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Grand Test	HSC Examination Maths Code - Set - A	SOLUTIONS

SECTION - A

1. $h^2 - ab = 0$ (formula) **Ans. [B]** **[1 M]**

2. Let $\bar{a} = 3\hat{i} - 5\hat{j} + \hat{k}$ and $\bar{b} = 9\hat{i} - 15\hat{j} + p\hat{k}$ are collinear vector.
Then there exist $\lambda \in R$ such that
$\bar{a} = \lambda\bar{b}$
$3\hat{i} - 5\hat{j} + \hat{k} = \lambda(9\hat{i} - 15\hat{j} + p\hat{k})$
$9\lambda = 3, -5 = -15\lambda, 1 = \lambda p$ **Ans. [C]** **[1 M]**
$\lambda = \frac{1}{3}, \quad 1 = \frac{1}{3}p \Rightarrow p = 3$

3. Direction ratio of line AB are $1 + 4, 3 - 2, -2 - 3$ i.e. $5, 1, -5$ **Ans. [B]** **[1 M]**

4. $y = \sin^{-1}\left(\frac{2x}{1 + x^2}\right)$

Put $x = \tan\theta, \theta = \tan^{-1} x \quad \frac{2x}{1+x^2} = \frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$
$\therefore \quad y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1} x$
$\frac{dy}{dx} = \frac{2}{1+x^2}$ **Ans. [A]** **[1 M]**

5. $\int e^x(\sec x + \sec x \tan x)\,dx = e^x \cdot \sec x + c$ **Ans. [C]** **[1 M]**

6. 2, 3 **Ans. [D]** **[1 M]**

SECTION - B

7.

(1)	(2)	(3)	(4)
p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

3 column $\to$ 1 Marks
4 column $\to$ 1 Marks

8. $\cos x = -\frac{1}{2}$

We know that

$\cos\frac{\pi}{3} = \frac{1}{2}$

We know that

$\cos(\pi + \theta) = -\cos\theta, \qquad \cos(\pi - \theta) = -\cos\theta \qquad ...(1)$

Put $x = \frac{\pi}{3}$ in (1)

$\cos\left(\pi + \frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3}$ [1M]

$\cos\left(\frac{4\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right) = -\cos\frac{\pi}{3}$ [1 M]

$\frac{4\pi}{3}, \frac{2\pi}{3} \in [0, 2\pi)$

$\therefore \quad x = \frac{4\pi}{3},\ x = \frac{2\pi}{3}$ are principal solution. [1 M]

9. In Δ ABC

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1 + 3 - 4}{2 \times 1 \times \sqrt{3}} = 0^o \Rightarrow A = \frac{\pi^c}{2}$ [1 M]

$\cos B = \frac{c^2 + a^2 - b^2}{2ca} = \frac{3 + 4 - 1}{2 \times \sqrt{3} \times 2} = \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow B = \frac{\pi^c}{6}$

$\cos^2 A + \cos^2 B + \cos^2 c = 1$

$0 + \frac{3}{4} + \cos^2 c = 1$

$\cos^2 c = 1 - \frac{3}{4} = \frac{1}{4}$

$\cos = \frac{1}{2} \Rightarrow c = \frac{\pi}{3}$ [1 M]

10. Let a, b, c are direction ratios of a vector prependicular to the two lines having direction ratios are –2, 1, –1 and –3, –4, 1.

$-2a + b - c = 0$ ...(1)

$-3a - 4b + c = 0$ ...(2) [1 M]

By Cramers' rule

$$\frac{a}{\begin{vmatrix} 1 & -1 \\ -4 & 1 \end{vmatrix}} = \frac{-b}{\begin{vmatrix} -2 & -1 \\ -3 & 1 \end{vmatrix}} = \frac{c}{\begin{vmatrix} -2 & 1 \\ -3 & -4 \end{vmatrix}}$$

$$\frac{a}{1-4} = \frac{-b}{-2-3} = \frac{c}{8+3}$$

$$\frac{a}{-3} = \frac{-b}{-5} = \frac{c}{11}$$

$$\frac{a}{-3} = \frac{b}{5} = \frac{c}{11}$$

$\therefore$ $a = -3, b = 3, c = 11$ [1 M]

11. $y = x^{e^x}$

taking log on both side

$$\log y = \log x^{e^x}$$

$\log y = e^x \log x$

differentiating w.r.t. x

$$\frac{1}{y}\frac{dy}{dx} = e^x \frac{d}{dx}\log x + \log x \frac{d}{dx}e^x$$ [1 M]

$$\frac{dy}{dx} = y\left[e^x \cdot \frac{1}{x} + \log x \cdot e^x\right]$$

$$= y \cdot e^x\left[\frac{1}{x} + \log x\right]$$

$$= e^{e^x} \cdot e^x\left[\frac{1}{x} + \log x\right]$$ [1 M]

12. $f(x) = x^2 + 2x - 5$ [1 M]

diff w.r.t x

$f'(x) = 2x + 2$ [1 M]

f(x) is increasing

$\therefore$ $f'(x) > 0$

$2x + 2 > 0$

$2x > -2$

$x > -1$ [1 M]

13. $I = \int \frac{e^{x-1} + x^{e-1}}{e^x + x^e} dx$

Let $\theta = e^x + x^e$

$\frac{d\theta}{dx} = e^x + e \cdot x^{e-1}$

$d\theta = e(e^{x-1} + x^{e-1})\, dx$

$\frac{1}{e} d\theta = (e^{x-1} + x^{e-1})dx$ [1 M]

