GuruAanklan Foundation / MHT-CET / Examination
Mathematics
Set - [A] - Solutions

ANSWER KEY

1.	(A)	2.	(A)	3.	(D)	4.	(D)	5.	(C)	6.	(C)
7.	(B)	8.	(A)	9.	(D)	10.	(A)	11.	(A)	12.	(B)
13.	(D)	14.	(D)	15.	(B)	16.	(A)	17.	(C)	18.	(C)
19.	(B)	20.	(A)	21.	(A)	22.	(A)	23.	(A)	24.	(C)
25.	(C)	26.	(C)	27.	(C)	28.	(C)	29.	(B)	30.	(C)
31.	(C)	32.	(C)	33.	(B)	34.	(B)	35.	(A)	36.	(D)
37.	(D)	38.	(D)	39.	(C)	40.	(A)	41.	(A)	42.	(B)
43.	(D)	44.	(D)	45.	(D)	46.	(D)	47.	(B)	48.	(D)
49.	(C)	50.	(A)								

MATHEMATICS

= =

1. (A)

$PS + PS' = 2a = 20 \qquad 4\sqrt{5}$ (Here major axis of an allipse is along y-axis)

2. (A)

$$fe^{x}\left(\sin^{-1x} + \frac{1}{\sqrt{1-x^{2}}}\right)dx = e^{x}\sin^{-1}x + c$$

$$[\because fe^{x}\left(f(x) + f^{1}(x)\right)dx = e^{x}f(x) + c]$$

3. (D)

$$\text{If } f(x) = e^{x^2} \text{ then } \int xe^{x^2}dx = \frac{1}{2}\int 2x\,e^{x^2}dx$$

$$= \frac{1}{2}e^{x^2} + c = \frac{1}{2}f(x) + c$$

By trial and error

Checking each choice and integrating in case of choice D

$$\text{Let } f(x) = e^{x^2}$$

$$\therefore \int x f(x)dx = \int xe^{x^2}dx = \frac{1}{2}\int 2xe^{x^2}dx$$

$$= \frac{e^{x^2}}{2} + c = \frac{f(x)}{2} + c$$

4. (D)

$$I = \int \frac{\sin x}{\sin 3x}dx$$

$$= \int \frac{\sin x}{3\sin x - 4\sin^{3}x}dx$$

$$= \int \frac{1}{3 - 4\sin^{2}x}dx$$

$$= \int \frac{\sec^{2}x}{3\sec^{2}x - 4\tan^{2}x}dx$$

$$= \int \frac{\sec^{2}x}{3 - \tan^{2}x}dx$$

$$\text{Let } \tan x = t \Rightarrow \sec^{2}x\,dx = dt$$

$$= \int \frac{1}{3 - t^{2}}dt = \int \frac{1}{\left(\sqrt{3}\right)^{2} - t^{2}}dt$$

$$=\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+c$$

5. (C)

$$\int_0^{2\pi} e^x\left(\frac{x}{2}+\frac{\pi}{4}\right)dx=\int_0^{2\pi}\left(\frac{x}{2}e^x+\frac{\pi}{4}e^x\right)dx$$

$$=\left[\left(\frac{x}{2}\right)\left(e^x\right)-\frac{1}{2}\left(e^x\right)+\frac{\pi}{4}e^x\right]_0^{2\pi}$$

$$=\pi e^{2\pi}-\frac{1}{2}e^{2\pi}+\frac{\pi}{4}e^{2\pi}-\left(-\frac{1}{2}+\frac{\pi}{4}\right)$$

$$=e^{2\pi}\left(\frac{5\pi}{4}-\frac{1}{2}\right)+\frac{1}{2}-\frac{\pi}{4}$$

$$\left[\text{since}\int xe^x dx=xe^x-e^x+c\right]$$

6. (C)

The distance between the given lines is equal to $\frac{2}{\sqrt{5}}$

$\therefore$ The required line must be $\perp$ to both lines let its equation be $2x-y+k=0$

