Guru Aanklan

GRAND TEST	SSC EXAMINATION ALGEBRA (SET-A)	SOLUTION

Q. 1 **[1M each] [5M]**

Ans.1 $S_{10} = 200,\ S_9 = 180.$

$t_{10} = S_{10} - S_9$ .... **[1/2M]**

$= 200 - 180$

$= 20$ .... **[1/2M]**

Ans.2 $x^2 + 5x - 2 = 0$

Compare with $ax^2 + bx + c = 0$

$\therefore a = 1, b = 5, c = -2$

$\therefore \alpha + \beta = \frac{-b}{a}$ .... **[1/2M]**

$= -5$ .... **[1/2M]**

Ans.3 $m^2 - 64 = 0$

$\therefore\ (m-8)(m+8) = 0$ .... **[1/2M]**

$\therefore\ m - 8 = 0$ or $m + 8 = 0$

$\therefore\ m = 8$ or $m = -8$ .... **[1/2M]**

$\therefore$ Solution set is $\{8, -8\}$

Ans.4 $2x + 3y = 5$ Add .... **[1/2M]**

$+3x + 2y = 10$

$5x + 5y = 15$

$\therefore\ x + y = 3.$ (Divide by 5) .... **[1/2M]**

Ans.5 $S = \{37, 39, 73, 79, 93, 97\}$ .... **[1M]**

$n(S) = 6$

Ans.6 $\bar{d} = \dfrac{\sum fidi}{\sum fi}$

$\therefore\ 2.18 = \dfrac{\sum fidi}{50}$ .... **[1/2M]**

$\therefore\ \sum fidi = 2.18 \times 50$

$= 109$ .... **[1/2M]**

Q. 2 **[2M each]** **[8M]**

Ans.1 $\begin{vmatrix} m & 2 \\ -5 & 7 \end{vmatrix} = 31$

$\therefore\ m \times 7 - (2) \times (-5) = 31$ .... **[1/2M]**

$7m + 10 = 31$ .... **[1/2M]**

$\therefore\ 7m = 31 - 10$

$\therefore\ 7m = 21$ .... **[1/2M]**

$\therefore\ m = 3$ .... **[1/2M]**

Ans.2 $\text{Mode} = L + \left(\dfrac{f_m - f_1}{2f_m - f_1 - f_2}\right) \times 4$ .... **[1/2M]**

$= 20 + \left(\dfrac{15 - 12}{2 \times 15 - 12 - 8}\right) \times 10$ .... **[1/2M]**

$= 20 + \left(\dfrac{3}{30 - 12 - 8}\right) \times 10$

$= 20 + \left(\dfrac{3}{10}\right) \times 10$ .... **[1/2M**

$= 20 + 3$

$= 23$ .... **[1/2M]**

Ans.3 $a + 2a + 3a + a = 70$ .... **[1/2M]**

$\therefore\ 7a = 70$

$\therefore\ a = \dfrac{70}{7}$

$a = 10$ .... **[1/2M]**

$\therefore$ class	20 – 30	30 – 40	40 – 50	50 – 60
frequency	10	20	30	10

.... **[1M]**

Ans.4 $x = 4$ is the solution of Q.E.

$\therefore$ It satisfy the equation .... **[1/2M]**

$\therefore$ Substitute in equation

$\therefore x^2 - 7x + K = 0$

$\therefore (4)^2 - 7 \times 4 + K = 0$ .... **[1/2M]**

$\therefore -16 - 28 + K = 0$

$\therefore -12 + K = 0$ .... **[1/2M]**

$\therefore K = 12$

$\therefore$ value of k is 12 .... **[1/2M]**

Ans.5 $S_{55} = 3300$

find t 28

$S_n = \frac{n}{2}[2a + (n-1)d]$ .... **[1/2M]**

$\therefore S_{55} = \frac{55}{2}[2a + (55-1)d]$

$\therefore 3300 = \frac{55}{2}[2a + 54d]$

$= \frac{55}{2} \times 2\,[a + 27d]$

$\therefore a + 27d = \frac{3300}{55}$ .... **[1/2M]**

$= 60$

But $t_n = a + (n-1)\,d$ .... **[1/2M]**

$\therefore t_{28} = a + 27d$

$\therefore t_{28} = 60$ .... **[1/2M]**

Ans.6 A coin is tossed, sample space is

$S = \{H, T\}$

$n\,(S) = 2$ .... **[1/2M]**

A is event of getting no head

$\therefore A = \{T\}$

$n\,(A) = 1$ .... **[1/2M]**

$\therefore P(A) = \frac{n(A)}{n(S)}$ .... [1/2M]

