Guru Aanklan

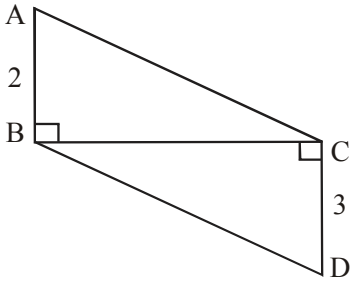
| GRAND TEST | SSC EXAMINATION GEOMETRY (SET-A) | SOLUTION |
|---|---|---|

**Q. 1 Solve any five sub-questions:** **[5M]**

**Ans.1** $\Delta ABC \,\&\, \Delta DCB$ have equal height

$$\frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{\frac{1}{2}\times AB \times BC}{\frac{1}{2}\times DC \times BC}$$ **[1/2M]**

$$= \frac{AB}{DC}$$




$$\boxed{\frac{A(\Delta ABC)}{A(\Delta DCB)} = \frac{2}{3}}$$ **[1/2M]**

**Ans.2** $\theta = 60°$

Slope of the line $= tan\,\theta$ ..... **[1/2M]**

$= tan\,60°$

$= \sqrt{3}$ ..... **[1/2M]**

**Ans.3** $\theta = -60°$

$\cos(-\theta) = \cos\theta$ ..... **[1/2M]**

$\cos(-60°) = \cos 60°$

$= \frac{1}{2}$

$$\boxed{\cos(-60°) = \frac{1}{2}}$$ ..... **[1/2M]**

**Ans.4** Diagonal of a square $= \sqrt{2} \times side$ ..... **[1/2M]**

$= \sqrt{2} \times 10$

$= 10\sqrt{2}\,cm$ ..... **[1/2M]**

**Ans.5** volume of a cube is $=1000\ cm^3$

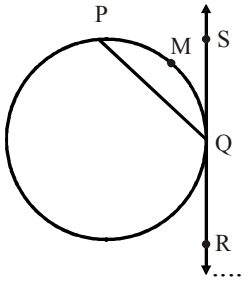
$l^3 = 1000$ ..... **[1/2M]**

$l = \sqrt[3]{1000}$ [Taking cube root]

$\boxed{l = 10cm}$ ..... **[1/2M]**

**Ans.6** RS is a tangent and PQ in a chord ,

$\angle PQS = \frac{1}{2} \times M(arc\ PMQ)$ [Tangent-Secant theorem] ..... **[1/2M]**




$M(arc\ PMQ) = 130°$ [given]

$\therefore \angle PQS = \frac{1}{2} \times 130$

$\boxed{PQS = 65°}$ ..... **[1/2M]**

**Q. 2 Solve any four sub-questions:** **[8M]**

**Ans.1** $sin\theta = \frac{7}{25}$ [given]

$sin^2\theta + cos^2\theta = 1$ [Trigonometric Identity] ..... **[1/2M]**

$cos^2\theta = 1 - \left(\frac{7}{25}\right)^2$ ..... **[1/2M]**

$= 1 - \left(\frac{49}{625}\right)$

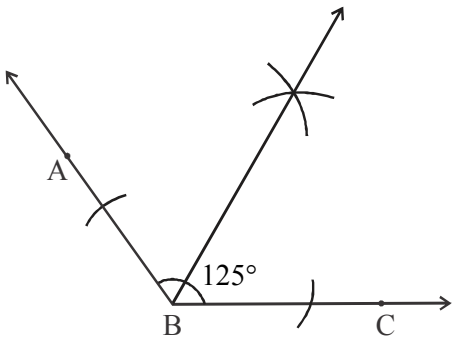
$= \frac{625 - 49}{625} = \frac{576}{625}$ ..... **[1/2M]**

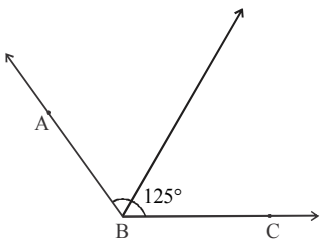
$cos\theta = \sqrt{\frac{576}{625}}$ [Taking square root]