As it passes through $(-5,4), \therefore -10-4+k=0$

Hence $k=14$

Therefore equation of lime is $2x-y+14=0$

7. (B)

$$\text{since}\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$$

$$I=\int_a^b x f(x)dx\ (\text{given})$$

$$\therefore I=\int_a^b (a+b-x)f(a+b-x)dx$$

$$=\int_a^b (a+b-x)f(x)dx$$

$$\therefore I=\int_a^b (a+b)f(x)dx-\int_a^b xf(x)dx$$

$$\Rightarrow I=(a+b)\int_a^b f(x)dx-I$$

$$\Rightarrow I = \frac{(a+b)}{2}\int_a^b f(x)\,dx$$

8. (A)

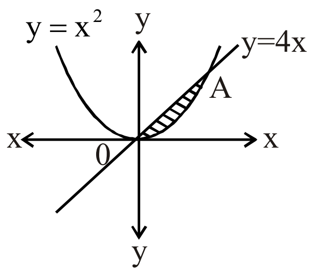
$x^2 = y$ and $y = 4x$

solving $x = 0, 4 \Rightarrow y = 0, 16$

$A(4,16)$

Required area $= A = \int_0^4 y\ dx = \int_0^4 y\,dx$ between line & parabola

$$= \int_0^4 4x\,dx - \int_0^4 x^2 dx = \int_0^4 (4x - x^2)\,dx$$



$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= 32 - \frac{64}{3} = \frac{32}{3}\ \text{sq. units}$$

9. (D)

$\frac{dy}{dx} = \cos^{-1} a$

Integrating both sides

$y = (\cos^{-1} a)x + c$

$\therefore 2 = 0 + c$

$x = 0,\ c = 2$

$y = (\cos^{-1} a)x + c$

$\frac{y-c}{x} = \cos^{-1} a$

$\cos\left(\frac{y-c}{x}\right) = a$

10. (A)

Let $4x+y+1=v$

Differentiating w.r.t. x

$$4+\frac{dy}{dx}=\frac{dv}{dx} \text{ but } \frac{dy}{dx}=(4x+y+1)^2=v^2$$

$$4+v^2=\frac{dv}{dx}$$

$$\frac{dv}{4+v^2}=dx$$

Integrating both sides

$$\frac{1}{2}\tan^{-1}\frac{v}{2}=x+c'$$

$$\frac{v}{2}=\tan(2x+2c') \quad \text{put } 2c'=c$$

$$\frac{v}{2}=\tan(2x+c)$$

$$v=2\tan(2x+c)$$

$$4x+y+1=2\tan(2x+c)$$

replace c by k

11. (A)

As the length of sides of a triangles in G.P.

Let they are 9, 9r and $9r^2\ (r>1)$

Now perimeter $=9+9r+9r^2=37$

$\therefore 9r^2+9r-28=0 \quad \therefore \quad (3r+7)(3r-4)=0$

$\therefore \quad r=\frac{-7}{3}$ or $r=\frac{4}{3}$ but $r>0, r=\frac{-7}{3}$ is rejected $\therefore \quad r=\frac{4}{3}$

$\therefore$ Hence the sides of the triangle are 9, 12, 16

$\therefore$ The lengths of the other two sides are 12 and 16

12. (B)

since $\frac{dx}{dt}=kx$

$$\log x=kt+c$$

$$\frac{dx}{x}=kdt$$

At $t = 0,\ x = x_0$, then $c = \log x_0$

$\Rightarrow \log x = kt + C$

$\log x = kt + \log x_0$

At $t = 2, x = 600$, then

$$\log 600 = 2k + \log x_0 \Rightarrow k = \frac{1}{2}\log\frac{600}{x_0}$$

$$\Rightarrow \log x = \frac{t}{2}\log\left(\frac{600}{x_0}\right) + \log x_0$$

At $t = 8, x = 75000$, then

$$\log 75000 = \frac{8}{2}\log\left(\frac{600}{x_0}\right) + \log x_0$$

$$\therefore \log 75000 = 4\log 600 - 4\log x_0 + \log x_0$$

$$\therefore 3\log x_0 = \log\left(\frac{(600)^4}{75000}\right)$$

$$= \log \frac{600\times600\times600\times600}{75000}$$

$$\therefore 3\log x_0 = \log\frac{6\times6\times6\times6\times100000}{3\times5\times5}$$

$$= \log(2\times6\times6\times6\times4\times1000)$$

$$= \log(2\times2\times2\times6\times6\times6\times10\times10\times10)$$

$$= 3\log 120$$

$\therefore x_0 = 120$

13. (D)