$\therefore \quad I = \int \frac{\frac{1}{e} d\theta}{\theta}$

$= \frac{1}{e} \int \frac{d\theta}{\theta}$

$= \frac{1}{e} \log|\theta| + c$

$= \frac{1}{e} \log|e^x + x^e| + c$ [1 M]

OR

$I = \int \frac{\sec\theta}{\sec\theta + \tan\theta} d\theta$

$= \int \frac{\sec\theta}{\sec\theta + \tan\theta} \times \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} d\theta$ [1 M]

$= \int \frac{\sec^2\theta - \sec\theta\tan\theta}{\sec^2\theta - \tan^2\theta} d\theta$

$= \int (\sec^2\theta - \sec\theta\,\tan\theta) d\theta$

$= \tan\theta - \sec\theta + c$ [1 M]

14. $\int_0^a (2x + 1)dx = 2$

$\left[\frac{2x^2}{2} + x\right]_0^a = 2$ [1 M]

$\left[x^2 + x\right]_0^a = 2$

$a^2 + a = 2$

$a^2 + a - 2 = 0$

$a^2 + 2a - a - 2 = 0$

$a(a + 2) - 1(a + 2) = 0$

$(a - 1)(a + 2) = 0$

$a - 1 = 0$ OR $a + 2 = 0$

$a = 1$ OR $a = -2$ [1 M]

SECTION - C

15. Equation of given lines are

$5x^2 - 8xy + 3y^2 = 0$

Comparing with $ax^2 + 2hxy + by^2 = 0$

$a = 5,\ 2h = -8,\ b = 3$

$a = 5,\ h = -4,\ b = 3$

Let m_1 and m_2 are slope of lines

$$m_1 + m_2 = \frac{-2h}{b},\quad m_1 m_2 = \frac{a}{b}$$

$$m_1 + m_2 = \frac{8}{3},\quad m_1 m_2 = \frac{5}{3}$$ [1 M]

Since required lines are $\perp^{ar}$ to given lines

$\therefore$ Slopes of required lines are $\frac{-1}{m_1}$ and $\frac{-1}{m_2}$.

Since required lines are passing through origin.

$\therefore$ Equation of lines are

$y = \frac{-1}{m_1}x$ and $y = \frac{-1}{m_2}x$

$m_1 y = -x$ and $m_2 y = -x$

$x + m_1 y = 0$ and $x + m_2 y = 0$ [1 M]

$\therefore$ Joint equation of lines is

$(x + m_1 y) \cdot (x + m_2 y) = 0$

$x^2 + m_2 xy + m_1 xy + m_1 m_2 y^2 = 0$

$x^2 + (m_1 + m_2)xy + m_1 m_2 y^2 = 0$

$$x^2 + \frac{8}{3}xy + \frac{5}{3}y^2 = 0$$

$3x^2 + 8xy + 5y^2 = 0$ [1 M]

This is required equation of lines.

16. Equation of lines are

$$L_1 : \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \quad ...(1)$$

$$L_2 : \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad ...(2)$$

From (1) and (2)

$x_1 = -1,\ y_1 = -1,\ z_1 = -1$

$a_1 = 7,\ b_1 = -6,\ c_1 = 1$

$x_2 = 3,\ y_2 = 5\ \ z_2 = 7$

$a_2 = 1,\ b_2 = -2,\ c_2 = 1$ [1 M]

$$\text{Shortest distance} = \left| \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(M_{11})^2 + (M_{12})^2 + (M_{13})^2}} \right|$$ [1 M]

$$= \left| \frac{\begin{vmatrix} 4 & 6 & 8 \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{\sqrt{(-4)^2 + (6)^2 + (-8)^2}} \right|$$

$$= \left| \frac{4(-4) - 6(6) + 8(-8)}{\sqrt{16 + 36 + 64}} \right|$$

$$= \left| \frac{-16 - 36 - 64}{\sqrt{116}} \right|$$

$$= \left| \frac{-116}{\sqrt{116}} \right|$$

$$= \left| -\sqrt{166} \right|$$

$$= \sqrt{166} \text{ unit}$$ [1 M]

17. Let $\bar{a} = \hat{i} + \hat{j} - 2\hat{k},\ \bar{b} = \hat{i} + 2\hat{j} + \hat{k},\ \bar{c} = 2\hat{i} - \hat{j} + \hat{k}$ are the three points through which plane passes.

