Combined Paper	Std. X - Geometry Chapter 1 and 3	Solutions

Q.1 Solve any four questions. **[4 M]**

(1) □ABCD is a trapezium

side $AB \parallel$ seg$PQ \parallel$ sideDC

$\therefore$ By intercepts madeby 3 parallel lines

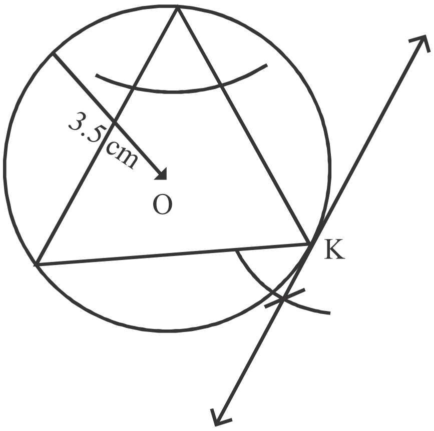
$\frac{AP}{PD} = \frac{BQ}{QC}$ ...**[½ M]**

$\frac{15}{12} = \frac{BQ}{14}$

$BQ = \frac{15 \times 14}{12}$

$BQ = 17.5$ units ...**[½ M]**

(2)



(i) To draw a circle with centre O an radius 3.5 cms ...**[½ M]**

(ii) To draw tangent through point A ...**[½ M]**

(3) $MN = MT + TN \quad [M - T - N]$

$MN = 2 + 4 = 6$ units

$$\frac{MT}{MN}=\frac{2}{6}=\frac{1}{3}$$

$$MK = MP + PK \qquad [M-P-K]$$

$$=3+6=9\text{ units}$$

$$\frac{MP}{MK}=\frac{3}{9}=\frac{1}{3}$$

$$\frac{TP}{NK}=\frac{2.5}{7.5}=\frac{1}{3}$$

In ΔMTP and ΔMNK

$$\frac{MT}{MN}=\frac{MP}{MK}=\frac{TP}{NK}$$ ...**[½ M]**

$\therefore \Delta MTP \sim \Delta MNK\,[S.S.S \text{ test of similarity}]$ ...**[½ M]**

OR

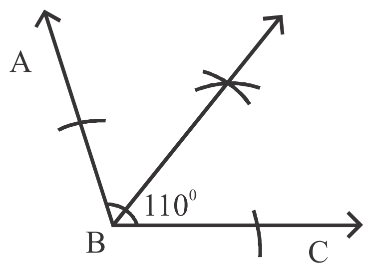
$$\frac{MT}{MN}=\frac{MP}{MK}$$ ...**[½ M]**

$\angle M = \angle M\,[\text{common angle}]$

$\therefore \Delta MTP \sim \Delta MNK\,[S.A.S \text{ test of similarity}]$ ...**[½ M]**

(4) (i) Draw an angle of 110^0

(ii) Bisect the angle



$RK = RP + PK$

$RK = 3k + 2k = 5k$

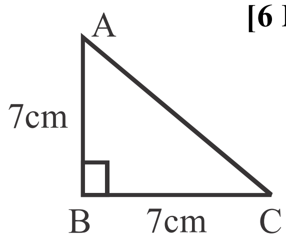
$$\frac{A(\Delta TRP)}{A(\Delta TRK)}=\frac{RP}{RK}\,[\text{Triangles having equal heights}]$$ ...**[½ M]**

$$=\frac{3k}{5k}$$

$$\frac{A(\Delta TRP)}{A(\Delta TRK)}=\frac{3}{5}$$ ...**[½ M]**

Q.2 Solve any three sub-questions. **[6 M]**

(1) In a right angled triangle



$ABC, AB = 7\,cms, s\,BC = 7\,cms$

By Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$ ...**[½ M]**

$AC^2 = (7)^2 + (7)^2$

$= 49 + 49$

$= 98$

$AC = \sqrt{98} = 7\sqrt{2}\ cms$ ...[½ M]

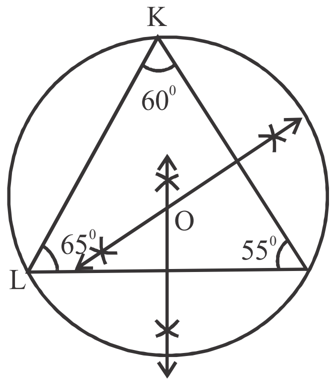
Perimeter of $\Delta ABC = AB + BC + AC$ ...[½ M]