$\boxed{cos\theta = \frac{24}{25}}$ ..... **[1/2M]**

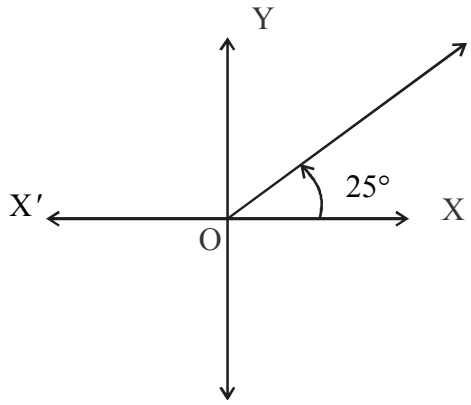
**Ans.2** **(i) Draw and angle of $125°$ [1M]**

**(ii) Bisect it [1M]**



**Ans.3**




..... **[1M]**

The terminal arm lies in I-quadrant. ..... **[1M]**

**Ans.4** $Length\ of\ an\ arc = l = 10cm$

$Rdius\,(r) = 5cm$

$Area\ of\ the\ sec tor = \frac{r}{2} \times length\ of\ an\ arc$ ..... **[1M]**

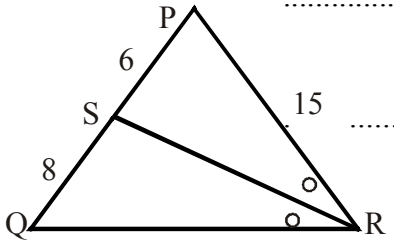
$= \frac{5}{2} \times 10$ ..... **[1/2M]**

$= 25cm$ ..... **[1/2M]**

**Ans.5** In $\Delta PQR$,

seg RS bisector $\Delta PRQ$ [given]

$\frac{PR}{QR} = \frac{PS}{SQ}$ [Angle bisector property] ..... **[1M]**




$\frac{15}{QR} = \frac{6}{8}$ ..... **[1/2M]**

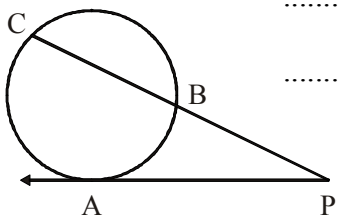
$QR = \frac{15 \times 8}{6}$

$\boxed{QR = 20}$ ..... **[1/2M]**

**Ans.6** $PA$ is a tangent and $PBC$ is a secant

By tangent secant property

$PA^2 = PB \times PC$ ..... **[1M]**



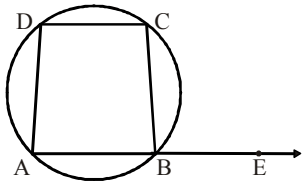

$(13)^2 = 6 \times PC$ ..... **[1/2M]**

$PC = \frac{169}{6} = 28.17$

$\boxed{PC = 28.17}$ ..... **[1/2M]**

**Q. 3 Solve any three sub-questions.** **[9M]**

**Ans.1** $\angle ADC = \frac{1}{2} \times M\left(arc\ ABC\right)$[Inscribed angle theorem] **[1/2M]**




$= \frac{1}{2} \times 210$

$\boxed{\angle ADC = 105°}$ *or* $\boxed{\angle CDA = 105°}$ **[1/2M]**

$\angle ADC + \angle ABC = 180°$ [Opposite angle of a cyclic quadrilateral are supplementary]..... **[1/2M]**