$\therefore$ Equation of plane is

$(\bar{r} - \bar{a}) \cdot (\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}) = 0$ ...(1) [1 M]

$$\bar{a} \times \bar{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{vmatrix} = \hat{i}(1 + 4) - \hat{j}(1 + 2) + \hat{k}(2 - 1) = 5\hat{i} - 3\hat{j} + \hat{k}$$

$$\bar{b} \times \bar{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{vmatrix} = \hat{i}(3) - \hat{j}(-1) + \hat{k}(-5) = 3\hat{i} + \hat{j} - 5\hat{k}$$

$$\bar{c} \times \bar{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \hat{i}(1) - \hat{j}(-5) + \hat{k}(2 + 1) = \hat{i} + 5\hat{j} + 3\hat{k}$$

$(\bar{a} \times \bar{b}) + (\bar{b} \times \bar{c}) + (\bar{c} \times \bar{a}) = (5\hat{i} - 3\hat{j} + \hat{k}) + (3\hat{i} + \hat{j} - 5\hat{k}) + (\hat{i} + 5\hat{j} + 3\hat{k})$ [1 M]

$= 9\hat{i} + 3\hat{j} - \hat{k}$

From (1)

$[\bar{r} - (\hat{i} + \hat{j} - 2\hat{k})] \cdot (9\hat{i} + 3\hat{j} - \hat{k}) = 0$

$\bar{r} \cdot (9\hat{i} + 3\hat{j} - \hat{k}) - (9 + 3 + 2) = 0$

$\bar{r} \cdot (9\hat{i} + 3\hat{j} - \hat{k}) - 14 = 0$

$\bar{r} \cdot (9\hat{i} + 3\hat{j} - \hat{k}) = 14$ [1 M]

OR

Equation of lines are

$L_1 : \frac{x-2}{1} = \frac{y-4}{4} = \frac{z-6}{7}$ ...(1)

$L_2 : \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ ...(2)

From (1)

$x_1 = 2,\quad y_1 = 4,\quad z_1 = 6,\quad a_1 = 1,\quad b_1 = 4,\quad c_1 = 7$

$x_2 = -1,\quad y_2 = -3,\quad z_2 = -5,\quad a_2 = 3,\quad b_2 = 5,\quad c_2 = 7$ [1 M]

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$= \begin{vmatrix} 2-(-1) & 4-(-3) & 6+5 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 7 & 11 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix}$$ [1 M]

$= 3(28 - 35) - 7(7 - 21) + 11(5 - 12)$

$= 3(-7) - 7(-14) + 11(-7)$

$= -21 + 98 - 77$

$= -98 + 98$

$= 0$ [1 M]

$\therefore$ Lines L_1 and L_2 are coplanar.

Equation of plane containing L_1 and L_2 are

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-2 & y-4 & z-6 \\ 1 & 4 & 7 \\ 3 & 5 & 7 \end{vmatrix} = 0$$

$(x-2)(28-35)-(y-4)(7-21)+(z-6)(5-12)=0$

$(x-2)(-7)-(y-4)(-14)+(z-6)(-7)=0$

$-7x+14+14y-56-7z+42=0$

$-7x+14y-7z=0$

$x-2y+z=0$ [1 M]

This is required equation of plane.

18. Given :

$x^5 \cdot y^7 = (x+y)^{12}$

taking log on both side, we get

$\log(x^5 \cdot y^7) = \log(x+y)^{12}$ [1 M]

$\log x^5 + \log y^7 = 12\log(x+y)$

$5\log x + 7\log y = 12\log(x+y)$

differentiating w.r.t. x

$$5\cdot\frac{1}{x} + 7\times\frac{1}{y}\frac{dy}{dx} = \frac{12}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\frac{5}{x} + \frac{7}{y}\frac{dy}{dx} = \frac{12}{x+y} + \frac{12}{x+y}\frac{dy}{dx}$$ [1 M]

$$\frac{7}{y}\frac{dy}{dx} - \frac{12}{x+y}\frac{dy}{dx} = \frac{12}{x+y} - \frac{5}{x}$$

$$\left(\frac{7}{y} - \frac{12}{x+y}\right)\frac{dy}{dx} = \left(\frac{12}{x+y} - \frac{5}{x}\right)$$

$$\frac{7x+7y-12y}{y(x+y)}\frac{dy}{dx} = \frac{12x-5x-5y}{x(x+y)}$$

$$\frac{7x-5y}{y}\frac{dy}{dx} = \frac{7x-5y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$ [1 M]

OR

Let $u = \cos^{-1}(\sin x)$ $\quad v = \tan^{-1} x$

$u = \frac{\pi}{2} - x$ $\quad v = \tan^{-1} x$ [1 M]

$\frac{du}{dx} = -1$ $\quad \frac{dv}{dx} = \frac{1}{1+x^2}$ [1 M]

By parametric differentiation

$$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-1}{1/(1+x^2)} = -(1+x^2)$$ [1 M]

19.

x	P(x)
–2	0.1
–1	k
0	0.2
1	2k
2	0.3
3	k
	0.6 + 4k

x	P(x)	x P(x)	x^2 P(x)
–2	0.1	–0.2	0.4
–1	0.1	–0.1	0.1
0	0.2	0	0
1	0.2	0.2	0.2
2	0.3	0.6	1.2
3	0.1	0.3	0.9
		E(x) = 0.8	$E(x^2) = 2.8$

Since P is p.m.t.