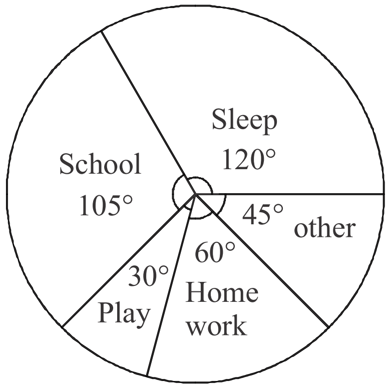
$= \frac{1}{2}$ .... [1/2M]

Q. 3 **[3M each]** **[9M]**

Ans.1 central angle $(\theta) = \frac{\text{Value of component}}{\text{Total value of component}} \times 360^\circ$ .... [1/2M]

Activity	No. of hours	θ
Sleep	8	$\frac{8}{24} \times 360 = 120^\circ$
School	7	$\frac{7}{24} \times 360^\circ = 105^\circ$
Play	2	$\frac{2}{24} \times 360^\circ = 30^\circ$
Homework	4	$\frac{4}{24} \times 360^\circ = 60^\circ$
other	3	$\frac{3}{24} \times 360 = 45^\circ$

[1M]



[1 ½ M]

pie diagram showing number of hours for different activities.

Ans.2 $4p^2 + 7 = 12p$

Divide throughout by 4

$\therefore \; p^2 + \frac{7}{4} = 3p.$

$\therefore \; p^2 - 3p = \frac{-7}{4}.$ (1)

Third Term $= \left[\frac{1}{2} \times \text{coefficient of } p\right]^2$ .... [1/2M]

$= \left[\frac{1}{2} \times -3\right]^2$

$= \left(\frac{-3}{2}\right)^2$ .... [1/2M]

$= \frac{9}{4}$

Add $\frac{9}{4}$ on both sides of equation (1)

$\therefore \; p^2 - 3p + \frac{9}{4} = \frac{-7}{4} + \frac{9}{4}$

$\therefore \; \left(p - \frac{3}{2}\right)^2 = \frac{2}{4}$ .... [1/2M]

Taking square root on bothsides

$$p-\frac{3}{2}=\frac{+\sqrt{2}}{2}$$

$\therefore\ p-\frac{3}{2}=\frac{+\sqrt{2}}{2}$ or $p-\frac{3}{2}=\frac{-\sqrt{2}}{2}$ .... **[1/2M]**

$\therefore\ p=\frac{\sqrt{2}}{2}+\frac{3}{2}$ or $p=\frac{-\sqrt{2}}{2}+\frac{3}{2}$

$\therefore p=\frac{3+\sqrt{2}}{2}$ or $p=\frac{3-\sqrt{2}}{2}$ .... **[1/2M]**

$\therefore$ Solutions of given equation are $\frac{3+\sqrt{2}}{2}$ and $\frac{3-\sqrt{2}}{2}$

OR Solution set is $\left\{\frac{3+\sqrt{2}}{2},\frac{3-\sqrt{2}}{2}\right\}$ .... **[1/2M]**

Ans.3 A committee of two is to be formed from three girls and two boys.

