)^2} = -\frac{V_h}{R+h}$

$$\Rightarrow \quad R + h = -\frac{V_h}{g_h}$$

$$\therefore \quad h = -\frac{V_h}{g_h} - R = -\frac{(-5.4 \times 10^7)}{6} - 6.4 \times 10^6$$

$$= 9 \times 10^6 - 6.4 \times 10^6 = 2600 \text{ km}$$

15. (d) : As escape velocity, $v = \sqrt{\frac{2GM}{R}}$

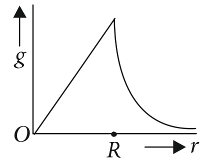
$$= \sqrt{\frac{2G}{R} \cdot \frac{4\pi R^3}{3}\rho} = R\sqrt{\frac{8\pi G}{3}\rho}$$

$$\therefore \frac{v_e}{v_p} = \frac{R_e}{R_p} \times \sqrt{\frac{\rho_e}{\rho_p}}$$

$$= \frac{1}{2} \times \sqrt{\frac{1}{2}} = \frac{1}{2\sqrt{2}} \quad (\because R_p = 2R_e \text{ and } \rho_p = 2\rho_e)$$

16. (b): Acceleration due to gravity is given by

$$g = \begin{cases} \frac{4}{3}\pi\rho G r & ; r \le R \\ \frac{4}{3}\frac{\pi\rho R^3 G}{r^2} & ; r > R \end{cases}$$



17. (b): Total energy of satellite at height h from the earth surface,

$$E = \text{PE} + \text{KE}$$

$$= -\frac{GMm}{(R+h)} + \frac{1}{2}mv^2 \quad \text{...(i)}$$

Also, $\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$

or, $v^2 = \frac{GM}{R+h}$ ...(ii)

From eqns. (i) and (ii),

$$E = -\frac{GMm}{(R+h)} + \frac{1}{2}\frac{GMm}{(R+h)} = -\frac{1}{2}\frac{GMm}{(R+h)}$$

$$= -\frac{1}{2}\frac{GM}{R^2} \times \frac{mR^2}{(R+h)}$$

$$= -\frac{mg_0 R^2}{2(R+h)} \qquad \left(\because g_0 = \frac{GM}{R^2}\right)$$

18. (d) : As, $-\frac{GMm}{2R_1} + \text{KE} = \frac{-GMm}{2R_2}$

$$\therefore \quad \text{KE} = \frac{1}{2}GMm\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

19. (c): The acceleration due to gravity at a height h is given as $g_h = g\left(1 - \frac{2h}{R_e}\right)$

where R_e is radius of earth.

The acceleration due to gravity at a depth d is given as $g_d = g\left(1 - \frac{d}{R_e}\right)$

Given, $g_h = g_d$

$$\therefore \quad g\left(1 - \frac{2h}{R_e}\right) = g\left(1 - \frac{d}{R_e}\right)$$

$\therefore d = 2h = 2 \times 1 = 2$ km $\quad (\because h = 1 \text{ km})$

20. (a): Since two astronauts are floating in gravitational free space. The only force acting on the two astronauts is the gravitational pull of their masses, $F = \frac{Gm_1 m_2}{r^2}$, which is attractive in nature.

Hence they move towards each other.

☺☺☺