$\therefore \quad \sum P_i = 1$

$0.6 + 4k = 1$

$4k = 1 - 0.6$

$4k = 0.4$

$k = 0.1$

$\therefore \quad E(x) = 0.8$

$V(x) = E(x^2) - (E(x))^2$

$= 2.8 - (0.8)^2$

$= 2.8 - 0.64$

$= 2.16$

20. Experiment : Hitting a target in 10 shots.

X : number of shots hit the target

P : Probability that a short hit the target

$p = 0.2,\ n = 10,\ q = 1 - P = 0.8$

$\therefore \quad X \sim B(10, 0.2)$

p.m.t. of x is

$P(x = r) = {}^{10}c_r\, (0.2)^r\, (0.8)^{10-r}$ ..(1)

Probability that target will be hit at least twice

$= P(x \geq 2)$

$= 1 - P(x < 2)$

$= 1 - \{P(x = 0) + P(x = 1)\}$

$= 1 - \{{}^{10}c_0\, (0.2)^0\, (0.8)^{10} + {}^{10}c_1\, (0.2)^1\, (0.8)^{10-1}\}$

$= 1 - \{(0.8)^{10} + (10 \times 0.2 \times 0.8^9)\}$

$= 1 - \{(0.8)^{10} + 2 \times (0.8)^9\}$

$= 1 - (0.8)^9\, (0.8 + 2)$

$= 1 - (0.8)^9\, (2.8)$

$= 1 - 0.3758$

$= 0.6242$

SECTION - D

21. p : switch s_1 is closed [1 M]

q : switch s_2 is closed

Symbolic form :

$(p \wedge \sim q) \vee (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ [1 M]

$= (p \wedge \sim q) \vee [(\sim p) \wedge (q \vee \sim q)]$ Distributive law

$= (p \wedge \sim q) \vee [\sim p \wedge T]$ Complement law

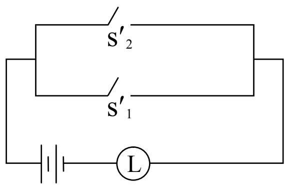
$= (p \wedge \sim q) \vee \sim p$ Identity law

$= (p \vee \sim p) \wedge (\sim q \vee \sim p)$ Distributive law

$= T \wedge (\sim q \vee \sim p)$ Complement law

$= \sim q \vee \sim p$ Identity law [1 M]

$\therefore$ simplified form : $\sim q \vee \sim p$



[1 M]

22. Let cost of one dozen pencil, pen and eraser are Rs x, y and z.

Given that

$4x + 3y + 2z = 60$

$2x + 4y + 6z = 90$

$6x + 2y + 3z = 70$

Matrix form

$$\begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$ **[1 M]**

$\left(\frac{1}{4}R_1\right)$

$$\begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 90 \\ 70 \end{bmatrix}$$

$R_2 - 2R_1,\ R_3 - 6R_1$

$$\begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} \\ 0 & \frac{5}{2} & 5 \\ 0 & -\frac{5}{2} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 60 \\ -20 \end{bmatrix}$$

R_2 $2R_1$	2 –2	4 $-\frac{3}{2}$	6 –1	90 –30
	0	$\frac{5}{2}$	5	60

R_3 $6R_1$	6 –6	2 $-\frac{9}{2}$	3 –3	70 –90
	0	$-\frac{5}{2}$	0	–20

[1M]

$R_3 + \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & \frac{3}{4} & \frac{1}{2} \\ 0 & \frac{5}{2} & 5 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 15 \\ 60 \\ 40 \end{bmatrix}$$ [1 M]

$\therefore \quad x + \frac{3}{4}y + \frac{z}{2} = 15 \quad$ ...(1)

$\frac{5}{2}y + 5z = 60 \quad$ ...(2)

$5z = 40 \quad$ ...(3)

From (3)

$\Rightarrow$ z = 8

From (2)

$\frac{5}{2}y + 40 = 60$

$\frac{5}{2}y = 60 - 40$

$\frac{5}{2}y = 20$

$y = \frac{40}{5} = 8$

From (1)

$x + \frac{3}{4} \times 8 + \frac{8}{2} = 15$

$x + 6 + 4 = 15$

$x + 10 = 15$

$x = 15 - 10$

$x = 5$

Thus cost of one dozen pencil, pen and eraser are Rs 5, 8 and 8. [1 M]

OR

Step 1 : Standard form

$x + y + z = -1$

$x - y + z = 2$

$x + y - z = 3$

Step 2 : Matrix form

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$ [1 M]

A X = B ...(1)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$ [1 M]

Step 3 : Calculation of A^{-1}

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$= 1(1-1) - 1(-1-1) + 1(1+1)$

$= 0 + 2 + 2$

$= 4$

$\neq 0$

(2) $A \cdot A^{-1} = I$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$, $R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$-\frac{1}{2}R_2, -\frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ +\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$R_1 - R_2$,

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} \frac{1}{2} & +\frac{1}{2} & 0 \\ +\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$

$R_1 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & +\frac{1}{2} & +\frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$ [1 M]

$$A^{-1} = \begin{bmatrix} 0 & +\frac{1}{2} & \frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}$$ [1 M]

(3) From (1)

$A X = B$

$A^{-1} A X = A^{-1} B$

$X = A^{-1} B$

$$= \begin{bmatrix} 0 & +\frac{1}{2} & \frac{1}{2} \\ +\frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & +1 & \frac{3}{2} \\ -\frac{1}{2} & -1 & +0 \\ -\frac{1}{2} & +0 & -\frac{3}{2} \end{bmatrix}$$