Sum of all probabilities is 1, $\sum p_i = 1$

$$\therefore P[x=-2] + P[x=-1] + P[x=0] + p[x=1] + p[x=2] + P[x=3] = 1$$

$$\therefore 0.1 + K + 0.2 + 2k + 0.3 + k = 1$$

$$4k + 0.6 = 1$$

$4k = 0.4 \qquad \therefore k = 0.1$

$x = 2n(ni)$	-2	-1	0	1	2	3
$P[f = n](pi)$	0.1	0.1	0.2	0.2	0.3	0.1

Expected value $= E(x) = \sum(x) = \sum x_i p_i$

$= (-2)(0.1) + (-1)(0.1) + 0(0.2) + 1(0.2) + 2(0.3) + 3(0.1)$

$= -0.2 - 0.1 + 0.2 + 0.6 + 0.3$

$= -0.3 + 0.2 + 0.6 + 0.3$

$= 0.8$

14. (D)

x_i	p_i	$x_i p_i$	$x_i^2 p_i$
0	0.45	0	0
1	0.35	0.35	0.35
2	0.15	0.30	0.60
3	0.03	0.09	0.27
4	0.02	0.08	0.32

$\sum x_i p_i = 0.82 \quad \sum x_i^2 p_i = 154$

$Var(x) = \sum x_i^2 p_i - (\mu)^2$ since expected value $E(x) = \mu = \sum x_i p_i = 0.82$

$= 1.54 - (0.82)^2 = 1.54 - 0.6724$

$= 0.8676$

15. (B)

Mean $=$ n.p variance $=$ npq

$$\frac{p[x = k]}{p[x = k-1]} = \frac{{}^nC_k p^k q^{n-k}}{{}^nC_{k-1} p^{k-1} q^{n-k+1}}$$

$$= \frac{{}^nC_k}{{}^nC_{k-1}} \frac{p^{k-k+I}}{q^{n-k+I-n+k}}$$

$$= \frac{{}^nC_k}{{}^nC_{k-1}} \frac{p}{q}$$

since ${}^nC_{k-1} = \dfrac{n!}{k!(n-k)!}$ and ${}^nC_{k-1} \dfrac{n!}{(n-k+1)!(k-1)!}$

$$=\frac{\frac{n!}{k!(n-k)!}}{\frac{n!}{(n-k+1)!(k-1)!}}\cdot\frac{p}{q}$$

$$=\frac{(n-k+1)!(k-1)!}{k!(n-k)!}\cdot\frac{p}{q}$$

$$=\frac{(n-k+1)(n-k)!(k-I)!}{k(k+I)!(n-k)!}\cdot\frac{p}{q}$$

$$=\frac{n-k+1}{k}\cdot\frac{p}{q}$$

16. (A)

$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

Equation of tangent to hyperbola having slope *m* is

$$y=mx+\sqrt{16m^2-9}$$

It touches the circle $\therefore$ Distance of this line from centre of the radius of the circle

$$\frac{\sqrt{16m^2-9}}{\sqrt{m^2+1}}=3$$

$$7m^2=18$$

$$m=3\sqrt{\frac{2}{7}}$$

Equation of tangents is $y=3\sqrt{\frac{2}{7}}x+\frac{15}{\sqrt{7}}$

17. (C)

Contrapositive of $(p\vee q)\rightarrow$ is $\sim r\rightarrow\sim(p\vee q)$

$\therefore\sim r\rightarrow\sim(p\vee q)\equiv\sim r\rightarrow(\sim p\wedge\sim q)$

18. (C)

p	q	$\sim q$	$p\vee\sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

19. (B)