$7 + 7 + 7\sqrt{2}$

$= 14 + 7\sqrt{2}$

$= 7\left(2 + 2\sqrt{2}\right)$ ...[½ M]

(2) Analysis



In ΔKLM

$\angle K + \angle L + \angle M = 180°$ [angle sum property]

$60° + \angle L + 55° = 180°$

$\angle L + 115° = 180°$

$\angle L = 180° - 115°$

$\angle L = 65°$

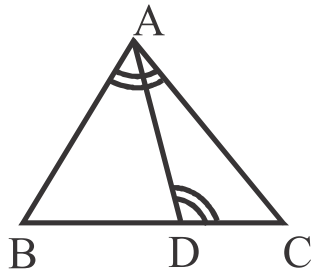
(i) Analysis ...[½ M]

(ii) Construction of ΔKLM ...[½ M]

(iii) Bisect any two sides ...[½ M]

(iv) Draw a circum-circle ...[½ M]

(3) Given : In $\Delta ABC, D$ is any point on BC



$\angle ADC = \angle BAC$

To prove that

$AC^2 = BC \times DC$

Proof

In ΔABC and ΔDAC

$\angle ADC = \angle BAC$

$\angle DCA = \angle ACB$

$\therefore\ \Delta ABC \sim \Delta DAC$ [A.A. test of similarity]

$\frac{AC}{DC} = \frac{BC}{AC}$ [$c.s.s.t$]

$AC^2 = BC \times DC$

(4) Longest side = 41

Square of the longest side $=(41)^2=1681$ ...**[½ M]**

Sum of squares of other two sides $=(9)^2+(40)^2$

$=81+1600$

$=1681$

Henc square of the longest side = sum of squares of other two sides

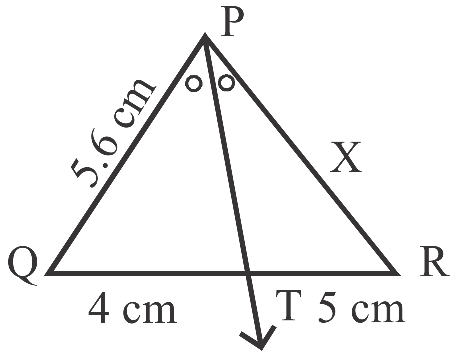
$(41)^2=(9)^2+(40)^2$

$\therefore$ By converse of pythagoras theorem

$\therefore$ The given sides are the sides of a right angled triangle.

Q.3 Solve any two sub-questions. **[6 M]**

(1) Given : In ΔPQR



Ray PT bisects $\angle QPR$

$PQ=5.6\,cms, QT=4\,cms, TR=5\,cms$

To find the value of x

In ΔPQR, Ray PT bisects $\angle \text{QPR}$ [given]

$\text{QR}=\text{QT}+\text{TR}=4+5=9\text{cm}\,[\text{Q}-\text{R}-\text{T}]$

By property of angle bisector

$$\frac{\text{PQ}}{\text{PR}}=\frac{\text{QT}}{\text{TR}}$$

$$\frac{5.6}{x}=\frac{4}{5}$$

$$x=\frac{5.6\times 5}{4}$$

$x=7\,cms$

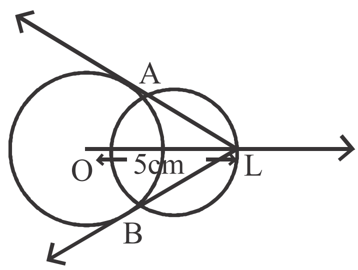
The value of $x=7\,cms$

(2) (i) Draw a circle of radius 2.8 cms ...**[½ M]**

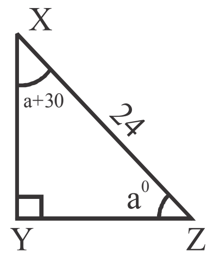
(ii) Take a distance of 5 cms from O to L ...**[½ M]**

(iii) Cutting the arcs on the circle ...**[1 M]**

(iv) Draw the tangents ...**[1 M]**



(3) In ΔXYZ



$\angle X = (a+30)^0$

$\angle Y = 90^0$

$\angle Z = a^0$

$\angle X + \angle Y + \angle Z = 180^0$ [angle sum property]