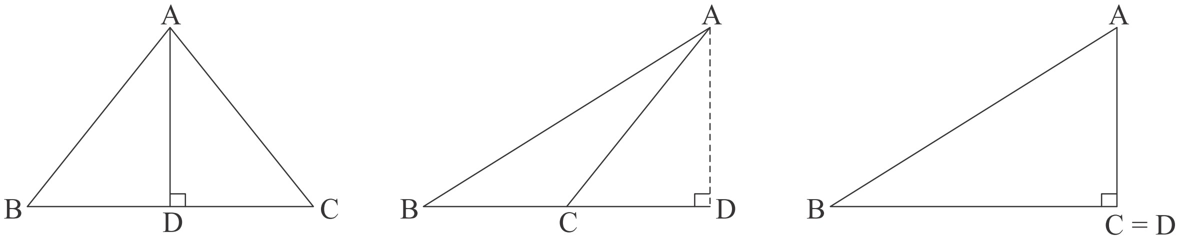
$$= \begin{bmatrix} \frac{5}{2} \\ -\frac{3}{2} \\ -2 \end{bmatrix}$$

$= \frac{5}{2},\ y = -\frac{3}{2},\ z = -2$ [1 M]

23. In Δ ABC, all angles are not obtuse.

Let $\angle$ B be acute. There are three cases on $\angle$ C.

(i) $\angle$ C can be acute (ii) $\angle$ C can be obtuse (iii) $\angle C = 90^\circ$



(1) (2) (3)

From (1), (2) and (3)

$\sin B = \frac{AD}{AB} \Rightarrow \sin B = \frac{AD}{c} \Rightarrow AD = c \sin B$ ...(1) [1 M]

From (1)

$\sin C = \frac{AD}{AC} \Rightarrow \sin C = \frac{AD}{b} \Rightarrow AD = b \sin C$ ...(2)

From (2)

$\sin(\pi - C) = \frac{AD}{AC} \Rightarrow \sin C = \frac{AD}{b} \Rightarrow AD = b \sin C$ ...(3)

From (3)

$\sin C = \sin 90^\circ = \frac{AD}{AD} = \frac{AD}{AC} \Rightarrow \sin C = \frac{AD}{b} \Rightarrow AD = b \sin C$ ...(4)

[1M]

From (1), (2), (3) and (4)

$c \sin B = b \sin C \quad \Rightarrow \quad \frac{b}{\sin B} = \frac{c}{\sin C}$ ...(5) [1 M]

$\|_y$ we can show that

$\frac{a}{\sin A} = \frac{b}{\sin B}$ ...(6)

From (5) and (6)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ [1 M]

24. Given $A(\bar{a}), B(\bar{b}), C(\bar{c})$ and $D(\bar{d})$ are coplanar.

$\Rightarrow \quad \overline{AB}\ \overline{AC}$ and $\overline{AD}$ are coplanar.

$\therefore \quad [\overline{AB}\ \overline{AC}\ \overline{AD}] = 0$ [1 M]

$[\bar{b}-\bar{a} \quad \bar{c}-\bar{a} \quad \bar{d}-\bar{a}] = 0$

$[\bar{b} \quad \bar{c}-\bar{a} \quad \bar{d}-\bar{a}] - [\bar{a} \quad \bar{c}-\bar{a} \quad \bar{d}-\bar{a}] = 0$

$[\bar{b} \quad \bar{c} \quad \bar{d}-\bar{a}] - [\bar{b} \quad \bar{a} \quad \bar{d}-\bar{a}] - [\bar{a} \quad \bar{c} \quad \bar{d}-\bar{a}] + [\bar{a} \quad \bar{a} \quad \bar{d}-\bar{a}] = 0$ [1 M]

$[\bar{b} \quad \bar{c} \quad \bar{d}] - [\bar{b} \quad \bar{c} \quad \bar{a}] - [\bar{b} \quad \bar{a} \quad \bar{d}] + [\bar{b} \quad \bar{a} \quad \bar{a}]$

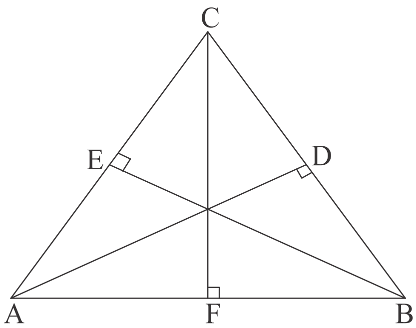
$- [\bar{a} \quad \bar{c} \quad \bar{d}] + [\bar{a} \quad \bar{c} \quad \bar{a}] + [\bar{a} \quad \bar{a} \quad \bar{d}-\bar{a}] = 0$

$[\bar{b} \quad \bar{c} \quad \bar{d}] - [\bar{b} \quad \bar{c} \quad \bar{a}] - [\bar{b} \quad \bar{a} \quad \bar{d}] - 0 - [\bar{a} \quad \bar{c} \quad \bar{d}] + 0 + 0 = 0$ [1 M]

$[\bar{b} \quad \bar{c} \quad \bar{d}] + [\bar{a} \quad \bar{b} \quad \bar{d}] + [\bar{c} \quad \bar{a} \quad \bar{d}] = [\bar{b} \quad \bar{c} \quad \bar{a}]$

$[\bar{b} \quad \bar{c} \quad \bar{d}] + [\bar{a} \quad \bar{b} \quad \bar{d}] + [\bar{c} \quad \bar{a} \quad \bar{d}] = [\bar{a} \quad \bar{b} \quad \bar{c}]$ [1 M]

OR



Let A, B and C be the vertices of a triangle.