Given expression

$\equiv\sim p\wedge(q\vee\sim q)]\vee(p\wedge\sim q)$

$\equiv \sim p \wedge T \vee (p \wedge \sim q)$

$\equiv \sim p \vee (p \wedge \sim q)$

$\equiv (\sim p \vee p) \wedge (\sim p \vee \sim q)$

$\equiv T \wedge (\sim p \vee \sim q)$

$\equiv \sim p \vee \sim q$

20. (A)

Given $A^2 = 0$

$\text{Now}, A(I+A)^n = A({}^nC_oI + {}^nC_1A + {}^nC_2A^2 + ---+{}^n({}_nA^n)$

$= A(I+nA)$

$= A + nA^2$

$= A$

21. (A)

Parabola is $y^2 = k\left(x - \frac{8}{k}\right)$ OR

$y^2 = 4AX$

Where $4A = k,\ Y = y,\ X = x - \frac{8}{k}$

Its directrix is $X = -A$ or $x - \frac{8}{k} = \frac{-k}{4}$ or $x = \frac{8}{k} - \frac{k}{4}$

Comparing with $x = 1$, we get $1 = \frac{32 - k^2}{4k}$

$k^2 + 4k - 32 = 0$

$(k+8)(k-4) = 0$

$k = 4$ or $k = -8$

22. (A)

Id $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

$Au_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad Au_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$Au_2 + Au_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$$\therefore A(u_2+u_3)=\begin{bmatrix}0\\1\\1\end{bmatrix}, \text{ let } u_2+u_3=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

$$\therefore A(u_2+u_3)=\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\1\\1\end{bmatrix}$$

$$\therefore x=0,\ 2x+y=1,\ 3x+2y+z=1$$

$$\therefore x=0,\ y=1,\ z=-1$$

$$\therefore u_2+u_3=\begin{bmatrix}0\\1\\-1\end{bmatrix}$$

23. (A)

$$\cos A=\frac{\sin B}{2\sin C}$$

$$\therefore \frac{b^2+c^2-a^2}{2bc}=\frac{bk}{2ck}\cdot\frac{b}{2c}$$

$$\therefore b^2+c^2-a^2=b^2$$

$$\therefore c=a$$

$\therefore \Delta$ is an isosceles Δ

24. (C)

$$c=a\cos B+b\cos A, b=c\cos A+a\cos C$$

$$\therefore \frac{c-b\cos A}{b-c\cos A}=\frac{a\cos B}{a\cos C}=\frac{\cos B}{\cos C}$$

25. (C)

$$\tan\left[\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}\right)\right]$$

$$=\tan\left[\tan^{-1}\left(\frac{1}{7}\right)\right]=\frac{1}{7}$$

26. (C)

Here, radius $\sqrt{\left(\frac{1-m}{2}\right)^2+\left(\frac{m}{2}\right)^2-5}\le 5$

$$2m^2 - 2m - 119 \le 0$$

$$\frac{1-\sqrt{239}}{2} \le m \le \frac{1+\sqrt{239}}{2}$$

$$-7.2 \le m \le 8.2 \text{ (approximately)}$$

$$-7.2 \le m \le 8.2 \text{ (approximately)}$$

$$m = -7, -6, ..., 5, 6, 7, 8$$

27. (C)

f(x) is continuous at $x = 0$

$$\therefore \lim_{x \to 0} f(x) = f(0)$$

$$\therefore f(0) = \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1 + 1 - \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \frac{1 - \cos x}{x^2}$$

$$= \log e + \frac{1}{2}$$

$$= 1 + \frac{1}{2}$$

$$= \frac{3}{2}$$

28. (C)

$$x^K \cdot y^T = (x+y)^{K+T}$$

Taking log on both sides

$$K \log x + T \log y = (K+T) \log(x+y)$$

Differentiating w.r.t. x

$$\therefore \frac{K}{x} + \frac{T}{y}\frac{dy}{dx} = \frac{K+T}{x+y}\left(1 + \frac{dy}{dx}\right)$$

$$\therefore \left(\frac{T}{y} - \frac{K+T}{x+y}\right)\frac{dy}{dx} = \frac{K+T}{x+y} - \frac{K}{x}$$

