GuruAanklan

Grand Test	HSC EXAMINATION SET - A MATHEMATICS	M.Marks : 80 Duration : 3 Hrs

Notes :
(i) All questions are compulsory.
(ii) Figures to the right indicate full marks.
(iii) Solution of LPP should be written on graph paper only.
(iv) Answers to both the sections should be written in the same answer book.
(v) Answer to every new question must be written on a new page.

SECTION – I

Q.1 (A) Select and write the correct answer from the given alternative in each of the following. (6)[12]

(i) If p, q, r are the statements with truth values T,F,T respectively. Then the truth value of $(p \vee q) \rightarrow (q \vee r)$ is (2)

a) T b) F c) T or F d) T and F

(ii) Find the value of 'k' if the following equation represents a pair of lines

$$3x^2 + 10xy + 3y^2 + 16y + k = 0$$ (2)

(a) 12 b) -12 c) 21 d) -21

(iii) Find the principal solution of the equation $\tan x = \sqrt{3}$ (2)

a) $\frac{\pi}{4}, \frac{3\pi}{4}$ b) $\frac{\pi}{3}, \frac{4\pi}{3}$ c) $\frac{\pi}{4}, \frac{5\pi}{4}$ d) $\frac{\pi}{2}, \frac{3\pi}{3}$

(B) Attempt any THREE of the following : (6)

(i) Find the distance between the parallel planes $\bar{r}.\left(2\bar{i} - 3\bar{j} + 6\bar{k}\right) = 5$ and $\bar{r}.\left(6\bar{i} - 9\bar{j} + 18\bar{k}\right) + 20 = 0$. (2)

(ii) The angle between the lines represented by $4x^2 + 5xy + y^2 = 0$ (2)

(iii) Write the negation of $r \rightarrow (\sim p \wedge q)$ (2)

(iv) The position vectors of the points A and B are $2\bar{i} - \bar{j} + 5\bar{k}$ and $-3\bar{i} + 2\bar{j}$ respectively. Find the position vector of the point which divides the line segment AB in the ratio 1 : 4 internally (2)

(v) The Cartesian equation of a line is $\frac{x+5}{3} = \frac{y+4}{5} = \frac{z+5}{6}$. Write its vector form. (2)

Q. 2 (A) Attempt any TWO of the following : **(6)[14]**

(i) Solve the given equations by the method of inversion : 2x + 3y = −5, 3x + y = 3 . (3)

(ii) Show that the lines and x + y = 10 contain the sides of an equilateral triangle. (3)

(iii) If the lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{4}$ *and* $\frac{x-3}{1}=\frac{y-k}{2}=\frac{z}{1}$ intersect, then find the value of ‘k’ . (3)

(B) Attempt any TWO of the following : **(8)**

(i) Determine whether the following statement pattern is a tautology or a contradiction or a contingency :

$[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$ (4)

(ii) In Δ ABC, if cos A = sin B – cos C then show that it is a right angled triangle. (4)

(iii) If $[\bar{u}]=3$ and vector $\bar{u}$ is equally inclined to the unit vectors $\bar{i}, \bar{j}$ *and* $\bar{k}$, find $\bar{u}$. (4)

Q. 3 (A) Attempt any TWO of the following : **(6) [14]**

(i) The cost of 4 dozen pencils, 3 dozen pens and 2 dozen erasers is Rs.60. The cost of 2 dozen pencils, 4 dozen pens and 6 dozen erasers is Rs 90 whereas the cost of 6 dozen pencils,2 dozen pens and 3 dozen erasers is Rs70. Find the cost of each item per dozen . (3)

(ii) Express $-\bar{i}-3\bar{j}+4\bar{k}$ as the linear combination of the vectors $2\bar{i}+\bar{j}-4\bar{k}$, $2\bar{i}-\bar{j}+3\bar{k}$ & $3\bar{i}+\bar{j}-2\bar{k}$ (3)

(iii) Prove that $(\bar{a}+2\bar{b}-\bar{c}).\left[(\bar{a}-\bar{b}) \text{ x } (\bar{a}-\bar{b}-\bar{c})\right]=3[\bar{a} \quad \bar{b} \quad \bar{c}]$ (3)

(B) Attempt any TWO of the following : **(8)**

(i) Maximize Z = 10 x + 25y subject to 0 ≤ x ≤ 3, 0≤y ≤ 3, x + y ≤ 5.Find the maximum value of z. (4)