$105° + \angle ABC = 180°$

$\angle ABC = 180° - 105°$

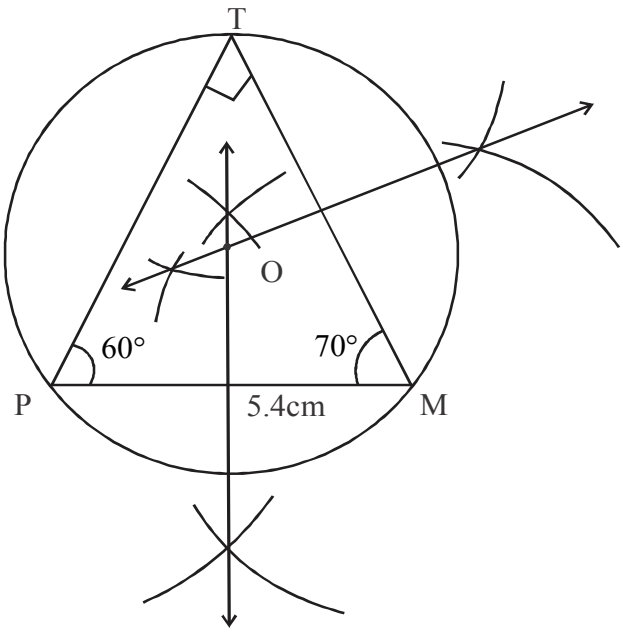
$\boxed{\angle ABC = 75°}$ ..... **[1/2M]**

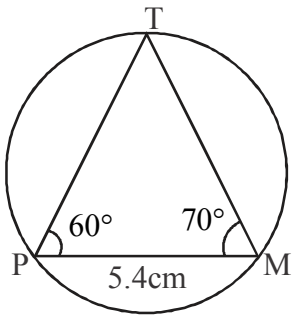
$\angle CDA = \angle CBE$ [Exterior angle of cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle] ..... **[1/2M]**

$\angle CDA = 105°$

$\boxed{\therefore \angle CBE = 105°}$ ..... **[1/2M]**

**Ans.2**



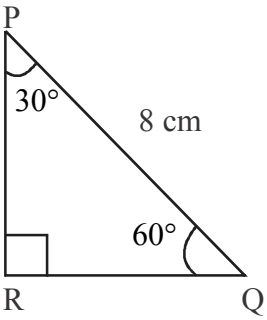




(i) Draw a triangle **[1M]**

(ii) Bisect any two sides **[1M]**

(iii) Circumcircle **[1M]**

3. In $\Delta PQR$




$\left.\begin{aligned}\angle P &= 30° \\ \angle Q &= 60° \\ \angle R &= 90°\end{aligned}\right\}$ **[1/2M]**

$\Delta PQR$ *is* $30° - 60° - 90°$ triangle **[1/2M]**

$PR = \frac{\sqrt{3}}{2} \times PQ$[side opposite to $60°$] ..... **[1/2M]**

$= \frac{\sqrt{3}}{2} \times 8$

$PR = 4\sqrt{3}cm$ ..... **[1/2M]**

$QR = \frac{1}{2} \times PQ\left[\text{side opposite to } 30°\right]$ ..... **[1/2M]**

$= \frac{1}{2} \times 8$

$\boxed{QR = 4cm}$ ..... **[1/2M]**

**Ans.4** $\sqrt{\frac{1+\sin x}{1-\sin x}} = \sec x + \tan x$

*L. H. S.*

$\sqrt{\frac{(1+sin\,x)}{(1-sin\,x)} \times \frac{(1+sin\,x)}{(1+sin\,x)}}$ ..... **[1/2M]**

$= \sqrt{\frac{(1+sin\,x)^2}{1-sin^2\,x}} \left[(a-b)(a+b) = a^2 - b^2\right]$ ..... **[1/2M]**

$= \sqrt{\frac{(1+sin\,x)^2}{cos^2\,x}} \left[1 - sin^2\,\theta = cos^2\,\theta\right]$

$= \frac{1+sin\,x}{cos\,x}$ ..... **[1/2M]**

$= \frac{1}{cos\,x} + \frac{sin\,x}{cos\,x}$ ..... **[1/2M]**

$= sec\,x + tan\,x \left[sec\,\theta = \frac{1}{cos\,\theta}, \frac{sin\,\theta}{cos\,\theta} = tan\,\theta\right]$ ..... **[1/2M]**

$= R.H.S$

$\therefore L.H.S. = R.H.S$

**Ans.5** Radius of sphere = $r = 4.2$ *cm.* ..... **[1/2M]**

Volume of a sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2$ ..... **[1/2M]**