$$\therefore \frac{xT + yT - Ky - yT}{y(x+y)} \frac{dy}{dx}$$

$$= \frac{xK + xT - Kx - Ky}{x(x+y)}$$

$$\therefore \frac{xT-Ky}{y}\frac{dy}{dx}=\frac{xT-Ky}{x}$$

$$\therefore \frac{dy}{dx}=\frac{y}{x}$$

29. (B)

$$y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+.....}}}$$

i.e. $y=\sqrt{\log x+y}$

squaring $y^2=\log x+y$

Differentiating w.r.t. x

$$2y\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx}$$

$$\therefore (2y-1)\frac{dy}{dx}=\frac{1}{x}$$

$$\therefore \frac{dy}{dx}=\frac{1}{x(2y-1)}$$

$$=\frac{1}{x(2y-1)}$$

30. (C)

Let $\cos^{-2}\left(\frac{x^2-y^2}{x^2+y^2}\right)=a$

put $\frac{y}{x}=\tan\theta$

$$\therefore \theta=\tan^{-1}\left(\frac{y}{x}\right)$$

$$\therefore \cos^{-1}\left(\frac{1-\left(\frac{y}{x}\right)^2}{1+\left(\frac{y}{x}\right)^2}\right)=a$$

$$\therefore \cos^{-1}\frac{(1-\tan^2\theta)}{1+\tan^2\theta}=a$$

$$\therefore \cos^{-1}(\cos 2\theta)=a$$

$\therefore 2\theta = a$

$\therefore \theta = \frac{a}{2}$

$\therefore \tan^{-1}\left(\frac{y}{x}\right) = \frac{a}{2}$

$\frac{y}{x} = \tan\frac{a}{2}$

Differentiating w.r.t. x, we get

$\frac{x \cdot \frac{dy}{dx} - y}{x^2} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} = \tan\frac{a}{2}$

$\therefore \frac{d^2y}{dx^2} = 0$

31. (C)

The probability of getting a defective bulb from the box is $\frac{1}{10}$. Hence using binomial distribution, the required probability is $(0.9)^5$

32. (C)

$f(x) = 2x^3 - 15x^2 - 144x - 7$

$\Rightarrow f'(x) = 6x^2 - 30x - 144$

$\Rightarrow f'(x) = 6(x-8)(x+3)$

For decreasing function $f'(x) < 0$

$\Rightarrow 6(x+3)(x-8) < 0 \Rightarrow (x+3)(x-8) < 0$

$\Rightarrow x - 8 < 0$ and $x + 3 > 0$

$\Rightarrow x < 8$ and $x > -3$

$\Rightarrow -3 < x < 8$

33. (B)

$x = 160t - 16t^2$

$\Rightarrow v = \frac{dx}{dt} = 160 - 32t$

at t = 1, v = 160 – 32 = 128

at t = 9, v = 160 – 288 = –128

velocities are equal and opposite.

34. (B)

Given general second degree equation represents a pair of lines if

$$\Delta = \begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 0 \\ 5 & 4 & 8 \\ 0 & 8 & k \end{bmatrix} = 0$$

$$\Rightarrow 3(3k-64)-5(5k-0)+0=0$$

$$\Rightarrow 9k-192-25k=0$$

$$\Rightarrow -16k=192$$

$$\Rightarrow k=-12$$

35. (A)

Required lines are x = 9 and x = –9

$\therefore$ joint equation is (x – 9) (x + 9) = 0

$x^2-81=0$

36. (D)

Suppose x families live in the town

A={families have scooter}

B= {families have car}

$$\therefore m(A)=\frac{30x}{100} \quad n(B)=\frac{40x}{100} \quad and\ n(A\cup B)'=\frac{50x}{100}$$

$$\therefore n(A\cup B)=\frac{50x}{100}$$

$$\therefore n(A\cup B)=\frac{20x}{100}$$

$$\therefore \frac{20x}{100}=2000$$

$$\therefore x=10000$$

37. (D)

Given equation $kx^2+4xy-y^2=0$

$a=k, 2h=4, b=-1$

$m_1+m_2=\frac{-2h}{b}=-\frac{4}{-1}=4,\ \ m_1m_2=\frac{a}{b}=\frac{k}{-1}=-k$

$m_1=m_2+8 \Rightarrow m_1-m_2=8$

$(m_1-m_2)^2=64$

$(m_1+m_2)^2-4m_1m_2=64$

$16-4(-k)=64$

$16+4k=64$

$4k=48$

$\therefore\ k=12$

38. (D)