(ii) Show that the line of intersection of the planes $\bar{r} \,.\, (\bar{i}+3\bar{j}-2\bar{k})=0$ and $\bar{r} \,.\, (2\bar{i}+4\bar{j}-3\bar{k})=0$ is equally inclined to $\bar{i}$ & $\bar{j}$. Also find the angle with which it makes with k. (4)

(iii) Show that $\frac{9\text{®}}{8}-\frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)=\frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$ (4)

SECTION – II

Q. 4 (A) Select and write the correct answer from the given alternatives in each of the following . (6)[12]

(i) If E (X) = 5 var (X) = 2.5 and $X \sim B(n, p)$, then find 'n' . (2)

a) 5 b) 15 c)10 d)20

(ii) Find 'k' if the function

$$\begin{aligned} f(x) &= k.x^2.(1-x),\ 0 < x < 1 \\ &= 0,\quad \text{otherwise} \end{aligned}$$ is p.d.f. of r.v. X. (2)

a) 12 b) 21 c) 15 d) 2

(iii) $y = ae^{x} + be^{-3x}$ is a solution of (2)

a) $\frac{d^2y}{dx^2} + y = 0$ b) $\frac{d^2y}{dx^2} + xy\frac{dy}{dx} + y = 0$ c) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$ d) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

(B) Attempt any THREE of the following : (6)

(i) Examine the continuity of the following functions :

$$\begin{aligned} f(x) &= \frac{3^x + 3^{-x} - 2}{x^2}, \quad for \quad x \neq 0 \\ &= (\log 3)^2, \quad for \quad x \neq 0, \qquad at \quad x = 0 \end{aligned}$$ (2)

(ii) If $u = e^{\log\cos 4x}$ & $v = e^{\log\sin 4x}$ show that $\frac{du}{dv} = \frac{-u}{v}$ (2)

(iii) Solve the differential equation $1 + \frac{dy}{dx} = \operatorname{cosec}(x+y)$, (2)

(iv) If $y = \cot^{-1}\left(\frac{1-3x^2}{3x - x^3}\right)$ then find $\frac{dy}{dx}$. (2)

(v) Evaluate : $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}}\ dx$ (2)

Q. 5.(A) Attempt any TWO of the following : (6) [14]

(i) Evaluate : $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + x^2 + a^2\right| + c$ (3)

(ii) Evaluate $\int x^5.\sqrt{a^3 + x^3}\,dx$ (3)

(iii) If mean of a binomial distribution is 3 and variance is $\frac{3}{2}$, find the probability of at least 4 successes. (3)

(B) Attempt any TWO of the following : (8)

(i) If f (x) is continuous on (0, 8), where

$$\begin{aligned} f(x) &= x^2 + ax + 6, \quad \text{for} \quad 0 \le x < 2 \\ &= 3x + 2, \quad \text{for} \quad 2 \le x < 4 \\ &= 2ax + 5b, \quad \text{for} \quad 4 < x \le 8.. \quad \text{Find a and b} \end{aligned}$$

(4)

(ii) Solve, $\frac{dy}{dx} + 2y\tan x = \sin x$, given that y =0, When $x = \frac{\text{®}}{3}$, (4)

(iii) Determine the maximum & minimum value of $f(x) = x^2 + \frac{16}{x^2}$ (4)

Q. 6 (A) Attempt any TWO of the following : (8) [14]

(i) Evaluate : $\int \frac{2x^3 + 3x^2 - 3}{2x^2 - x - 1} \, dx =$ (3)

(ii) Evaulate: $\int_0^{\text{®}/4} \frac{dx}{3\cos 2x + 5} \, dx$ (3)

(iii) Evaluate : $\int_{\text{®}/2}^{\text{®}} e^x \left(\frac{1 - \sin x}{1 - \cos x} \right) dx$ (3)

(B) Attempt any TWO of the following : (8)

(i) The p.d.f of continuous random variable X is given by

$$\begin{aligned} f(x) &= \frac{1}{2}, \quad 0 < x < 2 \\ &= 0, \quad \textit{otherwise.} \end{aligned}$$

Find P (X < 1.5), P (X > 1) (4)

(ii) If y is a differentiable function of u and u is differentiable function of x , then Prove that $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ Hence, find $\frac{dy}{dx}$, if $y = Sin(x^2 + 5)$ (4)

(iii) An aeroplane at an altitude of 1 km is flying horizontally at 800 km/hr, passes directly over an observer. Find the rate at which it is approaching the observer when it is 1250 meters away from him. (4)