Let three girls be G_1, G_2, G_3 and two boys be B_1, B_2

$\therefore$ Sample space = $S=\{G_1G_2,\ G_1G_3, G_2G_3,\ G_1B_1,\ G_1B_2,\ G_2B_1,\ G_2B_2,\ G_3B_1,\ G_3B_2,\ B_1B_2\}$

$n\,(S)=10$ .... **[1/2M]**

(i) A is event atleast one girls

$\therefore\ A=\{G_1G_2,\ G_1G_3, G_2G_3,\ G_1B_1,\ G_1B_2,\ G_2B_1,\ G_2B_2,\ G_3B_1,\ G_3B_2\}$ .... **[1/2M]**

$n\,(A)=9$

$$\therefore\ P(A)=\frac{n\,(A)}{n\,(S)}$$

$=\frac{9}{10}$ .... **[1/2M]**

(ii) only Boys be the event B

$\therefore\ B=\{B_1B_2\}$

$n\,(B)=1$ .... **[1/2M]**

$$\therefore\ P(B)=\frac{n\,(B)}{n\,(S)}$$

$=\frac{1}{10}$ .... **[1/2M]**

$\therefore\ P(A)=\frac{9}{10};\ P(B)=\frac{1}{10}\left[(\text{cut}\ \frac{1}{2}\ \text{if not written})\right]$ .... **[1/2M]**

Ans.4

Diameter (in mm)	Class mark = $\frac{UB+LB}{2}$ (x_i)	No. of screws (f_i)	$d_i = x_i - A$	$f_i d_i$
33-35	34	10	−6	−60
36-38	37	19	−3	−57
39-41	40	23	0	0
42-44	43	21	3	63
45-47	46	27	6	162
Total		100		108

Let Assumed Mean $= A = 40$

$$\bar{d} = \frac{\sum f_i d_i}{\sum f_i}$$

$$= \frac{108}{100}$$

$$= 1.08$$

$\therefore \quad \bar{x} = A + \bar{d}$

$$= 40 + 1.08$$

$$= 41.08$$

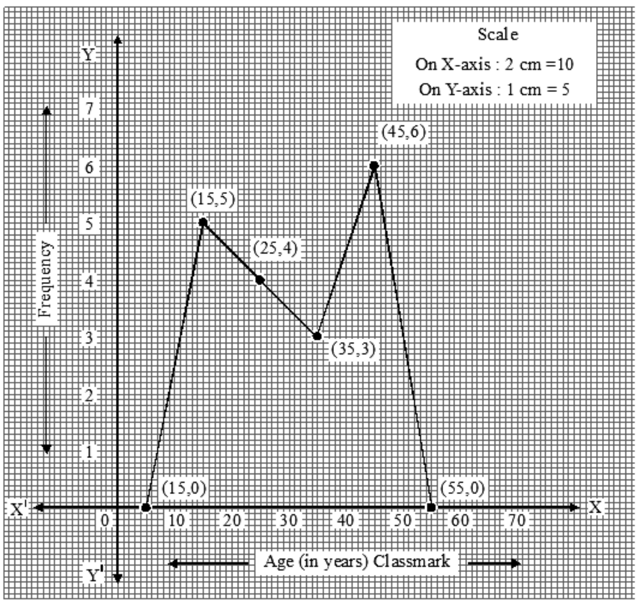
$\therefore$ Mean diameter of screws is 41.08 mm

Ans.5 The data given is cumulative frequency

$\therefore$ Prepare the table.

Age (in years)	frequency	classmark	(x_i, f_i)
$0-10$	0	5	$(5,0)$
$10-20$	5	15	$(15^{\circ},5)$
$20-30$	$9-5=4$	25	$(25,4)$
$30-40$	$12-9=3$	35	$(35,3)$
$40-50$	$18-12=6$	45	$(45,6)$
$50-60$	0	55	$(55,0)$

[1 1/2 M]



....................[1/2M]

Q. 4 **(4M each)** **[8M]**

Ans.1 Let probability of C winning race $= P(C) = x$

$\therefore$ Probability of Bwinning race $= P(B) = 2 \times P(c)$

$= 2x$

$\therefore$ Probability of A winning race $= P(A) = 2P(B)$

$= 2 \times 2x$

$= 4x$ .... **[1/2M]**

But events are mutualy exclasive .... **[1/2M]**

$\therefore \quad P(A) + P(B) + P(C) = 1$ .... **[1/2M]**

$\therefore \quad 4x + 2x + x = 1$

$\therefore \quad 7x = 1$

$\therefore \quad x = \dfrac{1}{7}$ .... **[1/2M]**

$\therefore$ Probability of C winning race is $= \dfrac{1}{7}$ .... **[1/2M]**

Probability of B winning race $= P(B)$

$= 2 \times x$

$= \dfrac{2}{7}$ .... **[1/2M]**

Probality of A winning race $= P(A)$

$= 4x$

$= \dfrac{4}{7}$ .... **[1/2M]**

$\therefore \quad P(A) = \dfrac{4}{7}; \; P(B) = \dfrac{2}{7}; \; P(C) = \dfrac{1}{7}$ .... **[1/2M]**