$\text{Volume of tetrahedron}=\frac{1}{6}\left[\overline{PQ}\ \overline{PR}\ \overline{PS}\right]$

$\overline{P}=-i+2j+3k$

$\overline{q}=3j-2j+k$

$\overline{r}=2i+j+3k$

$\overline{s}=i+2j+4k$

$\overline{PQ}=4i-4j-2k$ as $\overline{PQ}=\overline{q}-\overline{p}$

$\overline{PR}=3i-j-0k$ as $\overline{PR}=\overline{r}-\overline{p}$

$\overline{PS}=0i-0j-k$ as $\overline{PS}=\overline{s}-\overline{p}$

$\text{Volume of tetrahedron}=\frac{1}{6}\left[\overline{PQ}\ \overline{PR}\ \overline{PS}\right]$

$$=\frac{1}{6}\begin{bmatrix}4 & -4 & -2\\ 3 & -1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\frac{1}{6}\left[4(-1)+4(3)-2\times 0\right]$$

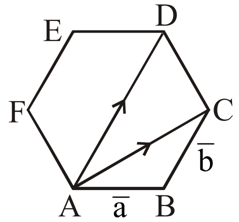
$$=\frac{8}{6}=\frac{4}{3}\text{cu.units}$$

39. (C)

$$[\overline{a}+\overline{b}+\overline{c}].[\overline{b}+\overline{q}+\overline{r}]=[\overline{a}+\overline{b}+\overline{c}].\frac{1}{[\overline{a}+\overline{b}+\overline{c}]}\left[\overline{b}\,x\,\overline{c}+\overline{c}\,x\,\overline{a}+\overline{c}\,x\,\overline{b}\right]$$

$$=\frac{3[\bar{a}\,\bar{b}\,\bar{c}]}{[\bar{a}\,\bar{b}\,\bar{c}]}=3$$

40. (A)



$$\overline{AC}=\overline{AB}+\overline{BC}=\bar{a}+\bar{b}$$

$$\overline{AD}=2\overline{BC}=2\bar{b}$$

and $\overline{AD}=\overline{AC}+\overline{CD}$

$$\overline{CD}=\overline{AD}-\overline{AC}$$

$$\overline{CD}=2\bar{b}-\bar{a}-\bar{b}$$

$$=\bar{b}-\bar{a}$$

41. (A)

$$f(x)=\sin^2 x+\left(\sin x\cos\frac{\pi}{3}+\cos x\sin\frac{\pi}{3}\right)^2+\cos x\left(\cos x\cos\frac{\pi}{3}-\sin x\sin\frac{\pi}{3}\right)$$

$$=\sin^2 x+\left(\frac{\sin x}{2}+\frac{\sqrt{3}}{2}\cos x\right)^2+\cos x\left(\frac{\cos x}{2}-\frac{\sqrt{3}}{2}\sin x\right)$$

$$=\frac{5}{4}\sin^2 x+\frac{5}{4}\cos^2 x=\frac{5}{4}$$

42. (B)

D.R.S. of line AB are $a+2$, $b-1$, $c+8$ line AB is parallel to the line whose D.R.S. are 6, 2, 3

then, $a+2=6,\ b-1=2,\ c+8=3$

$$a=4, b=3, c=-5$$

43. (D)

since $\alpha=\beta=\gamma$ and $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$$\therefore\ 3\cos^2\alpha=1$$

$$\therefore \cos^2\alpha=\frac{1}{3}\qquad \therefore \cos\alpha=\pm\frac{1}{\sqrt{3}}$$

44. (D)

Use dist $=d=\left|\dfrac{(\vec{a}_2-\vec{a}_1)\cdot(\vec{b}_1\times\vec{b}_2)}{|\vec{b}_1\times\vec{b}_2|}\right|$

$$\vec{a}_1=4i-j,\ \vec{a}_2=9-j+2k$$

$\vec{a}_2 - \vec{a}_1 = -3i + 2k$

$\vec{b}_1 = i + 2j - 3k$, $\vec{b}_2 = i + 4j - 5k$

$$\therefore \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} i & j & k \\ 1 & 2 & -3 \\ 1 & 4 & -5 \end{vmatrix}$$