Let AD, BE and CF be the altitudes of the triangle ABC, therefore AD ⊥ BC, BF ⊥ AC, CF ⊥ AB. [1 M]

Let $\bar{a}, \bar{b}, \bar{c}, \bar{d}, \bar{e}, \bar{f}$ be the position vectors of A, B, C, D, E, F respectively. Let P be the point of intersection of the altitudes AD BE with $\bar{P}$ as the position vectors. [1 M]

Therefore $\overline{AP} \perp \overline{BC}$, $\overline{BP} \perp \overline{AC}$ ...(1)

To show that the altitudes AD, BE and CF are concurrent, It is sufficient to show that the altitude CF passes through the point P. We will have to prove that $\overline{CF}$ and $\overline{CP}$ are collinear vectors. This can be achieved by showing $\overline{CP}$ and $\overline{AP}$.

Now from (1) we have

$\overline{AP} \perp \overline{BC}$ and $\overline{BP} \perp \overline{AC}$

$\overline{AP} \cdot \overline{BC} = 0$ and $\overline{BP} \cdot \overline{AC} = 0$

$(\bar{p} - \bar{a}) \cdot (\bar{c} - \bar{b}) = 0$ [1 M]

$(\overline{p} - \overline{b})\cdot(\overline{c} - \overline{a}) = 0$

$\overline{p} \cdot \overline{c} - \overline{p}\cdot\overline{b} - \overline{a}\cdot\overline{c} + \overline{a}\cdot\overline{b} = 0$ ...(2)

$\overline{p} \cdot \overline{c} - \overline{p}\cdot\overline{a} - \overline{b}\cdot\overline{c} + \overline{b}\cdot\overline{a} = 0$ ...(3)

Therefore, subtracting equation (2) from equation (3), we get,

$-\overline{p} \cdot \overline{a} + \overline{p}\cdot\overline{b} - \overline{b}\cdot\overline{c} + \overline{a}\cdot\overline{c} = 0$

$\overline{p}(\overline{b} - \overline{a}) - \overline{c}(\overline{b} - \overline{a}) = 0$

$(\overline{p} - \overline{c}) \cdot (\overline{b} - \overline{a}) = 0$

$\overline{CP} \cdot \overline{AB} = 0$

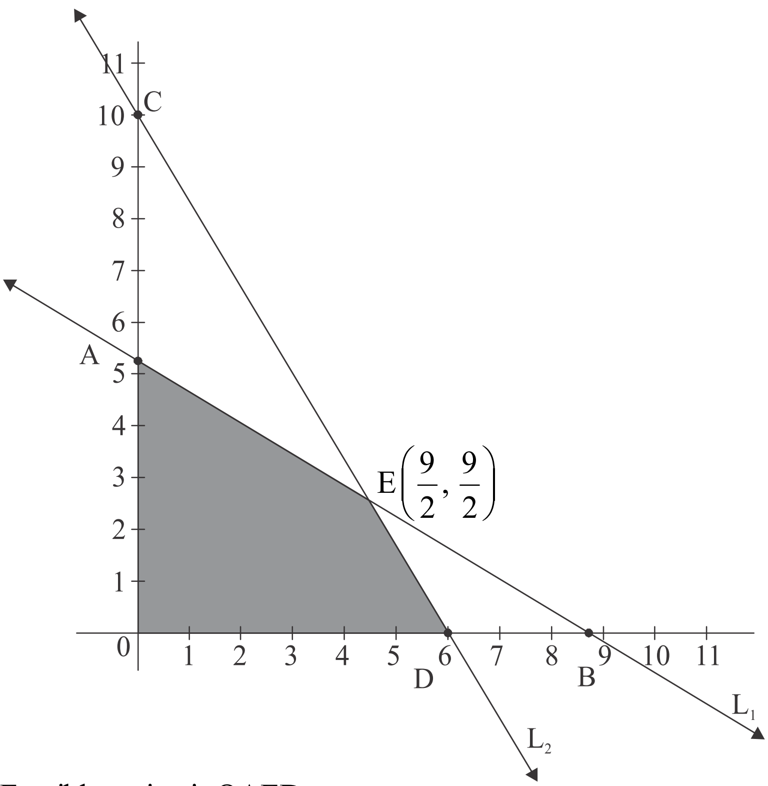
$\overline{CP} \perp \overline{AB}$

Hence the proof. [1 M]

25. Step 1 :

<table>
<tr><th>Inequation</th><th>Equation</th><th>Points</th><th>Region</th></tr>
<tr><td>$3x + 5y \leq 26$</td><td>$3x + 5y = 26$
L_1</td><td><table><tr><td>x
y</td><td>0
5.2</td><td>8.6
0</td></tr><tr><td>(x, y)</td><td>(0, 5.2)
A</td><td>(8.6, 0)
B</td></tr></table></td><td>Towards origin.</td></tr>
<tr><td>$5x + 3y \leq 30$</td><td>$5x + 3y = 30$
L_2</td><td><table><tr><td>x
y</td><td>0
10</td><td>6
0</td></tr><tr><td>(x, y)</td><td>(0, 10)
C</td><td>(6, 0)
D</td></tr></table></td><td>Towards origin</td></tr>
<tr><td>$x \geq 0$</td><td>$x = 0$</td><td>y-axis</td><td>+ve x-axis</td></tr>
<tr><td>$y \geq 0$</td><td>$y = 0$</td><td>x-axis</td><td>+ve y-axis</td></tr>
</table>

[1 M]



[2 M]