$= 4 \times 22 \times 0.2 \times 4.2 \times 4.2$

$= 310.464\,cm^3$ ..... **[1/2M]**

Surface area of a sphere $= 4\pi r^2$ ..... **[1/2M]**

$= 4 \times \frac{22}{7} \times 4.2 \times 4.2$ ..... **[1/2M]**

$= 4 \times 22 \times 0.6 \times 4.2$

$= 221.76 cm^2$ ..... **[1/2M]**

**Q.4 Solve any two sub-questions:** **[8M]**

**Ans.1** Given : $A$ circle with centre $O$ Fig - **[1/2M]**
and external point $P$
are given $AP$ and $BP$
are the two tangents drawn
from an external point $P$

**To prove :** $PA = PB$ ..... **[1/2M]**

**Construction :** Draw *seg OA, seg OB* and *seg OP* ..... **[1/2M]**

**Proof :** In $\Delta OBP$ and $\Delta OAP$

$OA = OB$ [Radii of the same circle] ..... **[1/2M]**

$OP = OP$ [common side] ..... **[1/2M]**

$\angle PAO = \angle PBO = 90°$ [Tangent Perpendicularity theorem] ..... **[1/2M]**

$\therefore \Delta OBP \cong \Delta OAP$ [Hypotenuse side test] ..... **[1/2M]**

$PA = PB$ [Corresponding sides of congruent triangle] ..... **[1/2M]**

Hence proved.

**Ans.2** Let

$AB$ represents the height of the first building = 30 cm

$DC$ represents the height of the second building

$BD$ represent the width of the road = 10 m

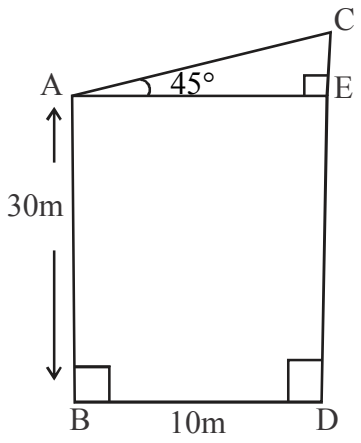



fig. **[1/2M]**

$\angle EAC = 45°\left[\text{Angle of elevation}\right]$ Expln. **[1/2M]**

In $\square ABDE$

$\angle ABD = \angle BDE = 90°\left[\text{objects are perpendicular to the ground}\right]$

$\angle AED = 90°$

$\angle BAE = 90°\left[\text{Remaining angle}\right]$

$\therefore \square ABDE$ is a rectangle $\left[\text{Definition of rectangle}\right]$ ..... **[1/2M]**

$AE = BD = 10m$ ———— (1)

$AB = ED = 30m$ ———— (2) } opposite sides of a rectangle. ..... **[1/2M]**

In $\Delta AEC$

$tan\,45° = \dfrac{CE}{AE}$ ..... **[1/2M]**

$1 = \dfrac{CE}{10}$ [From (1)]

$\boxed{CE = 10m}$ ———— (3) ..... **[1/2M]**

$CD = CE + ED\left[C - E - D\right]$

$= 10 + 30 \ \left[\, from\,(2)\,\&\,(3)\right]$

$CD = 40m$ ..... **[1/2M]**

$\therefore$ The height of the second building is 40 m ..... **[1/2M]**

**Ans.3** Length of ther semicircular tunnel $= 2\,km$

$h = 2000\ m$ ..... **[1/2M]**

Diameter of the semicircular tunnel $= 7\,m$

Radius of the tunnel $= r = \dfrac{7}{2}$

$r = 3.5\,m$ ..... **[1/2M]**

Volume of semicirluar tunnel $= \dfrac{1}{2} \times$ volume of circular tunnel

$= \dfrac{1}{2} \times \pi r^2 h$ ..... **[1/2M]**

$= \dfrac{1}{2} \times \dfrac{22}{7} \times 3.5 \times 3.5 \times 2000$

$= 11 \times 0.5 \times 3.5 \times 2000$

$= 11 \times 5 \times 35 \times 20$

$= 38500\ m^3$ ..... **[1/2M]**

$\therefore$ Expenditure for digging (semicircular) the tunnel at the rate Rs. 600 $m^3$

$= 38500 \times 600$

$= Rs.23100000$ ..... **[1/2M]**

Curved surface area of semicirlular tunnel

$= \frac{1}{2} \times 2\pi rh = \pi rh$ ..... **[1/2M]**

$= \frac{22}{7} \times 3.5 \times 2000$

$= 22 \times 0.5 \times 2000$

$= 22000\ m^2$ ..... **[1/2M]**

$\therefore$ Expenditure for plastering inner side of the tunnel at the rate of Rs.50 per sq.m.