$\vec{b}_1 \times \vec{b}_2 = 2i + 2j + 2k$ and $\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{4+4+4} = 2\sqrt{3}$

$\therefore \left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right) = -2$

$$\text{using distance} = \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|} \right|$$

$$= \left| \frac{-2}{2\sqrt{3}} \right| = \left| \frac{-1}{\sqrt{3}} \right| = \frac{1}{\sqrt{3}}$$

45. (D)

$A(5,5,\lambda), B(-1,3,2), C(-4,2,-2)$

$\therefore$ Equation of line BC.

$\vec{r} = \vec{b} + \mu\left(\vec{c} - \vec{b}\right)$

$\vec{r} = (i + 3j + 2k) + \mu(-3i - j - 4k)$

$\because$ A, B, C all collinear

$\therefore 5i + 5j + \lambda k$

$= (-i + 3j + 2k) + \mu(-3i - j - 4k)$

comparing coefficients of i

$5 = -1 - 3\mu$

$-3\mu = 6 \quad \ldots(i)$

comparing coefficient of k

$\lambda = 2 - 4\mu \quad \ldots(ii)$

$\mu = 2 - 4(-2)$

$\lambda = 10$

46. (D)

If θ is angle between line $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and plane $a_2x + b_2y + c_2z = d_2$

$$\text{then } \sin\theta = \left| \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

But $a_1, b_1, c_1 \equiv 1, 2, 2$

$\therefore \sqrt{a_1^2 + b_1^2 + c_1^2} = \sqrt{1 + 4 + 4} = 3$

$a_2, b_2, c_2 \equiv 2, -1, \sqrt{\lambda}$

$\sqrt{a_2^2 + b_2^2 + c_2^2} = \sqrt{4+1+\lambda} = \sqrt{5+\lambda}$

$\cos\theta = \frac{\sqrt{8}}{3} \qquad \therefore \sin\theta = \frac{1}{3}$

$\therefore \frac{1}{3} = \left|\frac{1(2)+2(-1)+2\sqrt{\lambda}}{3\sqrt{5+\lambda}}\right|$, squaring both sides

$\frac{1}{9} = \frac{4\lambda}{9(5+\lambda)}$

$5+\lambda = 4\lambda$

$5 = 3\lambda$

$\lambda = \frac{5}{3}$

47. (B)

Equation of plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\therefore A = (a,0,0)\ B = (0,b,0)\ C = (0,0,c)$

$\therefore$ centroid of ΔABC is $\left(\frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right)$

$\therefore \frac{1}{3} = \frac{a+0+0}{3}, \frac{2}{3} = \frac{0+b+0}{3}, \frac{4}{3} = \frac{0+0+c}{3}$

$\therefore$ a = 1, b = 2, c = 4

$\therefore$ equation of required plane

$\frac{x}{1} + \frac{y}{2} + \frac{z}{4} = 1$

$\therefore 4x + 2y + z = 4$

48. (D)

Distance between parallel planes

$ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$

$d = \left|\frac{d_1 - d_2}{\sqrt{a^2 + b^2 + c^2}}\right|$

Here $d_1 = 3 \qquad d_2 = \frac{5}{2}$

$a, b, c \equiv -2, -1, 2$

$\therefore \sqrt{a^2+b^2+c^2}=\sqrt{4+1+4}=3$

$\therefore d=\left|\dfrac{d-\frac{5}{2}}{3}\right|=\left|\dfrac{6-5}{2\times 3}\right|=\dfrac{1}{6}$

49. (C)

Equation of line AB is $x+y=20$

Equation of line CD is $2x+5y=80$

Feasible region is closed as per figure. Answer is (C)

50. (A)

Here A = {2, 4, 6} and B = {2, 3, 5}

$\therefore A\times B$ contains $3\times 3=9$ elements

No. of relations $=2^9$