Feasible region is OAED

Points $z = 7x + 11y$

A(0, 5.2) $z = 0 + 57.2 = 57.2$

$E\left(\frac{9}{2}, \frac{9}{2}\right)$ $\quad z = \frac{63}{2} + \frac{99}{2} = \frac{162}{2} = 81$

$D(6, 0)$ $\quad z = 42 + 0 = 42$

$Z_{Max} = 81$ at $\left(\frac{9}{2}, \frac{9}{2}\right)$

$\therefore \quad x = \frac{9}{2}, y = \frac{9}{2}$ are optimal solution. [1 M]

26. $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \qquad x \neq \frac{\pi}{3}$

$= k \qquad x = \frac{\pi}{3}$

Step 1 :

$$\lim_{x \to \frac{\pi}{3}} f(x) = \lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad ...(1)$$

put $x - \frac{\pi}{3} = t$

$x = \frac{\pi}{3} + t$

as $x \to \frac{\pi}{3} \quad t \to 0$

$-3x + \pi = -3t$

$\pi - 3x = -3t$ [1M]

$\therefore$ From (1)

$$\lim_{x \to \frac{\pi}{3}} f(x) = \lim_{t \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} + t\right)}{-3t}$$

$$= \lim_{t \to 0} \frac{\sqrt{3} - \left(\frac{\tan\frac{\pi}{3} + \tan t}{1 - \tan\frac{\pi}{3}\tan t}\right)}{-3t}$$

$$= \lim_{t \to 0} \frac{\sqrt{3} - \frac{\sqrt{3} + \tan t}{1 - \sqrt{3}\tan t}}{-3t}$$ [1M]

$= \lim_{t \to 0} \dfrac{\sqrt{3} - 3\tan t - \sqrt{3} - \tan t}{-3t}$

$= \lim_{t \to 0} \dfrac{-4\tan t}{-3t}$

$= \dfrac{4}{3}\lim_{t \to 0}\dfrac{\tan t}{t}$

$= \dfrac{4}{3} \times 1$

$= \dfrac{4}{3}$ [1 M]

Step 2 :

$f\left(\dfrac{\pi}{3}\right) = k$

Step 3 :

f(x) is at continuous at $x = \dfrac{\pi}{3}$

$\therefore \lim_{x \to \frac{\pi}{3}} f(x) = f\left(\dfrac{\pi}{3}\right)$

$\dfrac{4}{3} = k \Rightarrow k = \dfrac{4}{3}$ [1 M]

27. Equation of curve is

$y = x^2 + 4x + 1$ ...(1)

Differentiating w.r.t. x.

$\dfrac{dy}{dx} = 2x + 4$

$\therefore \left.\dfrac{dy}{dx}\right|_{(-1, -2)} = -2 + 4 = 2$ [1 M]

$\therefore$ Slope of tangent at $(-1, -2) = m = 2$

$\therefore$ Equation of tangent at $(-1, -2)$ is

$y + 2 = m(x + 1)$

$y + 2 = 2(x + 1)$

$y + 2 = 2x + 1$

$2x - y + 1 - 2 = 0$

$2x - y - 1 = 0$ [1 M]

This is the required equation of tangent

Slope of normal at $(-1, -2) = \dfrac{-1}{m} = \dfrac{-1}{2}$ [1 M]

$\therefore$ Equation of normal at $(-1, -2)$ is

$y + 2 = \dfrac{-1}{2}(x + 1)$

$2y + 4 = -x - 1$

$x + 2y + 4 + 1 = 0$

$x + 2y + 5 = 0$ [1 M]

This is the required equation of normal.

28. $\int \frac{3x+1}{(x-2)^2(x+2)} dx$

$$\frac{3x+1}{(x-2)^2(x+2)} = \frac{A}{(x-2)^2} + \frac{B}{(x-2)} + \frac{C}{(x+2)} \quad ...(1)$$

$$= \frac{A(x+2) + B(x-2)(x+2) + C(x-2)^2}{(x-2)^2(x+2)} \quad [1\text{ M}]$$

Comparing we get

$3x + 1 = A(x+2) + B(x-2)(x+2) + C(x-2)^2$

Let $x = 2$

$6 + 1 = 4A \Rightarrow 7 = 4A \Rightarrow A = \frac{7}{4}$

Let $x = -2$

$-6 + 1 = C(-4)^2 \Rightarrow -5 = 16C \Rightarrow C = \frac{-5}{16}$

Let $x = 0$

$1 = 2A - 4B + 4C$

$4B = 2A + 4C - 1$

$= 2 \times \frac{7}{4} + 4\left(\frac{-5}{16}\right) - 1$

$= \frac{7}{2} - \frac{5}{4} - 1$

$B = \frac{7}{8} - \frac{5}{16} - \frac{1}{4}$

$= \frac{14 - 5 - 4}{16}$

$= \frac{14 - 9}{16}$

$= \frac{5}{16}$

$\therefore$ From (1) [2 M]

$$\frac{3x+1}{(x-2)^2(x+2)} = \frac{\frac{7}{4}}{(x-2)^2} + \frac{\frac{5}{16}}{(x-2)} + \frac{\frac{-5}{16}}{(x+2)}$$