Ans.2 $\dfrac{14}{x+y} + \dfrac{3}{x-y} = 5 \; ; \; \dfrac{21}{x+y} - \dfrac{2}{x-y} = 1$

Let $\dfrac{1}{x+y} = m$ and $\dfrac{1}{x-y} = n$

$\therefore \quad 14m + 3n = 5 \qquad (1)$

$21m - 2n = 1 \qquad (2)$ .... **[1/2M]**

Multiply equation (1) by 2 and (2) by 3

$\therefore \quad 28m + 6n = 10 \qquad (3)$
$\quad\ \ 63m - 6n = 3 \qquad (4)$ } Add
$\quad\ \ 91m \quad = 13$ .... [1/2M]

$\therefore \quad m = \frac{13}{91}$

$\therefore \quad m = \frac{1}{7}$ .... [1/2M]

Substitute $m = \frac{1}{7}$ in equation (1)

$\therefore \quad 14 \times \frac{1}{7} + 3n = 5$

$\therefore \quad 3n = 5 - 2$

$\therefore \quad n = \frac{3}{3}$

$\therefore \quad n = 1$ .... [1/2M]

Resubstitute $m = \frac{1}{x+y}$ and $n = \frac{1}{x-y}$

$\therefore \quad \frac{1}{7} = \frac{1}{x+y}$ and $1 = \frac{1}{x-y}$

$\therefore \quad x + y = 7 \quad (5)$ and $x - y = 1 \quad (6)$ .... [1/2M]

Add both equations^n (5) and (6)

$x + y = 7$
$x - y = 1$
$2x = 8$

$\therefore \quad x = 4$ .... [1/2M]

Substitude in equation (5)

$4 + y = 7$

$\therefore \ y = 7 - 4$ .... [1/2M]

$y = 3$

$\therefore$ Value of x is 4 and y is 3. .... [1/2M]

Ans.3 The first 11 positive numbers which are multiple of 6 are

$6, 12, 18, --- 66$ .... [1/2M]

They are in A.P. with

first term $= a = 6$

common difference $d = t_2 - t_1$ .... [1/2M]

$= 12 - 6$

$= 6$ .... [1/2M]

and Last term $= t_n = 66$

$\therefore \quad t_n = a + (n-1)\, d$

$\therefore \quad 66 = 6 + (n-1) \times 6$

$\therefore \quad 66 = 6 + 6n - 6$

$\therefore \quad 66 = 6n$ .... [1/2M]

$\therefore \quad n = 11$

$S_n = \frac{n}{2}[t_1 + t_n]$ .... [1/2M]

$\therefore \quad S_{11} = \frac{11}{2}[6 + 66]$

$= \frac{11}{2}[72]$ .... [1/2M]

$= 11 \times 36$

$= 396$ .... [1/2M]

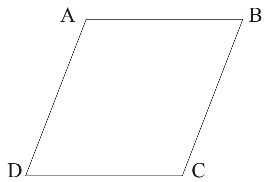
Sum of first 11 positive numbers multiple of 6 is 396 .... [1/2M]

Q.5 **(5M each)** **[10M]**

Ans.1 Let the four angles of quadrilaterals which are in A.P. be

$a-3d,\, a-d,\, a+d,\, a+3d$ .... [1/2M]

Sum of all the angles of quadrilateral is 360°



$\therefore \; a - 3d + a - d + a + d + a + 3d = 360^\circ$ .... [1/2M]

$\therefore \; 4a = 360$

$\therefore \quad a = \frac{360}{4}$

$a = 90^\circ \qquad (1)$ .... [1/2M]

Now, According to condition.