$(a+30)^0 + 90^0 + a^0 = 180^0$

$2a^0 + 120^0 = 180^0$

$2a^0 = 180^0 - 120^0$

$2a^0 = 60^0$

$a = 30^0$ …**[½ M]**

$\angle X = (a+30)^0 = (30+30)^0 = 60^0$ …**[½ M]**

$\angle Z = a^0 = 30^0$

In ΔXYZ, it is a $30^0 - 60^0 - 90^0$ triangle

$\text{XY} = \frac{1}{2} \times XZ$ (side opp to 30^0) …**[½ M]**

$XY = \frac{1}{2} \times 24 = 12$ units …**[½ M]**

$YZ = \frac{\sqrt{3}}{2} \times XZ$ (side opp to 60^0) …**[½ M]**

$YZ = \frac{\sqrt{3}}{2} \times 24 = 12\sqrt{3}$ units …**[½ M]**

Q.4 Solve any one sub-questions. **[4 M]**

(1) In ΔNED

$\angle N + \angle E + \angle D = 180^0$ [angle sum property]

$30^0 + \angle E + 30^0 = 180^0$

$\angle E + 50^0 = 180^0$

$\angle E = 180^0 - 50^0$

$\angle E = 130^0$ ...[½ M]

$\Delta RHP \sim \Delta NED$ [given]

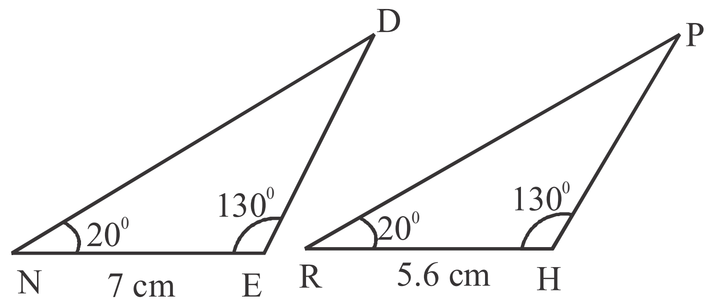
$\frac{RH}{NE} = \frac{HP}{ED} = \frac{RP}{ND}$ [C.S.S.T] ...[½ M]

$\frac{RH}{NE} = \frac{4}{5}$

$\frac{RH}{7} = \frac{4}{5}$

$RH = \frac{7 \times 4}{5} = \frac{28}{5}$

$RH = 5.6\,cms$ ...[½ M]



...[2 M]

By Corresponding angles of similar triangle ...[½ M]

$\angle H = \angle E = 130^0$

$\angle N = \angle R = 20^0$

(2) In ΔABC

$\angle C = 90^0, BC = a, CA = b, AB = c$ and $CD = p$

To Prove that

(1) cp = ab (2) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Proof:

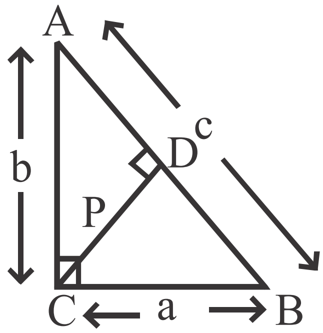
(1) Area of $\Delta ABC = \frac{1}{2} \times b \times h = \frac{1}{2} \times AB \times CD$ ...[½ M]

Area of $\Delta ABC = \frac{1}{2} \times c \times p$.....(1)

Area of $\Delta ABC = \frac{1}{2} \times b \times h = \frac{1}{2} \times CB \times AC$

$= \frac{1}{2} \times a \times b$ .....(2)

From (1) and (2)



$\frac{1}{2} \times c \times p = \frac{1}{2} \times a \times b$ ...**[½ M]**

$\therefore cp = ab$ ...**[½ M]**

(2) $cp = ab$

$p = \frac{ab}{c}$

Squaring both the sides

$p^2 = \frac{a^2b^2}{c^2}$

By invertendo $\frac{1}{p^2} = \frac{c^2}{a^2b^2}$ ......(3)

In $\Delta ABC, \angle C = 90^0$

By phythagoras theorem

$c^2 = a^2 + b^2$ ......(4) ...**[½ M]**

Substitute the values of (4) in (3)

$\frac{1}{p^2} = \frac{a^2 + b^2}{a^2b^2}$ ...**[½ M]**

$\frac{1}{p^2} = \frac{a^2}{a^2b^2} + \frac{b^2}{a^2b^2}$

$\frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{a^2}$

Hence $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ ...**[½ M]**