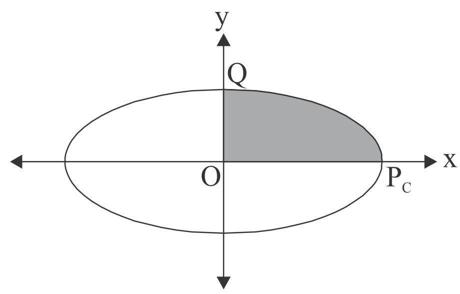
Integrating on both side

$$\int \frac{3x+1}{(x-2)^2(x+2)} dx = \frac{7}{4}\int \frac{1}{(x-2)^2} dx + \frac{5}{16}\int \frac{1}{(x-2)} dx - \frac{5}{16}\int \frac{1}{(x+2)}$$

$$= \frac{7}{4}\left(\frac{-1}{x-2}\right) + \frac{5}{16}\log|x-2| - \frac{5}{16}\log|x+2| + c \quad [1\text{ M}]$$

$$= \frac{-7}{4(x-2)} + \frac{5}{16}\log|x-2| - \frac{5}{16}\log|x+2| + c$$

29. By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO. For the region, the limits of integration are x = 0 and x = a.
From the equation



$$\frac{x^2}{4}+\frac{y^2}{25}=1$$

$$\frac{y^2}{25}=1-\frac{x^2}{4}$$

$$y^2=25\left[\frac{4-x^2}{4}\right]$$

$$y^2=\frac{25}{4}(4-x^2)$$

$$y=\pm\frac{5}{2}\sqrt{4-x^2}$$ [1 M]

In first quadrant, y > 0

$$\therefore \quad y=\frac{5}{2}\sqrt{4-x^2}$$

$$\therefore \quad A=4\int_0^2 y\,dx$$ [1 M]

$$=4\int_0^2\frac{5}{2}\sqrt{4-x^2}\,dx$$

$$=2\times 5\int_0^2\sqrt{4-x^2}\,dx$$

$$=10\left[\frac{x}{2}\sqrt{4-x^2}+\frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$ [1 M]

$$=20\times\frac{\pi}{2}$$

$= 10\pi$ sq. units [1 M]

30. Let θ be the temperature of body at a time t. Room temperature is given to be 25ºC.
By Newton’s laws of cooling, we have

$$\frac{d\theta}{dt}\propto(\theta-25)$$ [1 M]

$$\frac{d\theta}{dt}=-k(\theta-25)\quad k>0$$

$$\frac{d\theta}{\theta-25}=-kdt\qquad k>0$$

Integrating on both side

$$\int\frac{d\theta}{\theta-25}=-k\int dt$$

$\log|\theta - 25| = -kt + c$ ...(1) [1 M]

Initially $t = 0, \theta = 80^\circ C$

$\log|80 - 25| = -0 + c$

$c = \log 55^\circ$

$\therefore$ From (1)

$\log|\theta - 25| = -kt + \log 55^\circ$

$\log|\theta - 25| - \log 55^\circ = -kt$

$$\log\left|\frac{\theta - 25}{55}\right| = -kt \quad ..(2)$$ [1 M]

Put $t = 30$ mm, $\theta = 50^\circ C$

$$\log\left|\frac{50 - 25}{55}\right| = -k \times 30$$

$$-k = \frac{1}{30}\log\left|\frac{25}{55}\right| = \frac{1}{30}\log\left|\frac{5}{11}\right|$$

$\therefore$ From (2)

$$\log\left|\frac{\theta - 25}{55}\right| = \frac{t}{30}\log\left|\frac{5}{11}\right| \quad ...(3)$$

Put $t = 1$ hrs $= 60$ min

$$\log\left|\frac{\theta - 25}{55}\right| = \frac{60}{30}\log\left|\frac{5}{11}\right|$$

$$\log\left|\frac{\theta - 25}{55}\right| = 2\log\left|\frac{5}{11}\right|$$

$$\log\left|\frac{\theta - 25}{55}\right| = \log\left|\frac{5^2}{11^2}\right|$$

$$\frac{\theta - 25}{55} = \frac{5 \times 5}{11 \times 11}$$

$$\theta - 25 = \frac{5 \times 5 \times 55}{11 \times 11}$$

$$\theta - 25 = \frac{125}{11}$$

$$\theta = \frac{125}{11} + 25$$

$$\theta = \frac{125 + 275}{11}$$

$$\theta = \frac{400}{11}$$

$\theta = 36.36^\circ C$ [1 M]

OR

$$(1 + e^{x/y})\,dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$

Let $\frac{x}{y} = v$ [1 M]

$x = vy$

$dx = vdy + ydv$ [1 M]

$\therefore$ $(1 + e^v)(v\,dy + y\,dv) + e^v(1 - v)\,dy = 0$

$(1 + e^v)\,v\,dy + y(1 + e^v)\,dv + e^v(1 - v)\,dy = 0$

$(v + ve^v + e^v - ve^v)\,dy + y(1 + e^v)\,dv = 0$

$(v + e^v)\,dy = -y(1 + e^v)\,dv$

Integrating on both side

$$\int \frac{dy}{y} = -\int \frac{1 + e^v}{v + e^v}\,dv$$ [1 M]

$\log|y| = -\log|v + e^v| + \log|c|$

$\log|y| + \log|v + e^v| = \log|c|$

$\log|y(v + e^v)| = \log|c|$

$y(v + e^v) = c$

$$y\left[\frac{x}{y} + e^{x/y}\right] = c$$

$x + ye^{x/y} = c$ [1 M]