The greatest angle is $a + 3d$ and least is $a - 3d$

$\therefore \quad a + 3d = 2\,(a - 3d)$ .... [1/2M]

$\therefore \quad a + 3d = 2a - 6d$

$\therefore \quad 3d + 6d = 2a - a$

$\therefore \quad 9d = a$

$\therefore \quad 9d = 90 \quad \text{(from 1)}$

$\therefore \quad d = 10° \quad (2)$ .... **[1/2M]**

$\therefore$ The angles will be

$a - 3d = 90 - 3 \times 10$

$= 90° - 30°$

$= 60°$ .... **[1/2M]**

$a - d = 90 - 10$

$= 80°$ .... **[1/2M]**

$a + d = 90 + 10$

$= 100°$ .... **[1/2M]**

$a + 3d = 90 + 3 \times 10$

$= 90 + 30$

$= 120°$ .... **[1/2M]**

$\therefore$ The four angles of quadrilateral which are in A.P. are $60°, 80°, 100°, 120°$ .... **[1/2M]**

Ans.2 Let the four consecutive natural numbers which are multiples of 5 are

$x, x+5, x+10, x+15$ .... **[1/2M]**

According to condition

$x\,(x+5)\,(x+10)\,(x+15) = 15000$ .... **[1/2M]**

$\therefore \quad x\,(x+15)\,(x+5)\,(x+10) = 15000$

$\therefore \quad (x^2 + 15x)\,(x^2 + 15x + 50) = 15000$ .... **[1/2M]**

Substitute $x^2 + 5x = m$

We get $m\,(m + 50) = 15000$ .... **[1/2M]**

$\therefore \quad m^2 + 50m - 15000 = 0.$

$\therefore \quad m^2 + 150m - 100m - 15000 = 0$ .... **[1/2M]**

$\therefore \quad m\,(m + 150) - 100\,(m + 150) = 0$

$\therefore \quad (m + 150)\,(m - 100) = 0$ .... **[1/2M]**

$\therefore \quad m + 150 = 0 \quad or \quad m - 100 = 0$

$\therefore \quad m = -150 \quad or \quad m = 100$ .... **[1/2M]**

But it is natural number $\quad \therefore \quad m = 100$

Resubstitute $m = x^2 + 15x$

$x^2 + 15x = 100$ .... **[1/2M]**

$\therefore \quad x^2 + 15x - 100 = 0$

$\therefore \quad x^2+20x-5x-100=0$

$\therefore \quad x(x+20)-5(x+20)=0$

$(x+20)(x-5)=0$

$\therefore \quad x+20=0$ or $x-5=0$

$\therefore \quad x=-20$ or $x=5$ .... **[1/2M]**

But $x=-20$ not acceptable because number is natural number

$\therefore \quad x=5$

$\therefore$ The numbers are $5, x+5=10, x+10=15$ and $x+15=20$ .... **[1/2M]**

Ans.3 Let the persent age of father be $'x'$ years and that of son be $'y'$ years .... **[1/2M]**

After $(x-y)$ years sons age will be $'x'$ years i.e. he will be as old as his father...................... **[1/2M]**

After $(x-y)$ years father's age will be

$x+(x-y)='2x-y$ years. .... **[1/2M]**

According to first condition

$x+(x-y)+x=126$ .... **[1/2M]**

$\therefore \quad 3x-y=126 \qquad (1)$

$(x-y)$ years ago fathers age was $'y'$ years the father was as old as son today

$(x-y)$ years ago son's age was $y-(x-y)$

$=(2y-x)$ years .... **[1/2M]**

According to second condition.

$y+(2y-x)=38$

$\therefore \quad 3y-x=38 \qquad (2)$ .... **[1/2M]**

Multiply equation (2) by 3 and add to (1)

$\therefore \quad 3x-y=126$

$-\ 3x+9y=114$

$8y=240$

$\therefore \quad y=\frac{240}{8}$ .... **[1/2M]**

$=30$ .... **[1/2M]**

Substitute $y=30$ in equation (1)

$\therefore \quad 3x-30=126$

$\therefore \quad 3x=156$

$\therefore \quad x=\frac{156}{3}$

$=52$ .... **[1/2M]**

$\therefore$ The present age of father is 52 years and son is 30 years. .... **[1/2M]**