$= 22000 \times 50$

$= Rs.1100000$ ..... **[1/2M]**

**Q. 5 Solve any two sub-questions.** **[10M]**

**Ans.1** Given :-

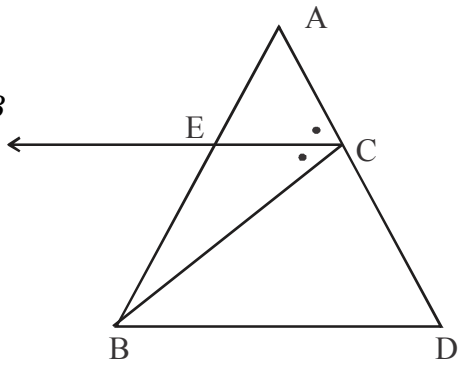
In $\Delta ABC$, fig. **[1/2M]**

ray CE bisects $\angle ACB$

To prove :-

$\frac{AE}{EB} = \frac{CA}{CB}$ **[1/2M]**

Construction :-




Draw a line parallel to ray $CE$,. passing through the point $B$. Extend $AC$ so as to meet it at $D$. **[1/2M]**

Proof : -

Line $CE$ || line $BD$ and $AD$ is its transversal.

$\angle ACE = \angle CDB$ ——— (1) [Converse of corresponding angle test].....**[1/2M]**

Line $CE$ || Line $BD$ and $BC$ as transversal

$\angle ECB = \angle CBD$ ——— (2) [Converse of alternate angles test] .....**[1/2M]**

But $\angle ACE = \angle ECB$ ——— (3) [given] ..... **[1/2M]**

But $\angle CBD = \angle CDB$ [from (1), (2) & (3)] ..... **[1/2M]**

In $\Delta CBD$

Side $CB \cong side\ CD$ ——— (4) [side opposite to congruent angles] ..... **[1/2M]**

In $\Delta ABD$

$seg\ EC \,||\, side\ BD$ $[Construction]$

$\frac{AE}{EB} = \frac{AC}{CD}$ ——— (5) [*B. P. T*] ..... **[1/2M]**

$\therefore \frac{AE}{EB} = \frac{AC}{CB}$ [from (4) & (5)] ..... **[1/2M]**

**Ans.2** $A \equiv (-2, 6) = (x_1, y_1)$

$B \equiv (3, -4) = (x_2, y_2)$

$P$ divides $seg\ AB$ internally in the ratio $2 : 3$

$m : n = 2 : 3$

Let $P \equiv (x, y)$

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{2(3)+3(-2)}{2+3} = \frac{0}{5} = 0$$ ..... **[1M]**

$$y = \frac{my_2 + ny_1}{m+n} = \frac{2(-4)+3(6)}{2+3} = \frac{10}{5} = 2$$ ..... **[1M]**

$P = (0, 2)$ ..... **[1/2M]**

Slope $= \frac{3}{2}$

Equation of the line by slope – point form

$y - y_1 = m(x - x_1)$ ..... **[1 M]**

$y - 2 = \frac{3}{2}(x - 0)$ ..... **[1/2M]**

$2(y-2) = 3(x)$

$2y - 4 = 3x$

$3x - 2y + 4 - 0$ ..... **[1/2M]**

$\therefore$ The equation of the required line is $3x - 2y + 4 = 0$ ..... **[1/2M]**

**Ans.3**

Analytical figure

(i) Analytical figure **[1M]**

(ii) Construct $\Delta AQB$ **[1M]**

(iii) Cutting 7 Parts & jointing $Q_7$ to B **[1M]**

(iv) Comtruct $-RQ_5 \| BQ_7$ **[1M]**

(v) Construct RP || AB **[1M]**